wolg

Lesson 8
A.C. Circuits
Examples

Nodal Analysis

Nodal analysis, as explained in DC circuits, carry over to A.C. Circuits. The difference being that we are dealing with phasors and impedance.

Example 8.1

Use nodal analysis to find $V_1(t)$ and $V_2(t)$ in the following circuit.

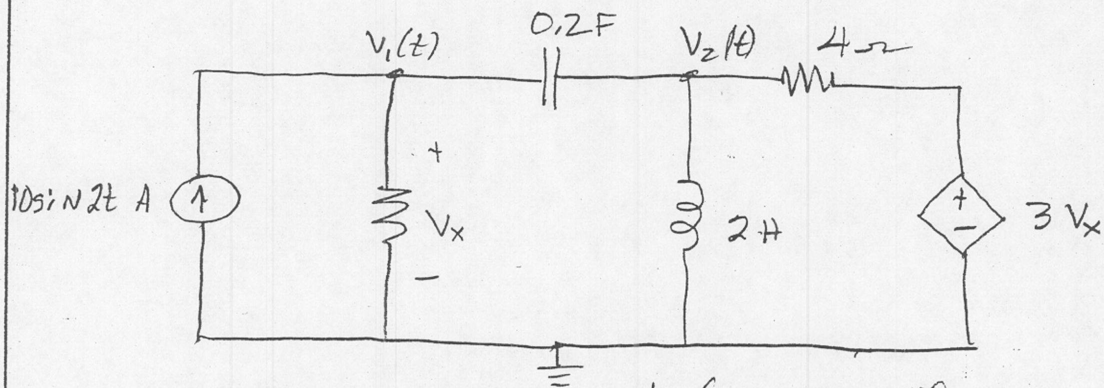

Figure 8.1: Circuit for Example 8.1

Three points of note here:

(1) We establish the impedance of the capacitor and inductor using the source $\omega = 2$ r/sec

(2) Rather than solve for $V_1(t)$, $V_2(t)$, $V_x(t)$ we use phasor voltages then change to the steady state time domain.

(3) We leave the source as a sine reference. Most of the time the reference is given as cosine.

Impedance

$.2F \longrightarrow \frac{-j}{2\times.2} = -j2.5\,\Omega$

$2H \longrightarrow j\,2\times2 = j4\,\Omega$

Draw The Phasor Circuit

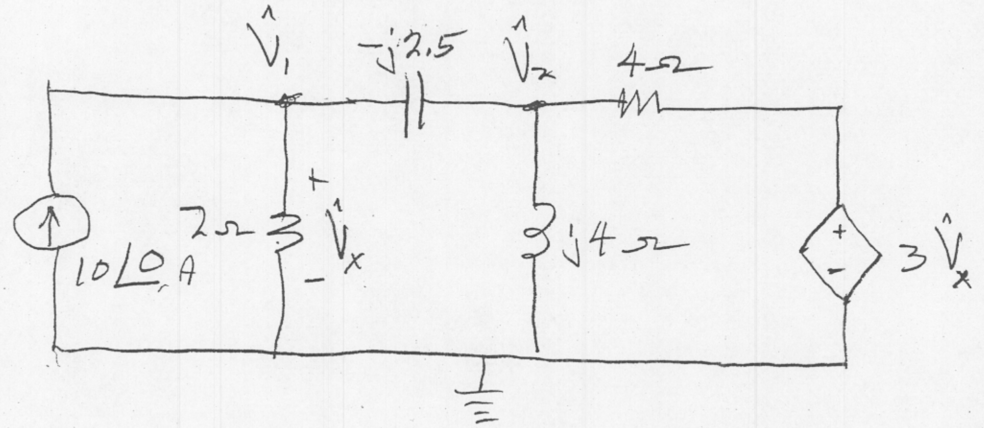

Figure 8.2: Phasor circuit for Example 8.1

At node $\hat{V}_1$:

$$\frac{\hat{V}_1}{2} + \frac{\hat{V}_1 - \hat{V}_2}{-j2.5} = 10$$

At node $\hat{V}_2$

note: $\hat{V}_x = \hat{V}_1$

$$\frac{\hat{V}_2 - V_1}{-j2.5} + \frac{\hat{V}_2}{j4} + \frac{V_2 - 3\hat{V}_1}{4} = 0$$

$$\left(\frac{1}{2} - \frac{1}{j2.5}\right)\hat{V}_1 + \frac{\hat{V}_2}{j2.5} = 10$$

