Desk Copy

ECE 300
Spring Semester, 2008
HW Set #10

Due: March 25, 2008

Name wlg
Print (last, first)

wlg

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers. Each problem 20 points.**

From the text:

(8.37) Ans: $i(t) = 5e^{-4t}\,u(t)$ A — 15 points

(8.49) Ans $i(t) = [3 + (3 + 6t)e^{-2t}]\,u(t)$ A — 15 points

8.51 This problem is modified from what is given in the text. Use these modifications. Total 30 points

Assume $R = 10\ \Omega$, $L = 0.04$ H, $C = 0.01$ F, i_o (source) = 1 A

(a) Develop and solve the differential equation for v(t), the voltage across the capacitor. 15 points
Note the magnitude swing and period of the waveform. Ans: On your own.

(b) Use P-spice to simulate the circuit for t greater than 0. Run the simulation out to t = 0.4 seconds. To get a nice smooth plot use 0.001 step size. This should give about 3 cycles if you have solved the v(t) correctly. Also, note the swing (peak to peak) of v(t) from the simulation plot. You should get –2 to +2. This should agree with your solution from (a). Turn in the P-spice circuit diagram with your name in the caption. Also turn in your plot of v(t). 15 points

ECE 300
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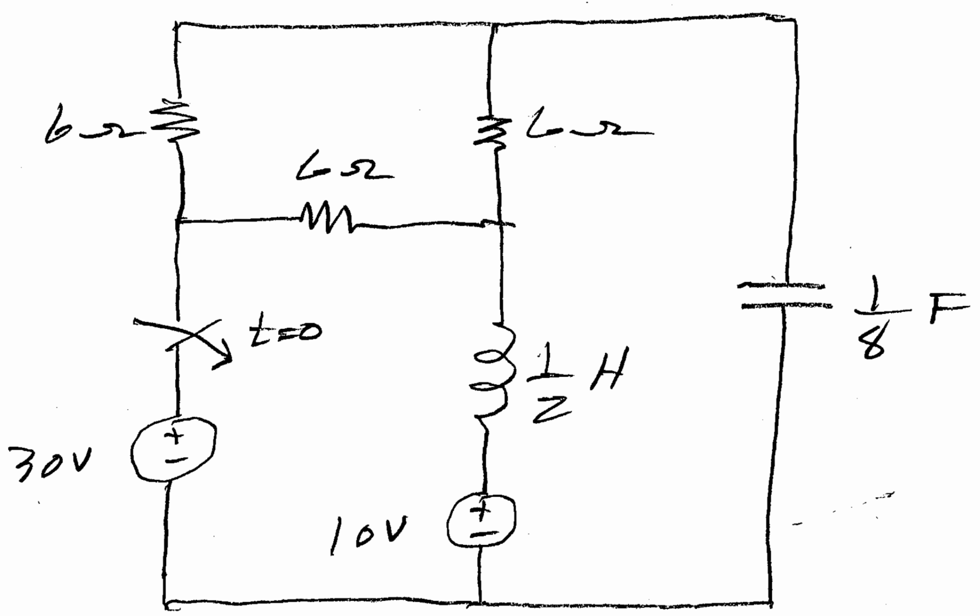
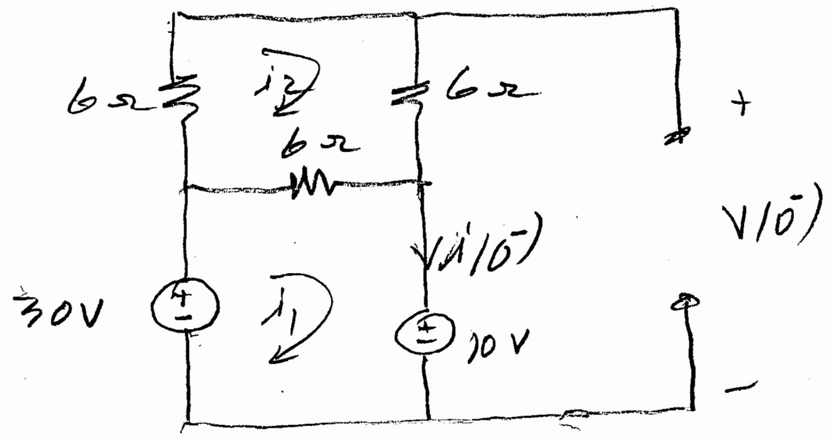
H.W. # 10

(8.37) For the circuit below solve for $i(t)$, $t > 0$.



