wlg

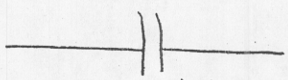
# Lesson 7

## An Introduction To AC Circuits.

In these notes the "how" of AC circuits will be explained. At a later time, the background to this work will be presented.

**The Capacitor:**

Anytime a capacitor is present in an AC circuit we use

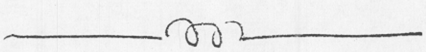


$$1/j\omega C = \frac{-j}{\omega C}$$ has units of ohms

as the "resistance". Note that $\omega$ is present. This is the $\omega$ in the source(s). $j$ is the same as $i$, the prefix used with imaginary numbers.

**The Inductor:**

Anytime an inductor is present in the circuit we use



$$j\omega L$$

as the "resistance". $j\omega L$ has units of ohms.

In AC circuits, we apply AC sources. That is why we call them AC circuits. The following is a typical AC circuit.

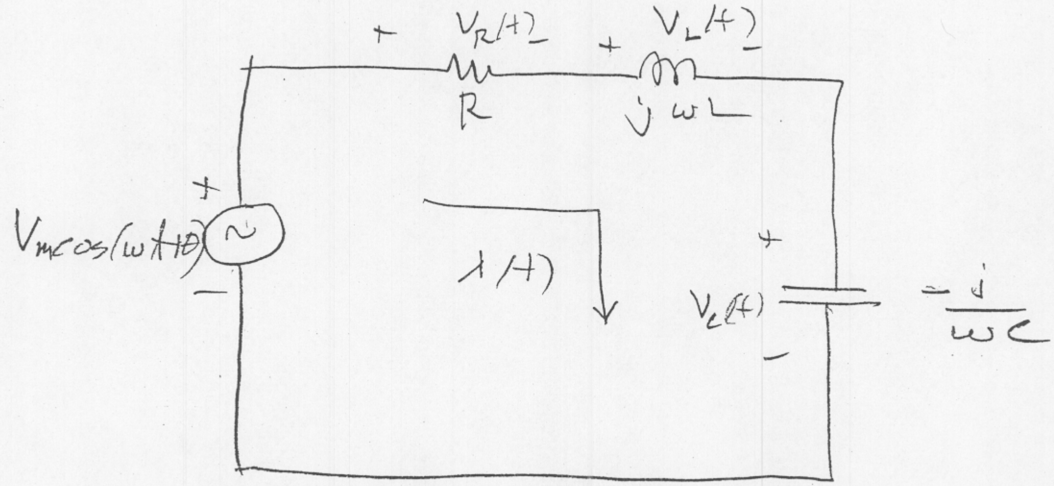
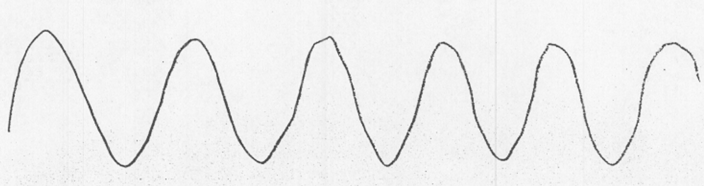



Figure 1: An AC circuit.

We would like to determine the steady state values of $i(t)$, $V_R(t)$, $V_C(t)$, and $V_L(t)$.

It makes sense that since we are applying a sinusoidal source the $i(t)$, $V_R(t)$, $V_C(t)$ and $V_L(t)$ will be sinusoidal in steady state. These waveforms will all appear as:

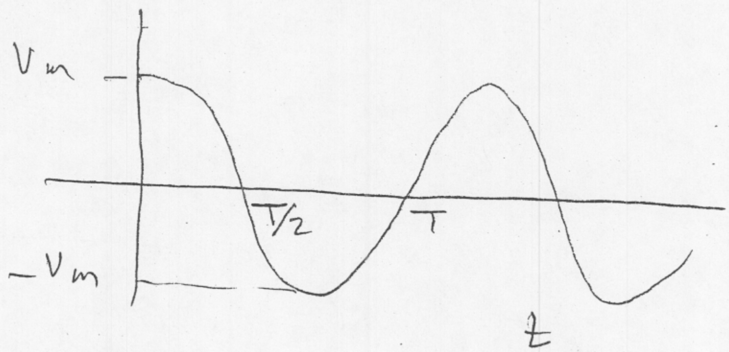


Can be verified using an oscilloscope.

