Desk Copy

ECE 300
Spring Semester, 2008
HW Set #9

Due: March 13, 2008

Name wlg

wlg
Version 2

Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers. Each problem 20 points.**

From the text.

(8.5) (a) Ans: $I(0^+) = 0$ A, $v(0^+) = 0$ V

(b) Ans: $\dfrac{di(0^+)}{dt} = 4A/s$, $\dfrac{dv(0^+)}{dt} = 0V/s$

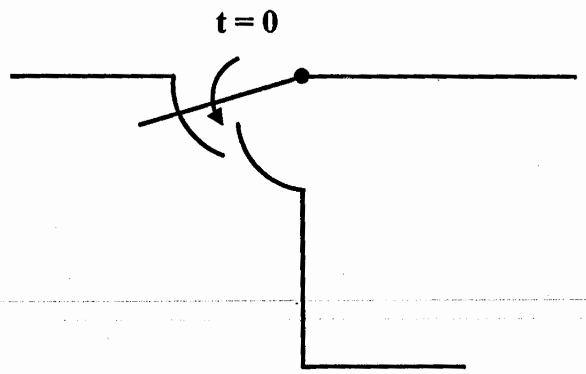
(c) Ans: $i(\infty) = 2.4A$, $v(\infty) = 9.6V$

(8.17) Ans: $v(t) = \left[64.65e^{-2.68t} - 4.64e^{-37.3t}\right]u(t)$ V

(8.21) Ans: $v(t) = ((18e^{-t} - 2e^{-9t}))u(t)$ V

(8.25) Work only for $v_o(t)$. The switch in this problem is a make before break switch as shown below.

Ans: $v_0(t) = e^{-0.25t}\left[24\cos 1.98t + 3.024\sin 1.98t\right]u(t)$ V



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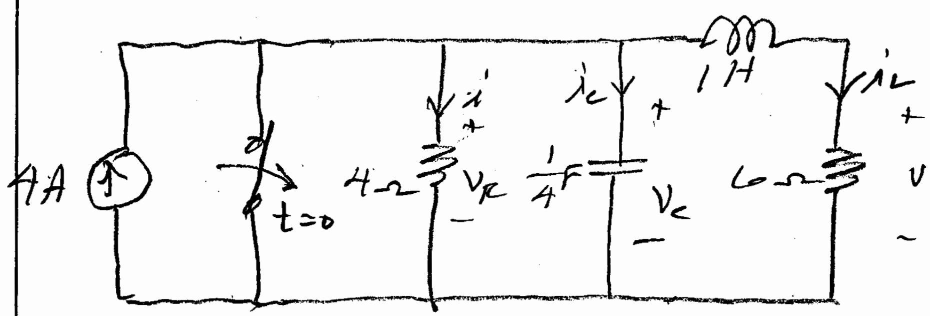
ECE 300
H.W. #9

8.5 For the circuit below, determine



(a) $i(0^+)$ and $V(0^+)$

For $t = 0^-$, $V(0^-) = 0$, $i_L(0^-) = 0$

$i(0^-) = 0$; $V_C(0^-) = 0$

For $t = 0^+$

$\underline{V(0^+) = 0}$ because $i_L(0^+) = i_L(0^-) = 0$

and $V = i_L 6 = 0$

Since $V_C(0^+) = V_C(0^-)$

the voltage across the resistor (4Ω)

is zero. $\therefore i = 0$

$\underline{V(0^+) = 0}$; $\underline{i(0^+) = 0}$

(b) $\dfrac{di(0^+)}{dt}$, and $\dfrac{dV(0^+)}{dt}$

$$i(t) = \frac{V_R(t)}{R}$$

$$\frac{di}{dt} = \frac{1}{R}\frac{dV_R}{dt}$$

8.5 cont.

since $V_R(t) = V_c(t)$

$$\frac{dV_R(t)}{dt} = \frac{dV_c}{dt}$$

so

$$\frac{di}{dt} = \frac{1}{R}\frac{dV_c}{dt}$$

since

$$i_c = C\frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = \frac{i_c}{C}$$

so

$$\frac{dV_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{4}{.25} = 16$$

the

$$\frac{di(0^+)}{dt} = \frac{1}{R}\frac{dV_c(0^+)}{dt} = \frac{1}{4}\times 16 = 4A/s$$

$$\boxed{\frac{di(0^+)}{dt} = 4\ A/s}$$

next, for $\frac{dV(0^+)}{dt}$

Now, $V(t) = 6\,i_L(t)$.