For $t < 0$ (in steady state)



18.37) cont.

$$6i_1 - 6i_2 = 20$$
$$-6i_1 + 18i_2 = 0$$

Add

$$12i_2 = 20$$

$$\boxed{i_2 = \frac{5}{3} A}$$

$$\boxed{i_1 = 5 A}$$

Now $i(0^-) = i_2(0^-) + i_1(0^-) - i_2(0^-)$

$i(0^-) = 5A$

so $\boxed{i(0^+) = i(0^-) = 5A}$ ok (1)

Also notice that

$V(0^-) = 10 + 6i_2(0^-) = 10 + 6 \times \frac{5}{3} =$

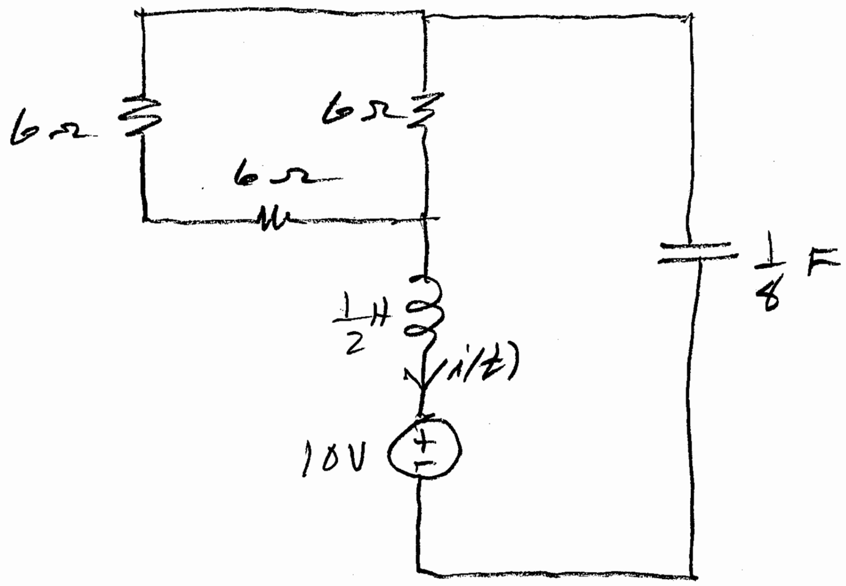
$V(0^-) = 20V$

so

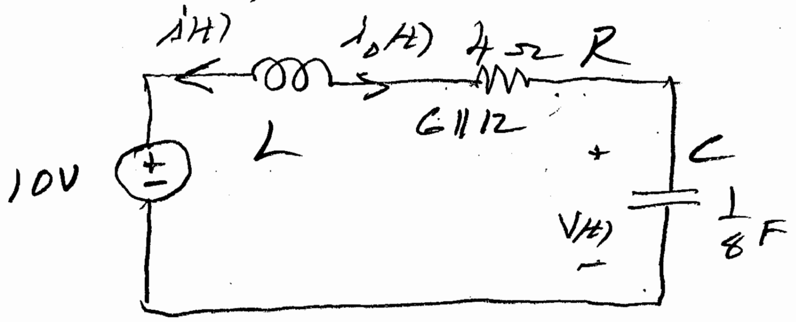
$\boxed{V(0^+) = V(0^-) = 20V}$ ok (2)

Now draw the circuit for $t > 0$

(8.37)



Becomes



$\frac{6 \times 12}{18} = 4\Omega$

series RLC ckt. solve for $i_0(t)$, then $i(t) = -i_0(t)$.

