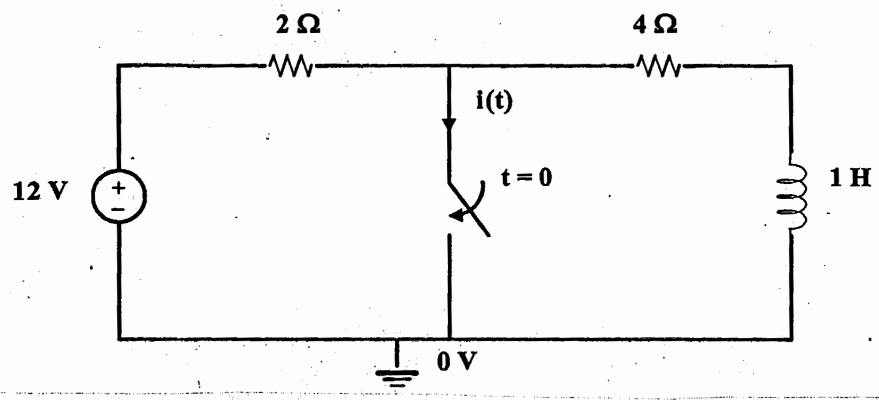
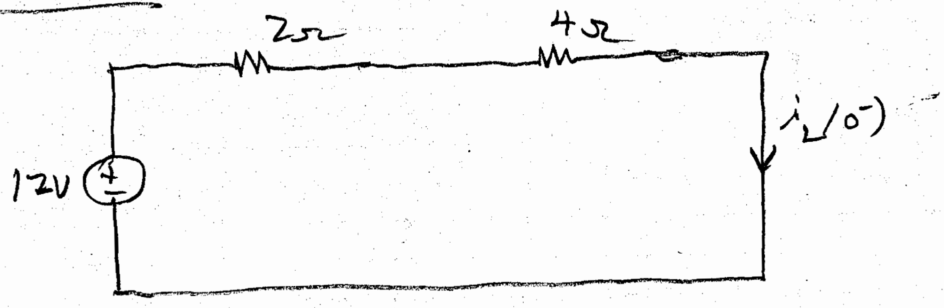
ECE 300
Spring Semester, 2008
Test #3

wlg: Test A ;

(1) The switch in the circuit of Figure 1 has been open for a long time. At t = 0 the switch is closed.
(a) Determine $i(0^+)$ and $i(\infty)$.
(b) Determine i(t) for $t \geq 0$. Use any method you desire.
(c) How many milliseconds after the switch has been closed will the current i(t) equal 5 A?

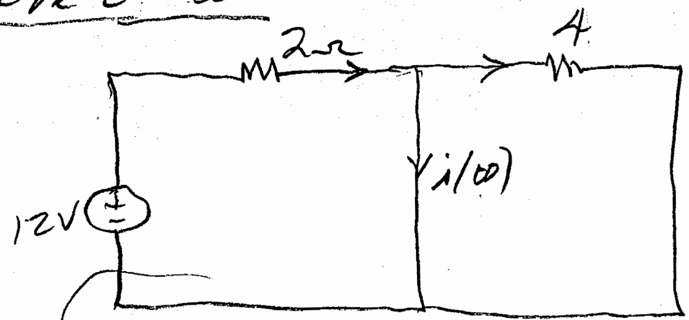


FOR $t<0$



$i_L(0^-) = \frac{12}{6} = 2A \quad \therefore \quad i_L(0^+) = 2A$ (1)