$$\frac{dV}{dt} = 6\frac{di_L}{dt} \qquad (A)$$

8.5 cont

Now $V_L = L \frac{di_L}{dt}$

or

$$\frac{di_L}{dt} = \frac{V_L}{L}$$

we have from (A)

$$\frac{dV(t)}{dt} = 6\,\frac{V_L(t)}{L}$$

and

$$\frac{dV(0^+)}{dt} = \frac{6}{L}\,V_L(0^+)$$

since $V_C(t) = V_L(t) + V(t)$

$V_L(t) = V_C(t) - V(t)$

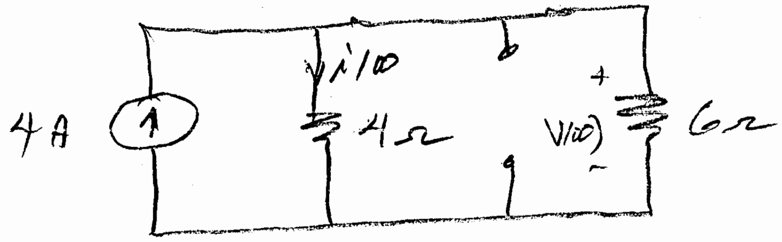
$V_L(0^+) = V_C(0^+) - V(0^+)$

but $V_C(0^+) = 0$ and $V(0^+) = 0$

so $V_L(0^+) = 0$

$$\therefore \boxed{\frac{dV(0^+)}{dt} = \frac{6}{L}\,V_L(0^+) = 0}$$

(c) As $t \to \infty$



8.5 cont.

The inductor looks like a short circuit.

The capacitor looks like an open circuit.

From the circuit

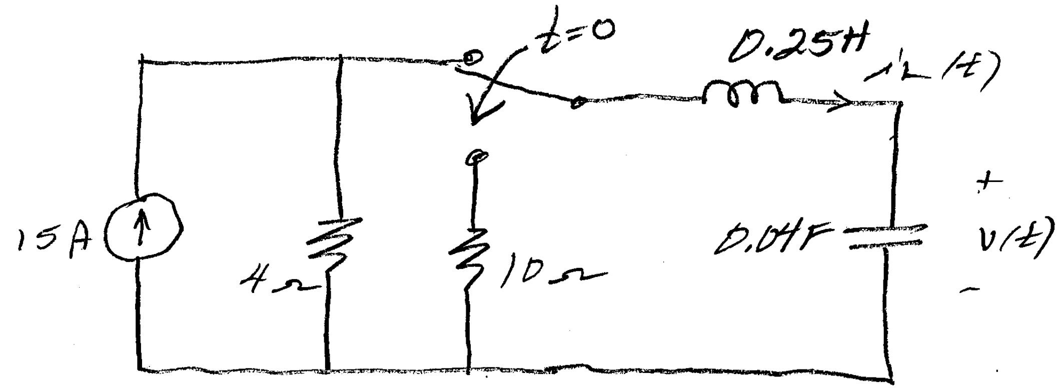
$$i(\infty) = \frac{4 \times 6}{6+4} = \frac{24}{10} = 2.4A$$

$$\boxed{i(\infty) = 2.4\ A}$$

$$V(\infty) = \left(\frac{4 \times 4}{10}\right) \times 6 = 9.6V$$

$$\boxed{V(\infty) = 9.6\ V}$$

8.17 Switch in the following circuit moves instantly from A to B at $t=0$.
Find $v(t)$ for $t \geq 0$.

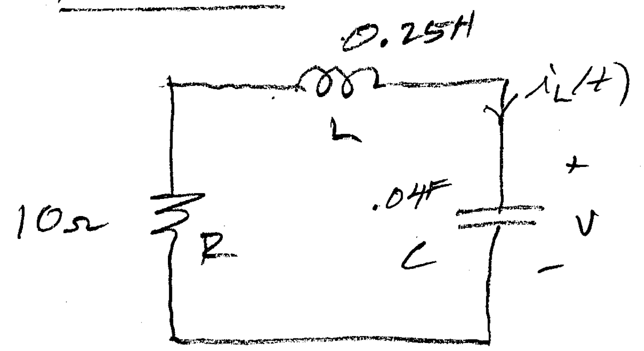


For $t<0$, the current through the inductor is zero, $i_L(0^-)=0$.
$V(0^-)$ must be the 15A through the 10Ω resistor, or

$$V(0^-) = 15 \times 4 = 60V$$

$$V(0^+) = 60V$$

FOR $t > 0$



$$R i_L(t) + L \frac{d i_L}{dt} + V(t) = 0 \qquad (1)$$

8.17 cont

but $i_L = C\frac{dv}{dt}$ (2)

substitute (2) into (1)

$$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v(t) = 0$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = 0$$

with numbers

$R = 10\,\Omega$, $L = 0.25H$, $C = .04F$

$$\frac{d^2v}{dt^2} + 40\frac{dv}{dt} + 100\,v(t) = 0$$

Char. Eq.

