POWER

#9

Instantaneous Power

$$p(t) = v(t)\, i(t)$$

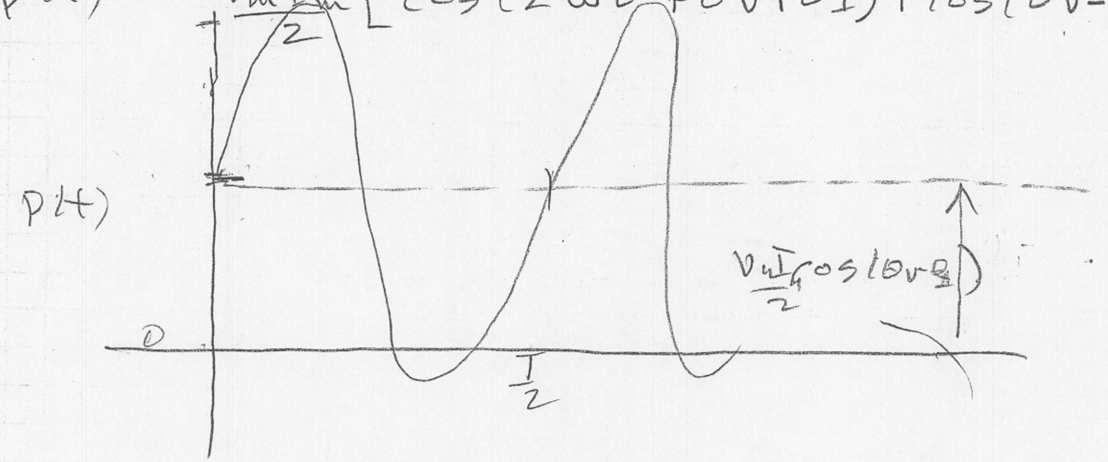
Let
$$v(t) = V_m \cos(\omega t + \theta_V)$$
$$i(t) = I_m \cos(\omega t + \theta_I)$$

$$p(t) = V_m I_m \cos(\underbrace{\omega t + \theta_V}_{A}) \cos(\underbrace{\omega t + \theta_I}_{B})$$

Recall:
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

so
$$V_m I_m \cos(\omega t + \theta_V) \cos(\omega t + \theta_I)$$
$$p(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta_V + \theta_I) + \cos(\theta_V - \theta_I)\right]$$



We are really interested in the <u>average power</u>

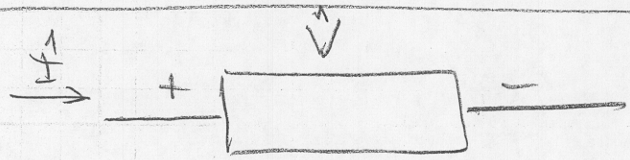
#9

$$P_{AVG} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cancelto{0}{\int \cos(2\omega t+\theta_V+\theta_I)\,dt} +$$

$$+ \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos(\theta_V-\theta_I)\,dt$$

$$\boxed{P_{AVG} = \frac{V_m I_m}{2}\cos(\theta_V-\theta_I)}$$



$\hat{V} = V_m \angle \theta_V$

$\hat{I} = I_m \angle \theta_I$

If we consider

$\frac{1}{2} V_m I_m \cos(\theta_V - \theta_I)$ we see this

is the same as

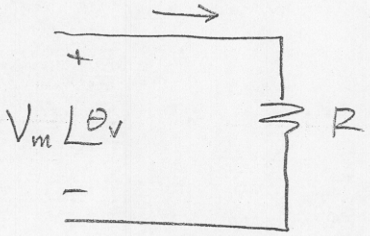
$$\frac{1}{2}\mathrm{Re}\left[\hat{V}\hat{I}^*\right] = \frac{1}{2}\mathrm{Re}\left[V_m \angle\theta_V \times I_m \angle{-\theta_I}\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[V_m I_m e^{j(\theta_V-\theta_I)}\right.$$

$$= \frac{1}{2}\mathrm{Re}\left[V_m I_m\left(\cos(\theta_V-\theta_I) + j\sin(\theta_V-\theta_I)\right)\right]$$

If we are talking about a resistor

$\theta_V = \theta_I$



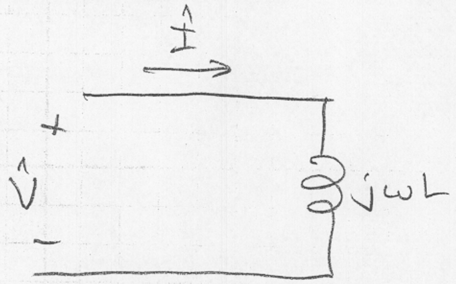
$$\hat{I} = \frac{\hat{V}}{R} = \frac{V_m}{R} \angle \theta_V$$

$$\hat{I} = I_m \angle \theta_I$$

$$I_m = \frac{V_m}{R}, \quad \theta_I = \theta_V$$

$$\boxed{P = \frac{V_m I_m}{2} = \frac{V_m^2}{2R} = \frac{I_m^2 R}{2}} \quad \text{(resistor)}$$

An Inductor



$$\hat{I} = \frac{\hat{V}}{j\omega L} = \frac{V_m \angle \theta_V \cdot \angle -90^\circ}{\omega L}$$