We have

$$R i_0 + L\frac{di_0}{dt} + \frac{1}{C}\int_0^t i_0(t)\,dt + V(0^+) = 10 \qquad \text{Eq (3)}$$

Take $\frac{d}{dt}[\cdot]$ of Eq (3)

(8.37)

$$R\frac{di_0}{dt} + L\frac{d^2 i_0}{dt^2} + \frac{i_0(t)}{C} = 0$$

$$\frac{d^2 i_0}{dt^2} + \frac{R}{L}\frac{di_0}{dt} + \frac{i_0}{LC} = 0$$

with numbers

$$\frac{d^2 i_0}{dt^2} + 8\frac{di_0}{dt} + 16 i_0(t) = 0$$

$\omega_n^2 = 16$ so $\omega_n = 4$

$2\xi\omega_n = 8$ so $\xi = 1$

critically damped

$$i_0(t) = \left[(A_1 + A_2 t)e^{-4t}\right]u(t)\ A$$

$$i_0(0^+) = -5 = A_1$$

$$i_0(t) = (-5 + A_2 t)e^{-4t} \qquad \text{Eq (4)}$$

Go to Eq (3) to solve for $\frac{di_0}{dt}$.

We have

$$4 i_0(0^+) + 0.5\frac{di_0}{dt} + 20 = 10 \qquad \text{Eq (5)}$$

$$i_0(0^+) = -5$$

(8.37) cont

$$0.5\frac{di_o}{dt} = 10 + 20 - 20$$

$$\frac{di_o}{dt} = \frac{10}{.5} = 20$$

Go to Eq (4) and find $\frac{d(\cdot)}{dt}$.

$$\frac{di_o}{dt} = -4(-5 + A_2 t)e^{-4t} + e^{-4t} \times A_2$$

so

$$20 = 20 + A_2$$

$$A_2 = 0$$

$$i_o = -5e^{-4t}u(t) \ A$$

but

$$i(t) = -i_o(t)$$

$\therefore$

$$\boxed{i(t) = 5e^{-4t}u(t) \ A}$$

(8.49)

Determine $i(t)$ for $t>0$ in the following circuit.