$$s^2 + 40s + 10 = 0$$

$$(s+37.32)(s+2.68) = 0$$

$$v(t) = \left(A_1 e^{-37.32t} + A_2 e^{-2.68t}\right)u(t)\ V \quad (3)$$

$$\boxed{v(0^+) = v(0^-) = 60V} \quad (4)$$

From Eq (2)

$$\frac{dv(0^+)}{dt} = \frac{i_L(0^+)}{C}$$

$$i_L(0^+) = i_L(0^-) = 0$$

$$\boxed{\frac{dv(0^+)}{dt} = 0} \quad (5)$$

using (4) in (3) gives

$$\boxed{60 = A_1 + A_2}$$

using (5) in $\frac{dv}{dt}$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = \left[-37.32A_1 e^{-37.32t} - 2.68e^{-2.68t}\right]\Big|_{t=0^+}$$

$$\boxed{0 = -37.32A_1 - 2.68A_2}$$

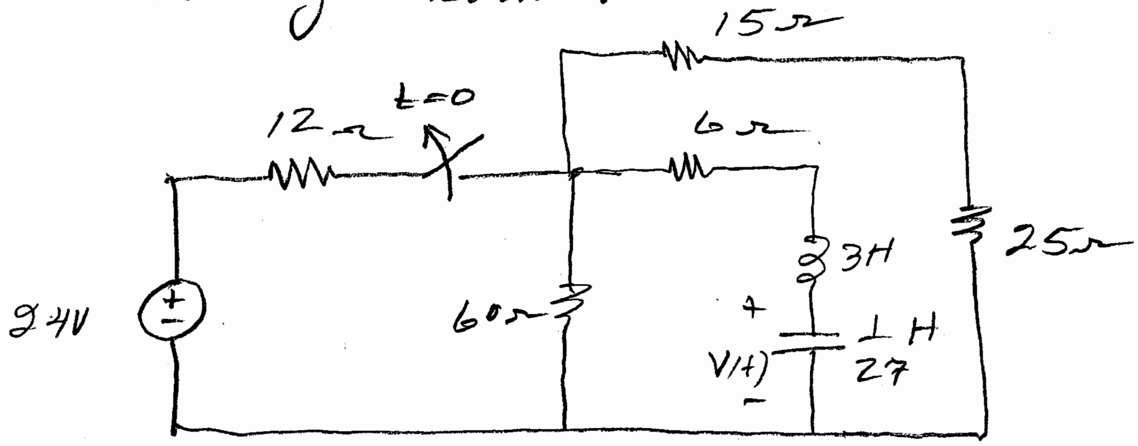
$$\begin{bmatrix} 1 & 1 \\ -37.32 & 2.68 \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \end{bmatrix}$$

$A_1 = -4.64, \quad A_2 = 64.64$

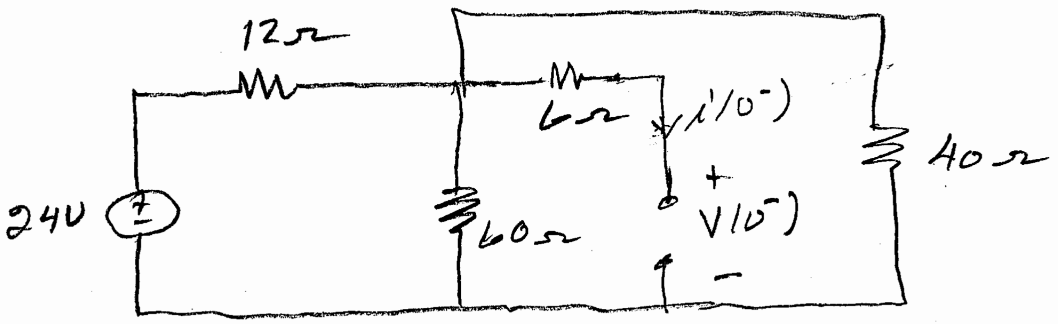
$$\therefore \boxed{v(t) = \left[-4.64e^{-37.32t} + 64.64e^{-2.68t}\right]u(t) \text{ V}}$$

8.21

Calculate $v(t)$ for $t>0$ in the following circuit.