Let us first consider

$$V_m \cos(\omega t + \theta)$$

Assume for now that $\theta = 0$. The wave appears as




$$\omega = 2\pi f = \frac{2\pi}{T}$$

$f$ is the frequency of the wave in hertz (Hz). $T$ is the period and is related to $f$ by $T = \frac{1}{f}$.

If $f = 60$ Hz (power line freq)

Then $\omega = 2\pi\, 60 = 377$ rad/sec

$$T = \frac{1}{f} = \frac{1}{60}.$$

Most of the time a cosine reference is used. If not, one would be informed that a sine reference is used.

We do not directly solve for the time varying signals $i(t)$, $v_R(t)$, $v_C(t)$ and $v_L(t)$ and furthermore we are seldom interested in the time varying form. We see why this is so, later.

When solving an AC circuit we use phasors. Phasors are similar to vectors, in mechanics, in that they have magnitude and angle. However, phasors are not vectors and do not adhere to such operations as $\hat{A} \times \hat{B}$, $\hat{A} \cdot \hat{B}$.

Suppose in Figure 1 that

$$V_m \cos(\omega t + \theta) = \boxed{200 \cos(100t + 30^\circ)}$$

As a phasor voltage we express this as

$$\hat{V}_s = 200 \angle 30 \qquad (1)$$

If we ever had reason to convert this back to the time domain we would need to know that we used a cosine and that $\omega = 100$.

Suppose for the circuit of Fig 1

$$R = 50\,\Omega$$
$$L = 0.7\ H$$
$$C = 300\,\mu F$$

The $\omega L = 100 \times .7 = 70\,\Omega$

$$\frac{1}{\omega C} = \frac{1}{100 \times 300\,\mu F} \doteq 33.3\,\Omega$$

Our circuit becomes;

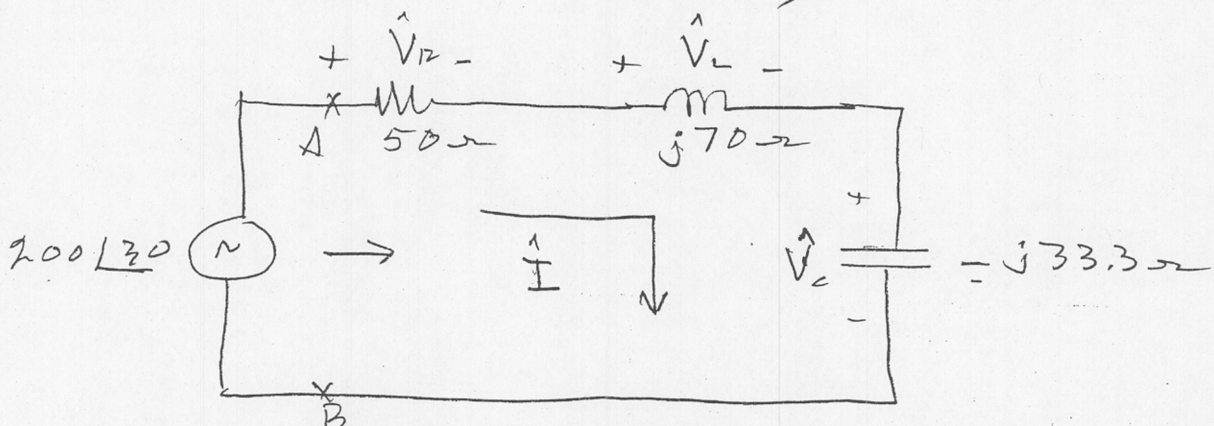



Figure 2: An AC circuit implemented with phasor notation.

The quantities $\hat{I}$, $\hat{V}_R$, $\hat{V}_L$ and $\hat{V}_c$ are phasors. In a text book they would show up as bold capital letters.

To find $\hat{I}$ we need to know the effective "resistance" seen looking to the right of A-B.