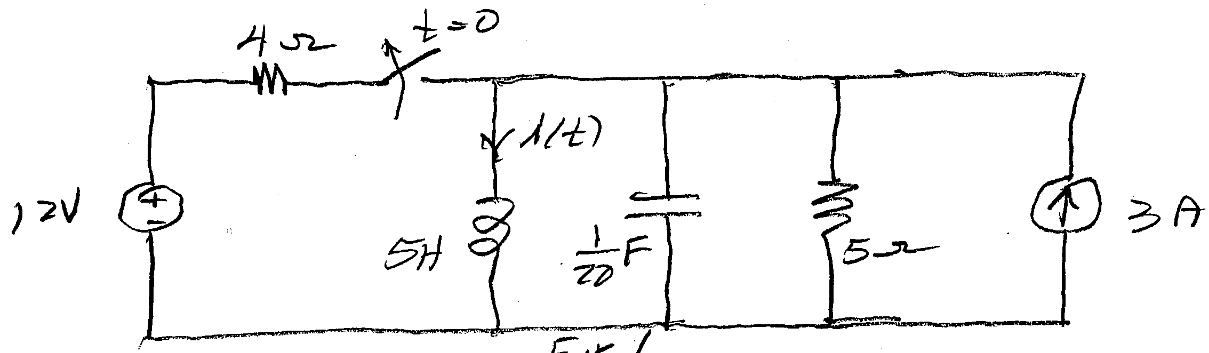


Fig 1

For $t<0$

Assume steady state

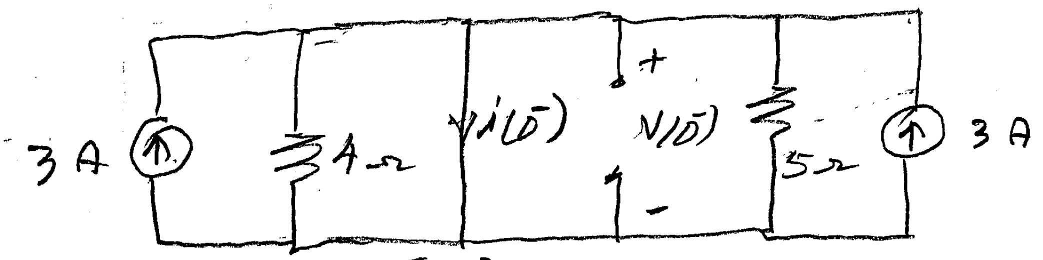


Fig 2

$i(0^-) = 6A$ $\quad v(0^-) = 0$ (by inspection)

Eq (1)

For $t>0$

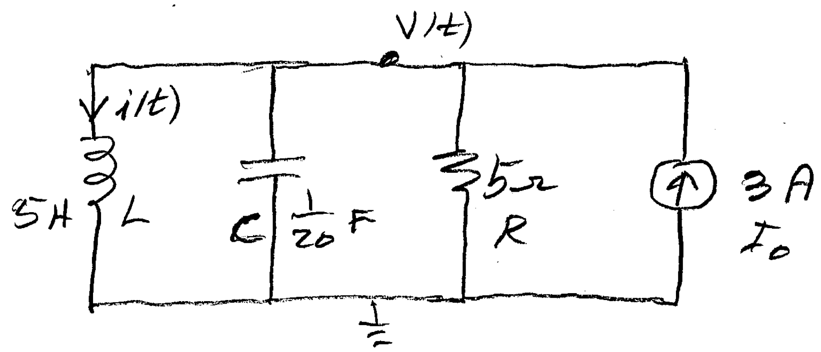


Fig 3

(8.49) cont

By nodal analysis;

$$i(t) + C\frac{dv}{dt} + \frac{V}{R} = I_0 \qquad (2)$$

but $V(t) = L\frac{di}{dt}$ (3)

Using (3) in (2)

$$i(t) + LC\frac{d^2i}{dt^2} + \frac{L}{R}\frac{di}{dt} = I_0 \qquad (4)$$

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i(t)}{LC} = \frac{I_0}{LC} \qquad (5)$$

Putting in numbers

$$\frac{d^2i}{dt^2} + 4\frac{di}{dt} + 4i(t) = 4I_0$$

Char. Eq.

$s^2 + 4s + 4 = 0$

$w_n^2 = 4 \qquad 2\zeta w_n = 4$

$w_n = 2 \ ; \quad \zeta = 1$ (comparing coef.)

critically damped

$i(t) = i_p + i_c$

18.49) cont.

$i_p = K$ substitute into (5) we get

$$K = I_0 = 3 = i_p \quad (6)$$

$$i_c = (A_1 + A_2 t)e^{-2t} \quad (7)$$

combining (6) and (7)

$$i(t) = 3 + (A_1 + A_2 t)e^{-2t} \quad (8)$$

$$i(0^+) = 6 = 3 + A_1$$

$$A_1 = 3$$

$$i(t) = 3 + (3 + A_2 t)e^{-2t} \quad (9)$$

$$\frac{di}{dt} = -2(3 + A_2 t)e^{-2t} + e^{-2t} \times A_2 \quad (10)$$

Now from (3)

$$V = L\frac{di}{dt}$$

$$V(0^+) = 0 \quad (\text{from (1)})$$

$$\frac{di(0^+)}{dt} = 0$$

using (10)

$$0 = -6 + A_2$$

$$\therefore i(t) = \left[3 + (3 + 6t)e^{-2t}\right]u(t) \quad A$$

(8.51)

(a) Find $v(t)$ for $t > 0$ in the following circuit.