For $t<0$, steady state



$$60 \parallel 40 = \frac{60 \times 40}{60+40} = 24\Omega$$

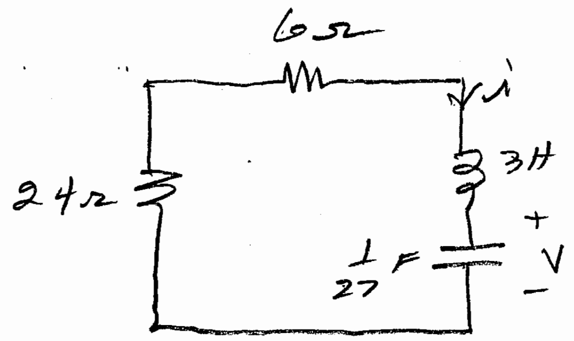
so

$$v(0^-) = \frac{24 \times 24}{12+24} = 16\text{ V}$$

$$i(0^-) = 0\text{ A}$$

8.21 cont

For $t > 0$



The equation for the above is

$$\frac{d^2 v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = 0$$

where $R = 30\Omega$, $L = 3H$, $C = \frac{1}{27}$ F

$$\frac{d^2 v}{dt^2} + 10\frac{dv}{dt} + 9\,v(t) = 0$$

$$s^2 + 10s + 1 = 0$$

$$(s+9)(s+1) = 0 \qquad \text{overdamped}$$

$$v(t) = (A_1 e^{-9t} + A_2 e^{-t})\,u(t) \quad V \qquad (1)$$

$$\boxed{v(0^+) = v(0^-) = 16\,V} \qquad (2)$$

$$i(0^+) = C\,\frac{dv(0^+)}{dt}$$

$$i(0^+) = i(0^-) = 0$$

$$\boxed{\frac{dv(0^+)}{dt} = 0} \qquad (3)$$

8.21 cont

using (2) in (1) gives

$\boxed{A_1 + A_2 = 16}$

using (3) in (1) gives

$\boxed{-9A_1 - A_2 = 0}$

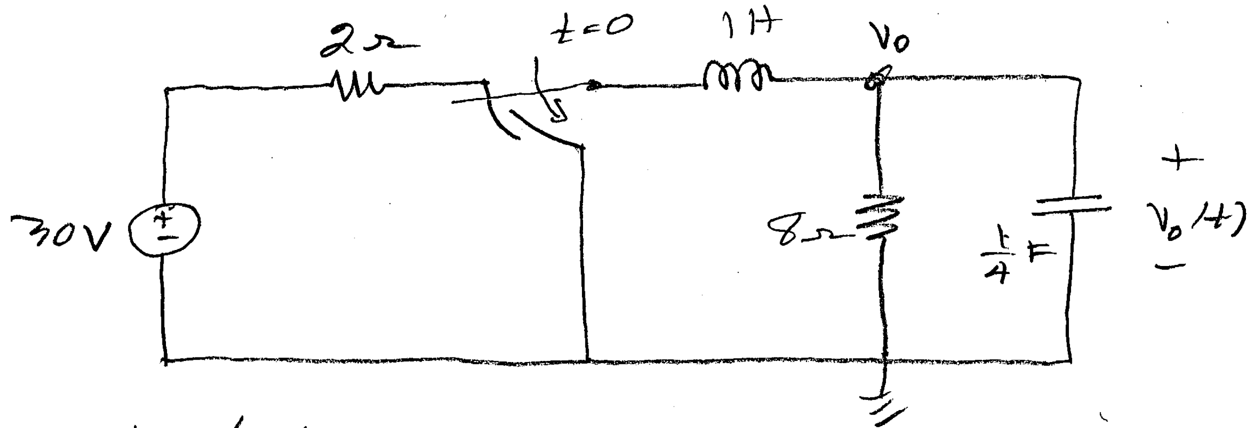
$$\begin{bmatrix} 1 & 1 \\ 9 & 1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \end{bmatrix}$$