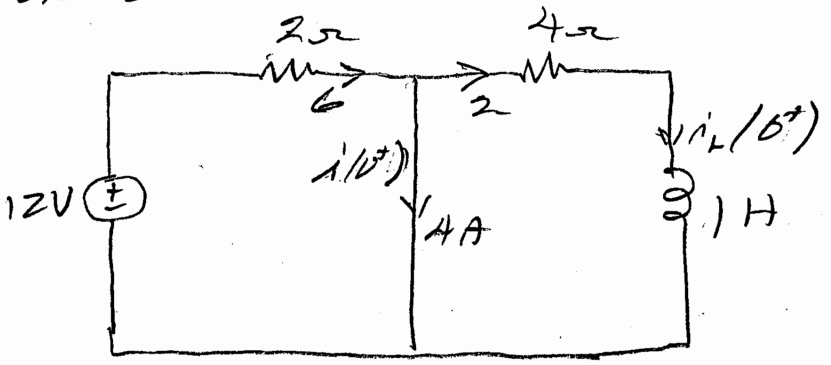
1a) FOR $t = \infty$



$i(\infty) = 6A$ (2)

(1) cont

For $t = 0^+$



$$\tau = \frac{L}{R} = \frac{1}{4} = 0.25 \text{ sec}$$

$$i(0^+) = 6 - 2 = 4A$$

$$i(\infty) = 6A$$

(b)

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$$

$$i(t) = 6 + [4 - 6]e^{-4t}$$

$$\boxed{i(t) = (6 - 2e^{-4t})\,u(t) \quad A}$$

(c)

$$5 = 6 - 2e^{-4t}$$

$$-1 = -2e^{-4t}$$

$$0.5 = e^{-4t}$$

$$\ln .5 = -4t \ln e = -4t$$

$$t = \frac{\ln .5}{-4}$$

$$\boxed{\begin{aligned} t &= 0.173 \text{ sec} \\ t &= 173 \text{ msec} \end{aligned}}$$

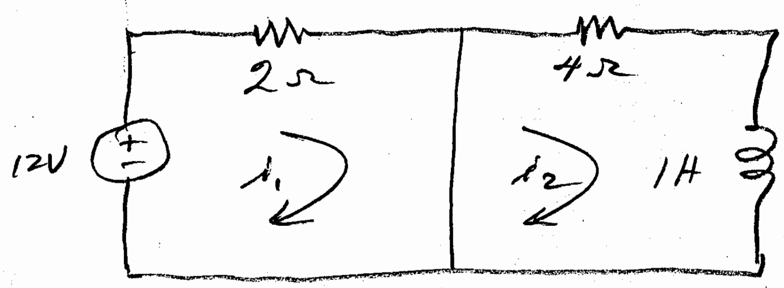
(1) cont

Alternate method

Determine $i_L(0^+)$ as before

$i_L(0^+) = i_2(0^+) = 2A$

For $t > 0$



$12 = 2i_1$

$i_1 = 6\ A$

$$4i_2 + \frac{di_2}{dt} = 0$$

$$i_2 = Ke^{-4t}$$

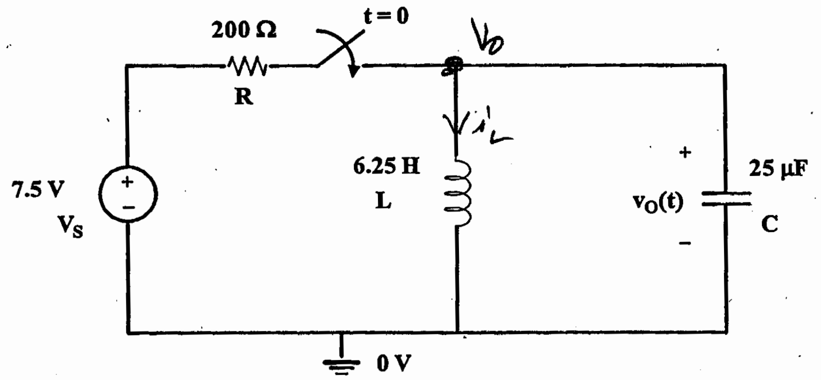
$i_2(0^+) = 2 \quad K = 4$

$$i_2 = 2e^{-4t}$$

$i(t) = i_1(t) - i_2(t)$

$$\boxed{i(t) = 6 - 2e^{-4t}\ A \quad u(t)}$$

(2) There is no energy stored in the circuit of Figure 2 when the switch is closed at t = 0.
(a) Find $v_o(t)$ for $t \geq 0$. To do this develop a second order differential equation in $v_o(t)$ and solve.
(b) Determine the time at which $v_o(t)$ will be maximum.