$$\left(-\frac{3}{4} + \frac{1}{j2.5}\right)\hat{V}_1 + \left(\frac{1}{4} + \frac{1}{j4} - \frac{1}{j2.5}\right)\hat{V}_2 = 0$$

$$(0.5 + j0.4)\hat{V}_1 - j0.4\hat{V}_2 = 10$$

$$(-0.75 - j0.4)\hat{V}_1 + (0.25 + j0.15)\hat{V}_2 = 0$$

$$\begin{bmatrix} (.5 + j0.4) & (0 - j0.4) \\ (-0.75 - j0.4) & (0.25 + j0.15) \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\hat{V}_1 = 11.3\angle 60^\circ \text{ V} \qquad \hat{V}_2 = 33\angle 57.1 \text{ V}$$

$$\therefore \boxed{v_1(t) = 11.3 \sin(2t + 60^\circ) \text{ V}}; \boxed{v_2(t) = 33 \sin(2t + 57.1) \text{ V}}$$

Example 8.2

This problem illustrates mesh analysis of a 3 mesh circuit that does not have current sources or dependent sources.
We will later work a problem with current source(s).

Consider the circuit below. Find $\hat{I}_1$, $\hat{I}_2$, and $\hat{I}_3$.

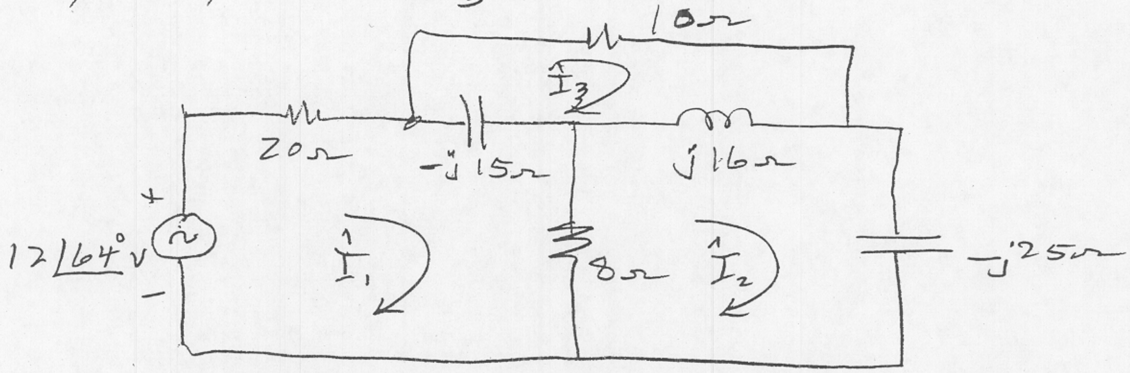

Figure 8.3: Circuit for Example 8.2.

mesh #1

$$20\hat{I}_1 - j15(\hat{I}_1 - \hat{I}_3) + 8(\hat{I}_1 - I_2) = 12\angle 64$$

or

$$\boxed{(28 - j15)\hat{I}_1 - 8\hat{I}_2 + j15 I_3 = 12\angle 64}$$

mesh #2

$$8(\hat{I}_2 - \hat{I}_1) + j16(\hat{I}_2 - I_3) - j25(\hat{I}_2) = 0$$

or

$$\boxed{-8\hat{I}_1 + (8 - j9)\hat{I}_2 - j16 I_3 = 0}$$

mesh #3

$$-j15(\hat{I}_3 - \hat{I}_1) + 10\hat{I}_3 + j16(\hat{I}_3 - \hat{I}_2) = 0$$

$$\boxed{j15 I_1 - j16\hat{I}_2 + (10 + j1) I_3 = 0}$$

$$\begin{bmatrix} 28-j15 & -8 & j15 \\ -8 & 8-j9 & -j16 \\ j15 & -j16 & 10+j1 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_3 \end{bmatrix} = \begin{bmatrix} 12\angle 64^\circ \\ 0 \\ 0 \end{bmatrix}$$

$\hat{I}_1 = 0.38\angle 109.6^\circ$ A, $\hat{I}_2 = 0.344\angle 124.4^\circ$ A

$\hat{I}_3 = 0.146\angle -60.4^\circ$ A

Example 8.3

This problem illustrates the mesh analysis method when both independent and dependent sources are present.
Consider the circuit of Figure 8.4.
Find $\hat{V}_o$.