$$A_1 = -2, \quad A_2 = 18$$

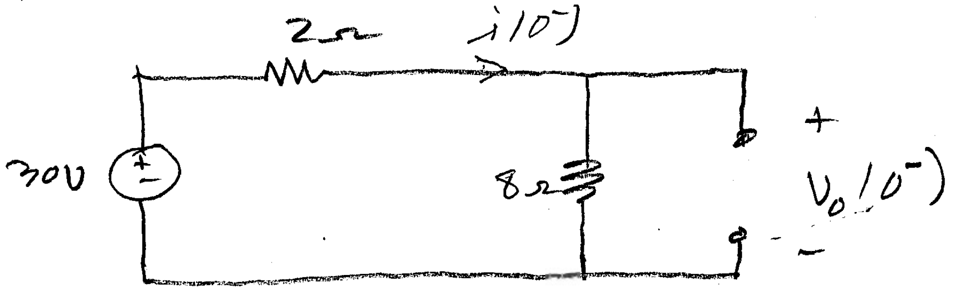
$$\therefore \boxed{v(t) = [-2e^{-9t} + 18e^{-t}]u(t) \quad V}$$

8.25

For the following circuit, determine $V_o(t)$ for $t \geq 0$.



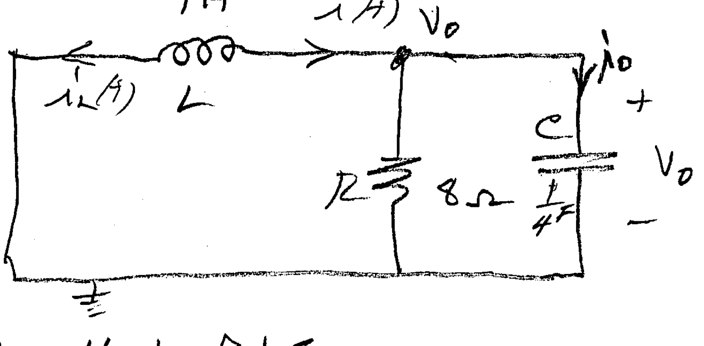
For $t < 0$



$$V_o(0^-) = \frac{30 \times 8}{8+2} = 24V = V_o(0^+) \quad (1)$$

$$i(0^-) = \frac{30}{10} = 3A = i(0^+) = -i_L(0^+) \quad (2)$$

For $t > 0$



Parallel RLC

8.25

The d.e. for the parallel RLC is

$$\frac{d^2 v_o}{dt^2} + \frac{1}{RC}\frac{dv_o}{dt} + \frac{v_o}{LC} = 0 \qquad (3)$$

going to need

$$\frac{v_o}{R} + i_L + C\frac{dv_o}{dt} = 0 \qquad (4)$$

$R = 8\,\Omega$, $L = 1H$, $C = 0.25F$

so

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 0.5s + 4 = 0$$

$$(s+.25+j\,1.98)(s+.25-j\,1.98) = 0$$

$$v_o(t) = e^{-0.25t}\left[B_1\cos 1.98t + B_2\sin 1.98t\right] \qquad (5)$$

$$\boxed{v_o(0^+) = v(0^-) = 24\,V} \quad \text{from (1)*} \qquad (6)$$

$$C\frac{dv_o(0^+)}{dt} = -i_L(0^+) - \frac{v_o(0^+)}{R} \quad \text{from (4)}$$

$$C\frac{dv_o(0^+)}{dt} = 3 - \frac{24}{8} = 0$$

$$\boxed{\frac{dv_o(0^+)}{dt} = 0} \qquad (7)$$

* We also know $i(0^+) = 3 = \frac{v_o(0^+)}{8} = 3$

$\therefore i_o(0^+) = 0 \quad i_o = C\,dv_o/dt \stackrel{\circ}{=} 0$

(8.25) cont

From (5) using (6)

$$24 = B_1 \cos 0 + B_2 \sin 0 = B_1$$

$$B_1 = 24$$

$$v_o(t) = e^{-0.25t}\left[24\cos 1.98t + B_2 \sin 1.98t\right]$$

take $\frac{dv_o}{dt}$

$$\frac{dv_o}{dt} = e^{-0.25t}\left[-1.98 \times 24 \sin 1.98t + 1.98 B_2 \cos 1.98t\right]$$
$$-0.25 e^{-.25t}\left[24\cos 1.98t + B_2 \sin 1.98t\right] \quad (8)$$

Evaluate (8) at $t=0^+$ using $\frac{dv_o(0^+)}{dt} = 0$

$$0 = 1.98 B_2 - 0.25 \times 24$$

$$B_2 = \frac{0.25 \times 24}{1.98} = \frac{6}{1.98} = 3.03$$

$$v_o(t) = e^{-0.25t}\left[24\cos 1.98t + 3.03 \sin 1.98t\right] u(t) \checkmark$$