$$\frac{V_o - V_s}{R} + C\frac{dV_o}{dt} + \frac{1}{L}\int_0^t V_o(t)\,dt + i_L(0^+) = 0 \quad (1)$$

Take $\frac{d[\cdot]}{dt}$ of (1)

$$\frac{1}{R}\frac{dV_o}{dt} + C\frac{d^2V_o}{dt^2} + \frac{V_o}{L} = 0$$

$$\frac{d^2V_o}{dt^2} + \frac{1}{RC}\frac{dV_o}{dt} + \frac{V_o}{LC} = 0 \quad (2)$$

Char. Eq.

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

with numbers

$$s^2 + 200s + 6400 = 0$$

$$(s+40)(s+160) \quad (3)$$

(2) cont

$$V_o(t) = A_1 e^{-40t} + A_2 e^{-160t} \qquad (4)$$

$$V_o(0^+) = 0 = \left(A_1 e^{-40t} + A_2 e^{-160t}\right)\Big|_{t=0}$$

$$A_1 + A_2 = 0 \qquad (5)$$

Need $\frac{dV_o(0^+)}{dt}$

From the ckt

$$i(0^+) = \frac{7.5}{200} = C\frac{dV_o(0^+)}{dt}$$

current thru inductor cannot change instantaneously so $i_{cap} = \frac{7.5}{200}$

$$\frac{dV_o(0^+)}{dt} = \frac{7.5}{200 \times 25 \times 10^{-6}} = 1500 \qquad (6)$$

Take $\frac{d}{dt}[\cdot]$ of (4)

$$\frac{dV_o(t)}{dt} = -40 A_1 e^{-40t} - 160 A_2 e^{-160t} \qquad (7)$$

Evaluate (7) at $t=0^+$

$$\frac{dV(0^+)}{dt} = 1500 = -40 A_1 - 160 A_2$$

$$\begin{bmatrix} 1 & 1 \\ -40 & -160 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1500 \end{bmatrix}$$

$A_1 = 12.5 \qquad A_2 = -12.5$

(2) cont

$$v_o(t) = \left[12.5e^{-40t} - 12.5e^{-160t}\right]u(t) \quad V$$

1b)

$$\frac{d v_o(t)}{dt} = -40 \times 12.5 e^{-40t} + 12.5 \times 160 e^{-160t}$$

$$0 = -40e^{-40t} + 160e^{-160t}$$

$$0 = -e^{-40t} + 4e^{-160t}$$

$$0 = -1 + 4e^{-120t}$$

$$4e^{-120t} = 1$$

$$e^{-120t} = 0.25$$

$$-120t \ln e = \ln .25 = -1.386$$

$$t = \frac{1.386}{120} = 11.6 \text{ msec}$$

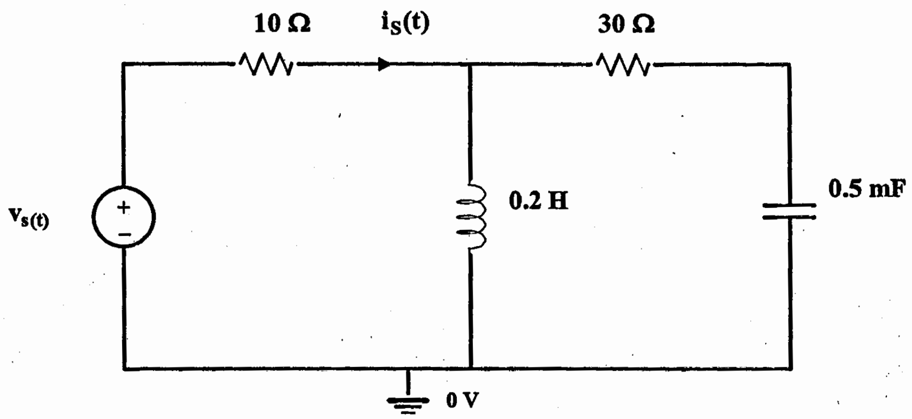
Checks with P-spice simulation

** Profile: "SCHEMATIC1-bias3"   [C:\OrCAD\OrCAD_10.5_Demo\NEL8_21-PSpiceFiles\SCHEMATIC1\bias3.sim]
Date/Time run: 03/31/08 17:20:29   Temperature: 27.0