We do not call this resistance but rather impedance because of the inductor and capacitor being present. We will say more about impedance later

We write

$$\hat{I} = \frac{200\angle 30°}{50 + j70 - j33.3}$$

We treat the resistor, inductor and capacitor in series as if they were 3 resistors in series.

$$\hat{I} = \frac{200\angle 30}{50 + j36.7}$$

$$\hat{I} = \frac{200\angle 30}{62.02\angle 36.2} = 3.22\angle -6.3 \text{ A}$$

OR

$$\hat{I} = 3.21 - j0.348 \quad \text{A}$$

$$\hat{V}_R = \hat{I} \times 50 = 161.2\angle -6.3° \quad \text{V}$$

$$\hat{V}_L = \hat{I} \times 70\angle 90 = 225.4\angle 83.7 \quad \text{V}$$

$$\hat{V}_C = \hat{I} \times (33.3\angle -90) = 107.2\angle -96.3 \quad \text{V}$$

Several questions may abound at this point. First, how can $\hat{V}_L$ be larger than the source voltage? Second, what happened to KVL? Is

$$\hat{V}_S = \hat{V}_R + \hat{V}_L + \hat{V}_C \quad ?$$

First, we are dealing with phasors and we add the voltages as we would vectors. Note

$$\hat{V}_R + \hat{V}_L + \hat{V}_C = 161.2\angle{-6.2} + 225.4\angle{83.7} + 107.2\angle{-96.3}$$

We perform the operations on the right to find;

$$\hat{V}_S = 173.2 + j100.1 \doteq 200\angle{30} \quad V$$

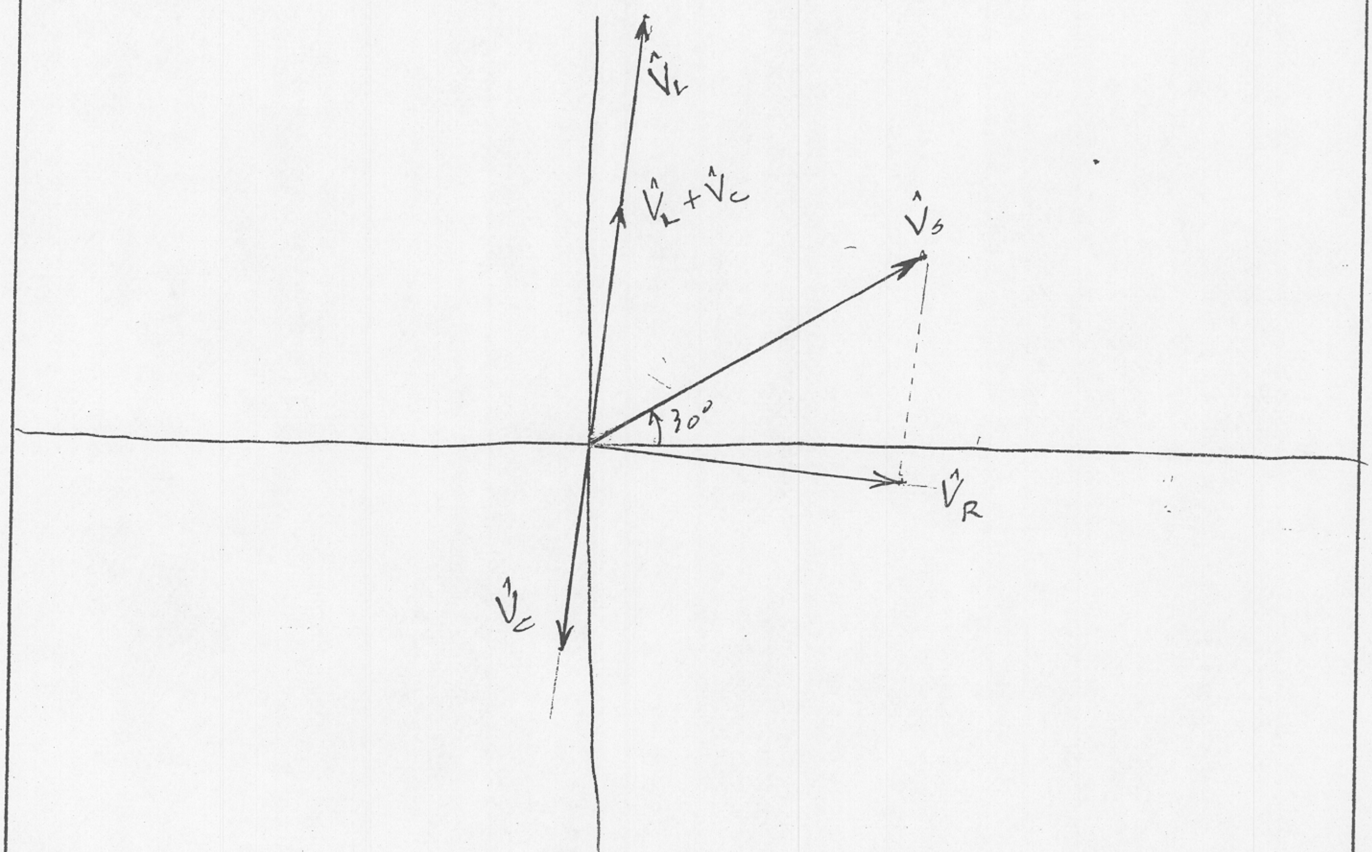
So we see that the three voltages do add up to be the source voltage.