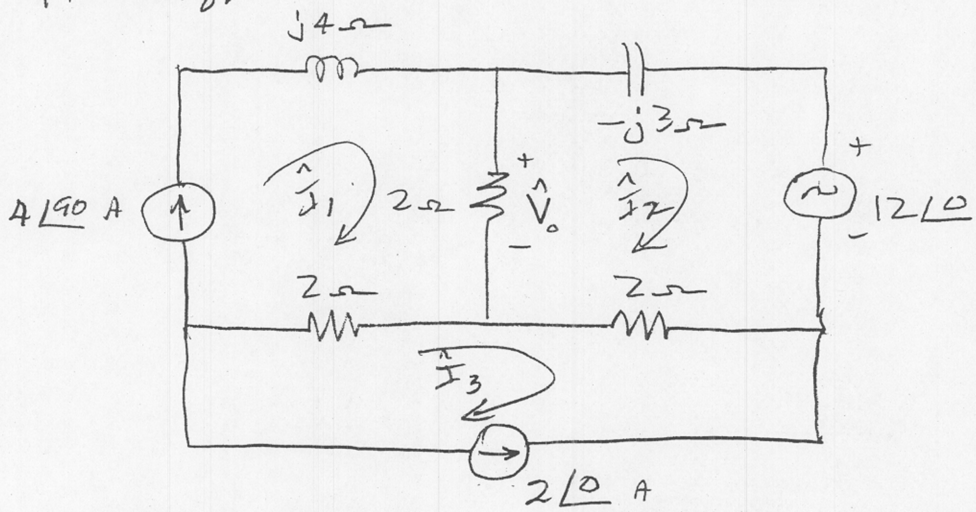

Figure 8.4: Circuit for Example 8.3.

We assign mesh currents as shown.
We open those sources that are current sources.
We not the relationships between the mesh currents and the current sources. Here,

$$\hat{I}_1 = 4\angle 90^\circ A, \quad \hat{I}_3 = -2\angle 0$$

We redraw the circuit as shown in Figure 8.5.

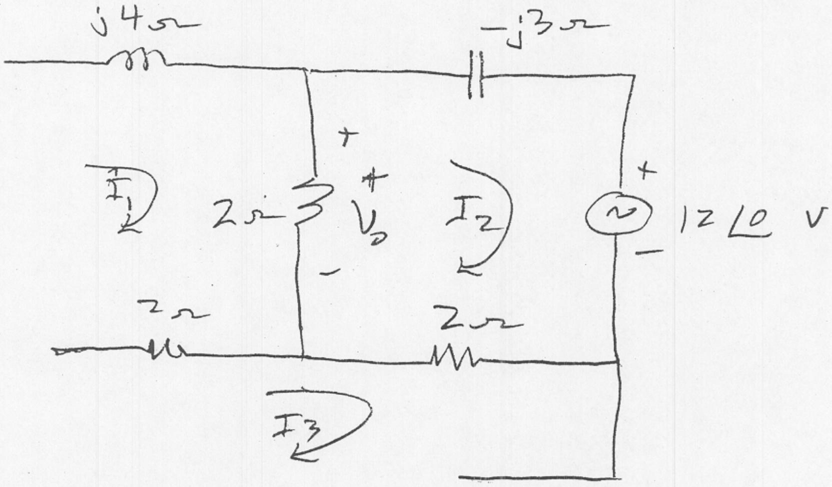

Figure 8.5: Circuit for analysis, Ex 8.3

We write

$$2(\hat{I}_2-\hat{I}_1)-j3I_2+12\angle 0+2(I_2-I_3)=0$$

or

$$-2\hat{I}_1+(4-j3)\hat{I}_2-2\hat{I}_3=-12\angle 0$$

using

$$\hat{I}_1=4\angle 90^\circ,\quad I_3=-2$$

$$-2(j4)+(4-j3)\hat{I}_2-2(-2)=-12\angle 0$$

$$(4-j3)\hat{I}_2=-12-4+j8=-16+j8$$

$$\hat{I}_2=\frac{-16+j8}{4-j3}=-3.52-j0.64$$

$$\hat{V}_0=(\hat{I}_1-I_2)2=(j4+3.52+j0.64)2$$

$$\hat{V}_0=(3.52+j4.64)2$$

$$\boxed{\hat{V}_0=11.65\angle 52.8^\circ\ \text{V}}$$ Ans.