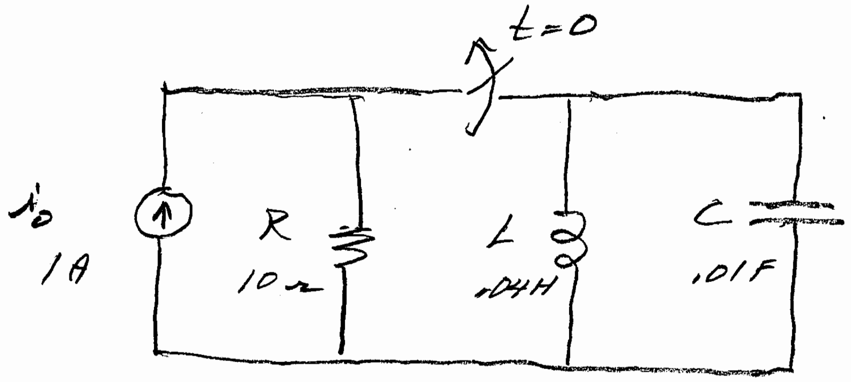


Fig 1:

<u>For $t < 0$</u>

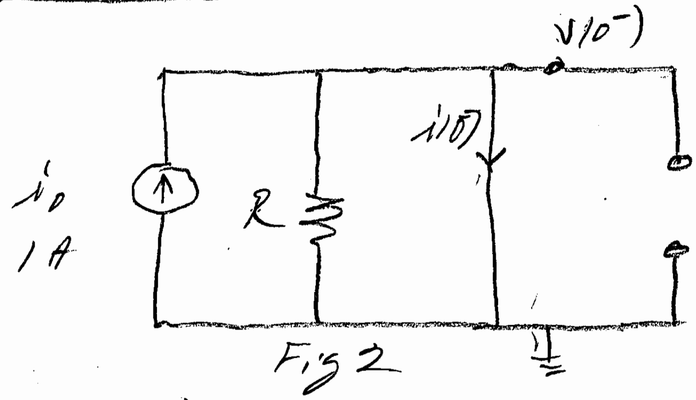


Fig 2

$$v(0^-) = 0 \quad ; \quad i(0^-) = 1A$$

$$\therefore \quad v(0^+) = v(0^-) = 0 \quad ; \quad i(0^+) = i(0^-) = 1A \qquad (1)$$

<u>For $t > 0$</u>

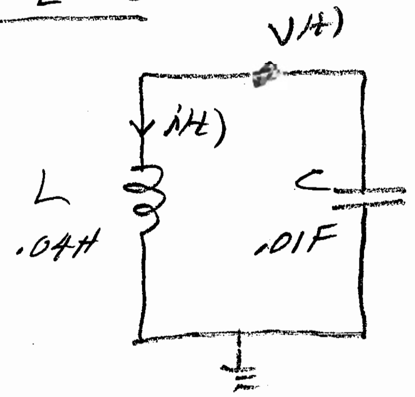


Fig 3

(8.51) using nodal analysis, Fig. 3

$$C\frac{dv}{dt} + \frac{1}{L}\int_0^t v(t)\,dt + i(0^+) = 0 \qquad (2)$$

Taking $\frac{d}{dt}(\cdot)$ of (2)

$$C\frac{d^2v}{dt^2} + \frac{v(t)}{L} = 0$$

$$\frac{d^2v}{dt^2} + \frac{v(t)}{LC} = 0 \qquad \underline{\text{undamped}}$$

For the undamped case, $\xi = 0$

This case $\omega_d = \omega_n = \frac{1}{\sqrt{LC}}$

$$\omega_n = \frac{1}{\sqrt{.04 \times .01}} = 50 = \omega_d$$

Now in general,

$$v(t) = e^{-\xi\omega_n t}\left[A_1\cos\omega_d t + A_2\sin\omega_d t\right] \qquad (3)$$

but $\xi = 0$

$$v(t) = (A_1\cos 50t + A_2\sin 50t) \qquad (4)$$

$v(0^+) = 0$ use this in (4)