It is of interest to show all the above voltages and the current graphically, in a diagram. We call such a diagram, a phasor diagram.

With

$\hat{V}_s = 200\angle 30$, $\hat{V}_R = 161.2\angle -6.3$, $V_L = 225.4\angle 83.7$,

$\hat{V}_c = 107.2\angle -96.3$, $\hat{I} = 3.22\angle -6.3$




If we want the time domain expression for the steady state voltages and current, we have;

$$i(t) = 3.22 \cos(100t - 6.3°) \text{ A}$$

$$V_R(t) = 161.2 \cos(100t - 6.3) \text{ V}$$

$$V_L(t) = 225.4 \cos(100t + 83.7°)$$

$$V_c(t) = 107.2 \cos(100t - 96.3°)$$

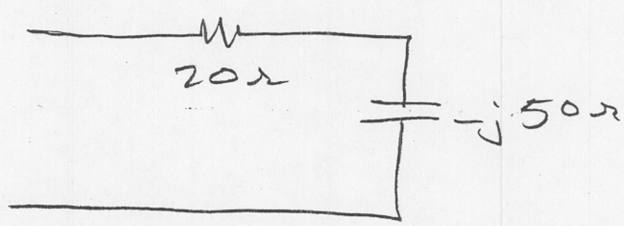
## About Impedance:

One might expect that the name (word) impedance is used in circuits analogous to resistance. Resistance "cuts-down" or hinders current flow in a circuit. Similarly, resistors, inductors and capacitors (connected in various arrangements) impede the current flow - hence we call it impedance. We express impedance as Z.

(Some short Examples)

Example 1

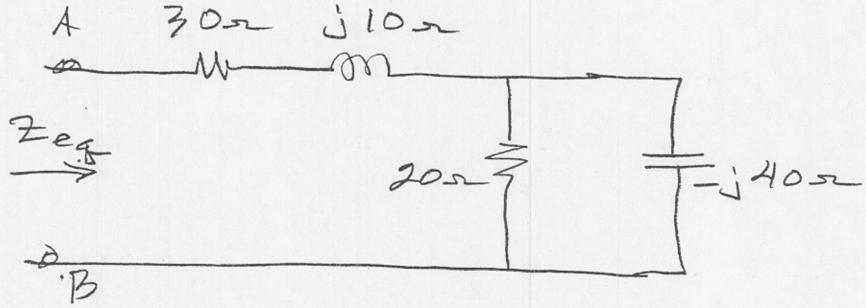
Find the impedance to the right of terminals A-B




$$Z = \underbrace{20 - j50}_{\text{rectangular form}} = \underbrace{53.85 \angle -68.2^\circ \ \Omega}_{\text{polar form}}$$