Example 8.4

This example illustrates how to find the Thevenin equivalent circuit of a AC circuit. Consider the circuit shown in Figure 8.7.

Find the Thevenin and Norton circuit, looking in terminals a-b. How much current will flow through a 10Ω resistor placed between a-b?

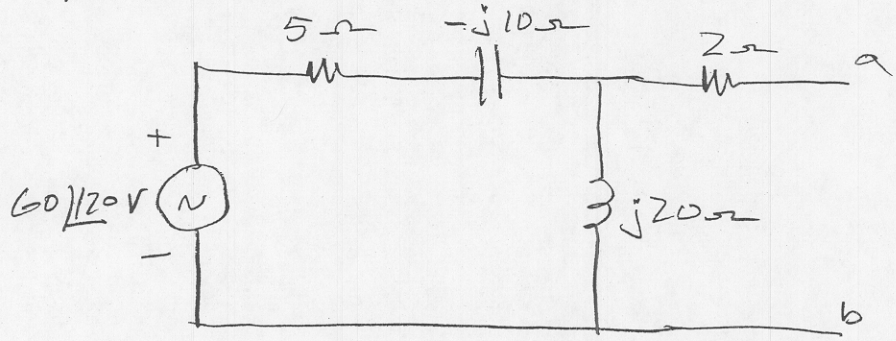

Figure 8.6: Circuit for Example 8.4.

$\underline{Z_{TH}}$

We replace the $60\angle 120^\circ$ V source with a short and find the impedance looking in a-b.

$$Z_{TH} = 2 + (j20) \| (5 - j10)$$

$$= 2 + \frac{(0 + j20)(5 - j10)}{j20 + 5 - j10}$$

$$Z_{TH} = 2 + \frac{(20\angle 90)(5 - j10)}{5 + j10}$$

$Z_{TH} = 2 + (16 - j12)$

$Z_{TH} = 18 - j12 = 21.6\angle{-33.7}^\circ \ \Omega$

To find V_{TH} we apply the voltage division rule, Thus,

$$V_{TH} = \frac{(60\angle 120)(j20)}{5 - j10 + j20}$$

$$V_{TH} = \frac{(60\angle 120)(20\angle 90)}{5 + j10}$$

Note:

V_{TH} is the open circuit voltage across terminals a-b. Since no current flows through the $2\,\Omega$ resistor when a-b is open, there is then no voltage (0 volts) across the $2\,\Omega$ resistor. Thus, V_{TH} becomes the voltage across the inductor. Therefore we apply the voltage divider rule to find the voltage across the inductor and hence, V_{TH},

$$\hat{V}_{TH} = 107.33\angle 146.6^\circ \text{ V}$$

The Thevenin circuit is shown below.

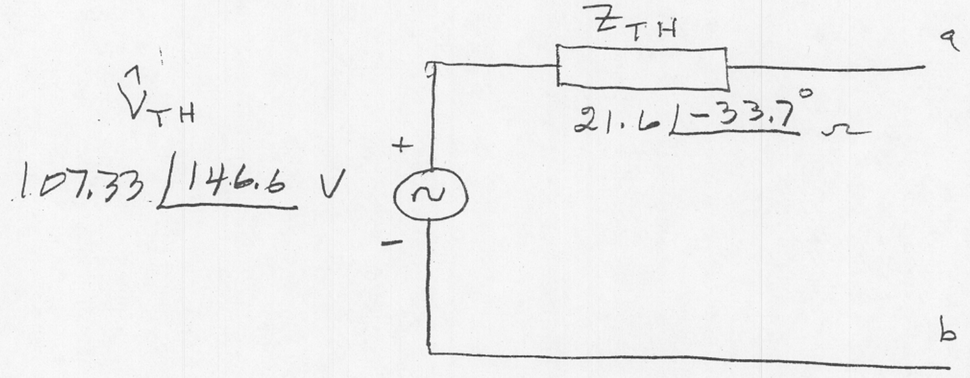

Figure 8.7: Thevenin circuit for Example 8.4.

When we place a 10 Ω resistor across a-b we solve for the current $\hat{I}$, which is the current in the 10 Ω resistor.