$$0 = A_1$$

$\therefore$

$$v(t) = A_2\sin 50t \qquad (5)$$

(8.51) cont. (3)

using equation (2)

$$\frac{dV(0^+)}{dt} = -\frac{i(0^+)}{C} = -\frac{1}{0.01} = -100$$

$$\frac{dV(0^+)}{dt} = -100 \text{ V/s} \qquad (6)$$

Take the derivative of (5) wrt t

$$\frac{dv}{dt} = A_2 \times 50 \cos 50t$$

so

$$-100 = A_2 \times 50 \cos 50t \Big|_{t=0}$$

$$A_2 = \frac{-100}{50} = -2$$

$$\therefore \boxed{V(t) = -2 \sin 50t}$$

(b) To simulate with P-spice

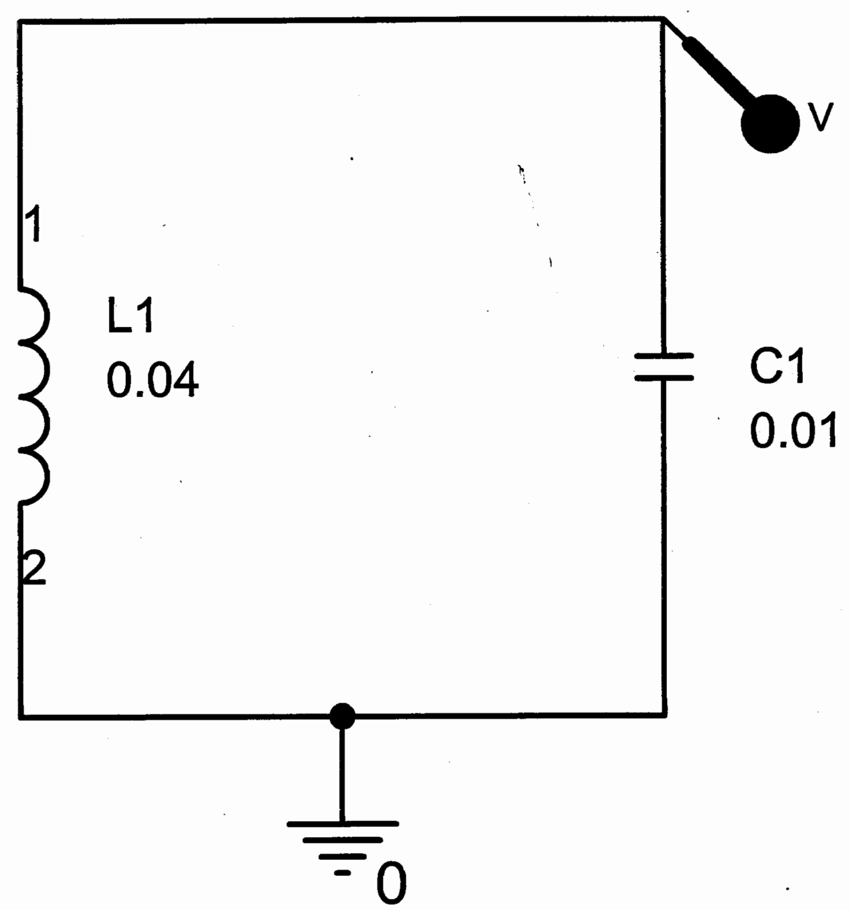
$$\omega = 50 = 2\pi f = \frac{2\pi}{T}$$