Date: March 31, 2008   Page 1   Time: 17:20:43

TEST II

(3) The circuit in Figure 3 is in steady state. Assume $v_S(t) = 20\sin(100t - 40^\circ)$ V.
- (a) Determine the steady state current $i_S(t)$ using a cosine reference.
- (b) Draw the phasor diagram for $\mathbf{V_S}$ and $\mathbf{I_S}$.
- (c) Is the voltage $\mathbf{V_S}$ leading the current $\mathbf{I_S}$ or is $\mathbf{I_S}$ leading $\mathbf{V_S}$ and by what angle?



$V_S = 20\sin(100t - 40^\circ)$ change to cosine ref.

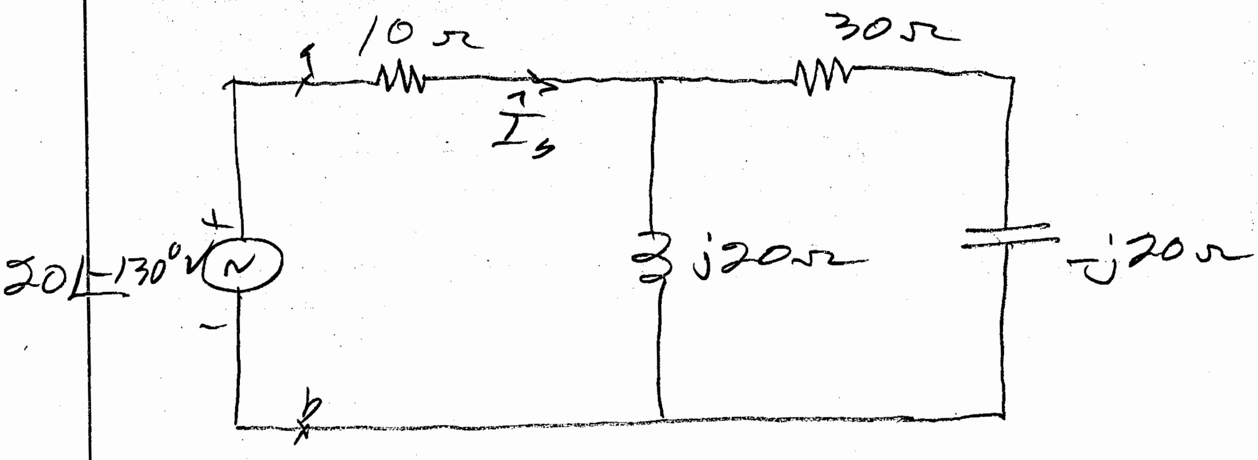
$V_S = 20\cos(100t - 130^\circ)$

$\vec{V}_S = 20\angle{-130^\circ}$ V

$0.2\,H \rightarrow j\,100 \times .2 \rightarrow j\,20\ \Omega$

$0.5\,mF \rightarrow \dfrac{-j}{100 \times .5 \times 10^{-3}} \rightarrow -j\,20\ \Omega$