## Example 2

Find the impedance to the right of terminals A-B

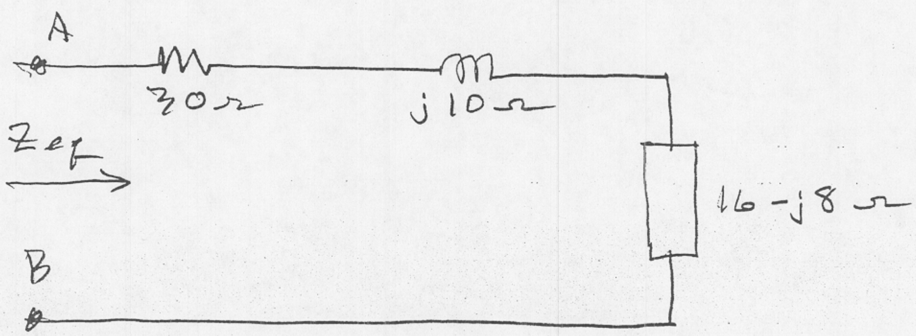



We treat components in parallel, (be they resistors, capacitors or inductors) the same as resistors in ~~series~~ parallel. We treat components in series (be they resistors, capacitors or inductors) the same as resistors in series.

For the above,

$$20 \| (-j40) = \frac{20 \times 40 \angle -90}{20 - j40} = 16 - j8$$

So we have



Thus,

$$Z_{eq} = 30 + j10 + 16 - j8$$

$$Z_{eq} = 46 + j2$$

In general we express impedance as

$$Z = R + jX \quad \text{(units, ohms)}$$

$R \rightarrow$ resistance (units, ohms)

$X \rightarrow$ reactance (units, ohms)

As we did with resistance, in writing

$$G = \frac{1}{R} \rightarrow \text{conductance}$$

We write

$$Y = \frac{1}{Z} \longrightarrow \text{admittance}$$

$$Y = G + jB \quad \text{(units, S)}$$

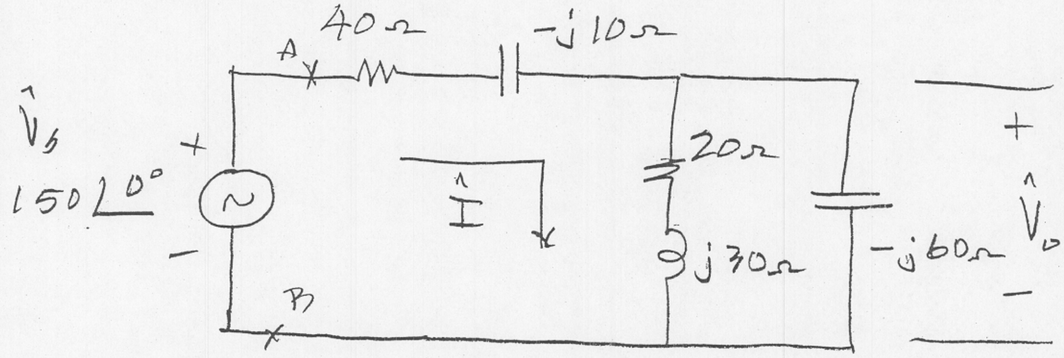
$G \rightarrow$ conductance (units, S)

$B \rightarrow$ susceptance (units, S)

One should try to remember the above.

## Circuit Example 2

Find $\hat{I}$ for the following circuit. Also find $\hat{V}_o$.




We have

$$\hat{Z}_L = \frac{(60\angle{-40})(20+j30)}{20+j30-j60} = 55.4 + j23.1$$

$$Z_{AB} = 40 - j10 + 55.4 + j23.1$$

$$Z_{AB} = 95.4 + j13.1 = 96.3\angle 7.8^\circ \ \Omega$$

$$\underline{\hat{I}} = \frac{\hat{V}_s}{\hat{Z}_{AB}} = \frac{150\angle 0}{96.3\angle 7.8} = \underline{1.56\angle{-7.8^\circ}} \ A$$

$$\hat{V}_o = \hat{I} \times \hat{Z}_L = (1.56\angle{-7.8})(55.4 + j23.1)$$

$$\underline{\hat{V}_o = 93.6\angle 14.9^\circ \ V}$$ 64∠-21.8

QED