$$\text{so } T = \frac{2\pi}{50} = 0.1256 \text{ sec}$$

3 periods = 0.3768

simulate to $t = 0.4$ sec

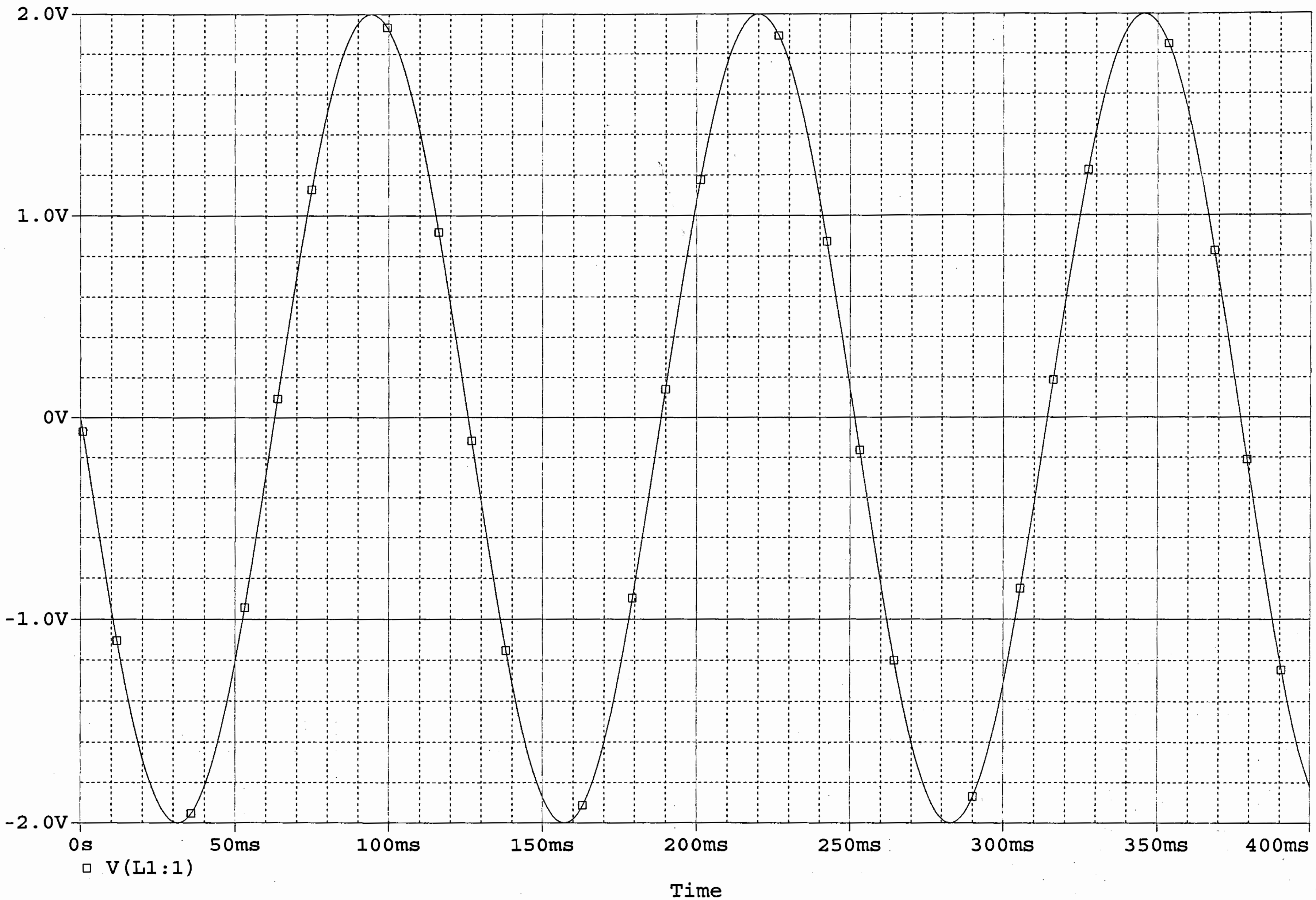
used 0.001 minimum step size
skip initial transient bias point.
Results follow. Agree with analytical work.
QED



Problem 8_51: W. Green

** Profile: "SCHEMATIC1-bias15" [C:\ORCAD\ORCAD_10.0_DEMO\LC8_51-PSpiceFiles\SCHEMATIC1\bias15.sim]
Date/Time run: 03/11/08 Temperature: 27.0

(A) bias15 (active)



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