DRAW Phasor ckt



(3) cont

$$Z_{ab} = 10 + (30 - j20) \parallel (j20)$$

$$Z_{ab} = 10 + \frac{(30 - j20)(j20)}{30 - j20 + j20}$$

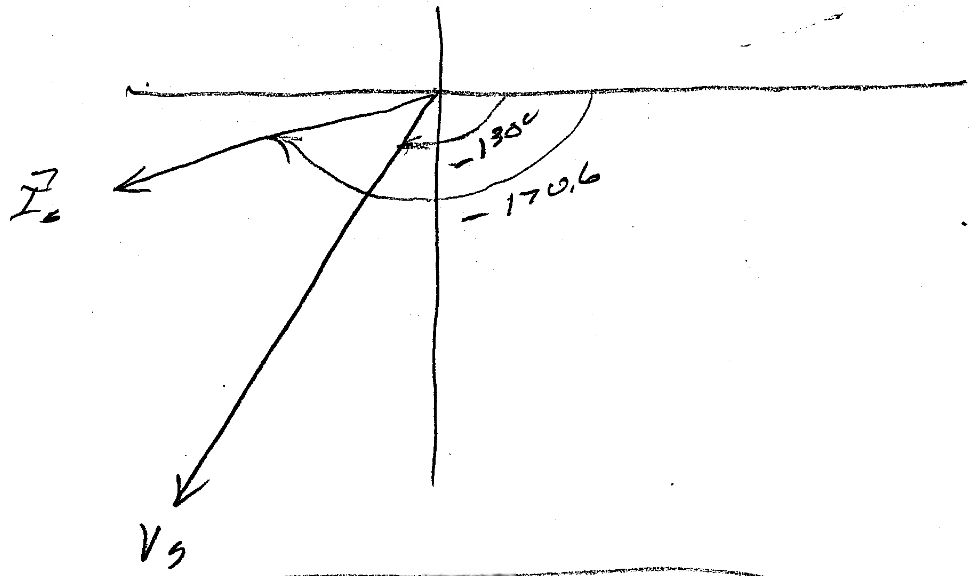
$$Z_{ab} = 30.73 \angle 40.6 \ \Omega$$

$$I_s = \frac{V_s}{Z_{ab}} = \frac{20 \angle -130}{30.73 \angle 40.6}$$

$$I_s = 0.65 \angle -170.6^\circ \ A$$

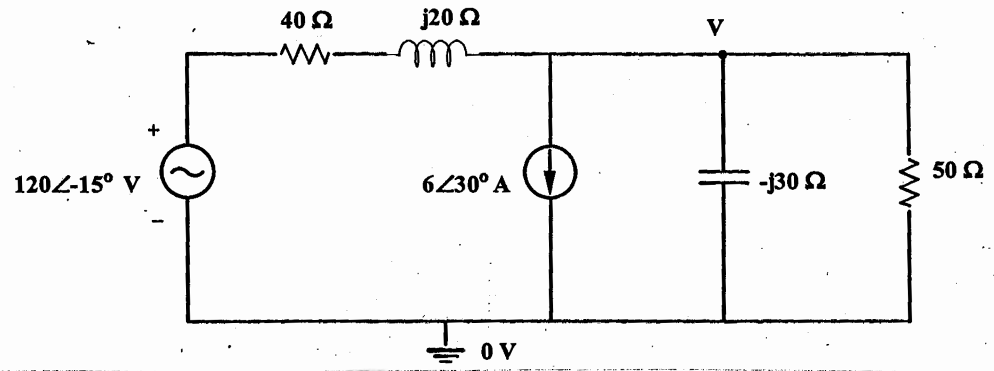
$$i_s(t) = 0.65 \cos(100t - 170.6^\circ) \ A$$

(5)



V_s leads I_s by 40°

(4) Use nodal analysis to find the phasor voltage **V** in the circuit of Figure 4.



$$\frac{\vec{V} - 120\angle{-15}}{40 + j20} + \frac{\vec{V}}{50} - \frac{\vec{V}}{j30} = -6\angle 30$$

$$(0.02 - j0.01)\vec{V} + 2.68\angle 133.4 - j0.0333V + 0.2V = -6\angle 30$$

$$(0.04 + j0.0233)\vec{V} = 6\angle{-150} - 2.68\angle 133.4$$

$$\vec{V} = 124.5\angle{-154^{\circ}} \quad V$$