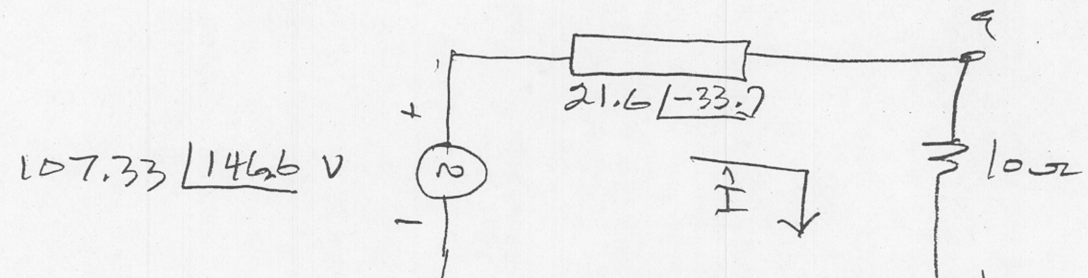

Figure 8.8: Circuit of Example 8.4 with a 10 Ω load.

$$\hat{I} = \frac{107.33\angle 146.6}{21.6\angle -33.7 + 10} = 3.53\angle 169.8^\circ \text{ A}$$

To find the Norton circuit, we can divide $\hat{V}_{TH}/Z_{TH} = \hat{I}_N$

$$\hat{I}_N = \frac{107.33 \angle 146.6}{21.6 \angle -33.7}$$

$$\hat{I}_N = 4.97 \angle -179.7^\circ \text{ A}$$

Then the Norton circuit is as shown in Figure 8.9.

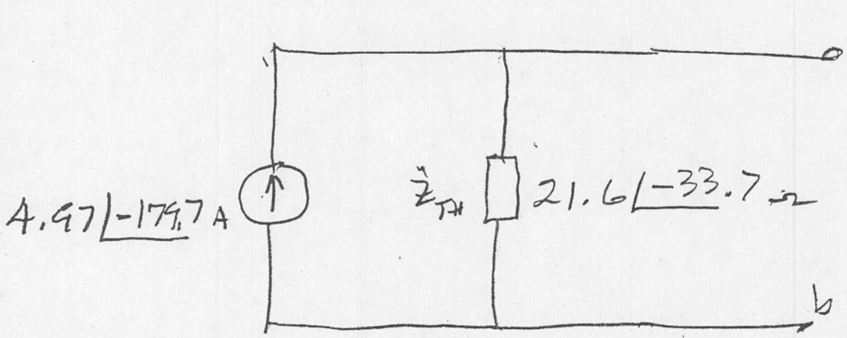

Figure 8.9: The Norton equivalent circuit for Example 8.4.

We also know that the Norton current source is the short circuit that flows from a-b in the circuit shown in Figure 8.10.

In this case, a source transformation has been made on the $60\angle 120^\circ$ V source and the $(5-j10)\,\Omega$ impedance.

This will allow direct application of the current splitting rule to find $\hat{I}_N$.

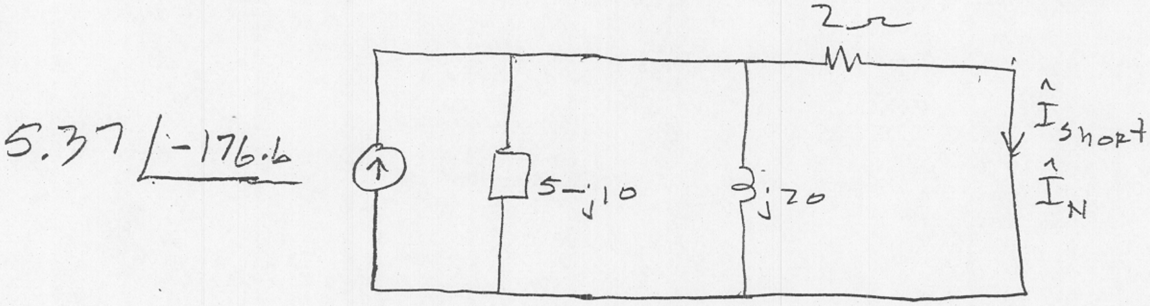

Figure 8.10: Circuit for finding $\hat{I}_N$ in Example 8.4.

$$\hat{I}_N = \frac{(5.37\angle -176.6)\,(5-j10)\,\|\,(j20)}{2+(5-j10)\,\|\,(j20)}$$

$$\hat{I}_N = \frac{5.37\angle -176.6 \;(16-j12)}{2+16-j12}$$

$$\hat{I}_N = 4.97\angle -179.8^\circ \text{ A}$$

This agrees with the previous ckt.

QED