ELI

$$\theta_V = \theta_V$$
$$\theta_I = \theta_V - 90^\circ$$

so

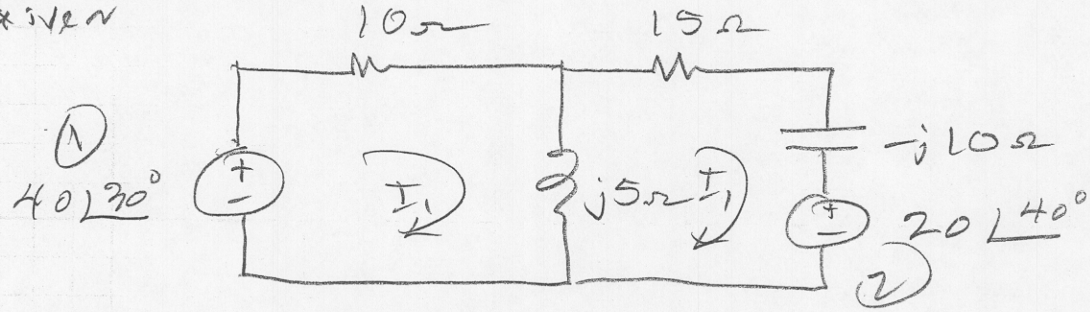
$$\cos(\theta_V - \theta_I) = \cos(\theta_V - (\theta_V - 90)) = \cos 90^\circ = 0$$

$$P_L = \frac{V_m I_m}{2} \cos(90^\circ) = 0$$ No Average power

Same for the capacitor, No Average Power

Example

Given



Determine the power

(a) supplied to each resistor

(b) power supplied by each source

$$\begin{bmatrix} 10+j5, & -j5 \\ -j5 & 15-j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 40\angle 30 \\ -20\angle 40 = 20\angle -140 \end{bmatrix}$$

$$I_1 = 3.398 - j0.1738 = 3.4024 \angle -2.928^\circ$$

$$I_2 = -0.9498 - j0.0411 = 0.9507 \angle -177.52$$

$$P_{10} = \frac{(3.4024)^2}{2} \times 10 = 57.88\,W$$

$$P_{15} = \frac{(.9507)^2}{2} \times 15 = 6.779$$

$$P_{TOTAL\ CKT} = 57.88 + 6.779 = \underline{64.66\,W}$$

$$P_1 = \frac{40 \times 3.4024}{2} \cos(30 + 2.928) = 57.12\,W$$

$$P_2 = -\frac{20 \times .9507}{2} \cos(40 + 177.52) = +\ 7.54$$

check $\quad\quad$ $\overline{64.66\,W}$

Maximum Power Transfer

For maximum power transfer we find V_{TH} as before, i.e., open circuit voltage,

$$\hat{Z}_L = Z_{TH}^*$$

If we are constrained to the load being purely real

$$R_L = |Z_{TH}| = \sqrt{R_{TH}^2 + X_{TH}^2}$$

RMS - The Effective Value

The effective value of a periodic voltage or current is the dc current (voltage) that delivers the same average power to a resistor as the periodic current (voltage)

Power Absorbed by a resistor, d.c. current;

$$P = I_{eff}^2 R$$

Power absorbed by a periodic signal

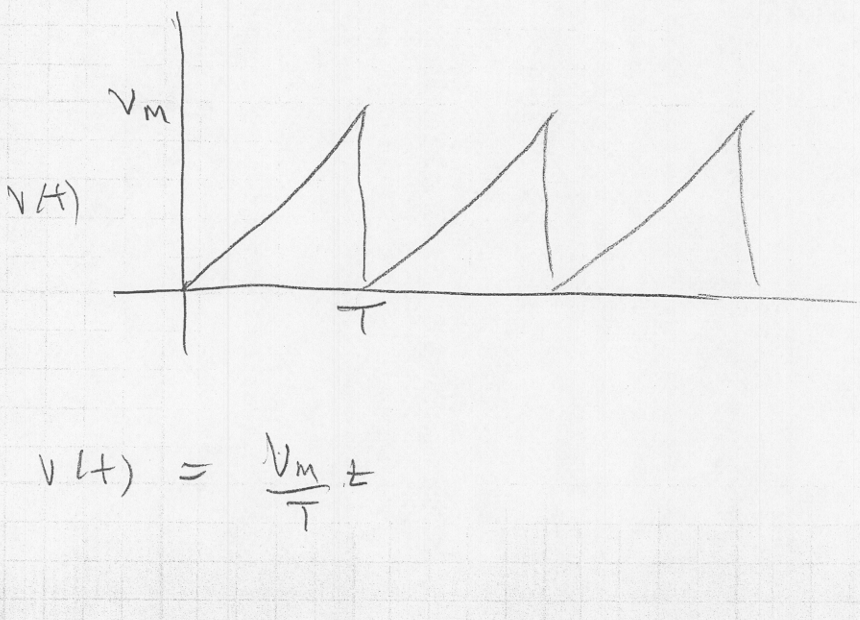
$$P = \frac{R}{T}\int_0^T i^2 dt$$

$$R I_{eff}^2 = \frac{R}{T}\int_0^T i^2 dt$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

Also

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$



$$v(t) = \frac{V_M}{T} t$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T \frac{V_m^2}{T^2} t^2 \, dt}$$

$$= \sqrt{\frac{V_m^2}{T^3} \left.\frac{t^3}{3}\right|_0^T}$$

$$V_{rms} = \sqrt{\frac{V_m^2 \cancel{T^3}}{\cancel{T^3} \times 3}} = \frac{V_m}{\sqrt{3}}$$

Apparent Power

$$P = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_I) = V_{rms} I_{rms} \cos(\theta_V - \theta_I)$$

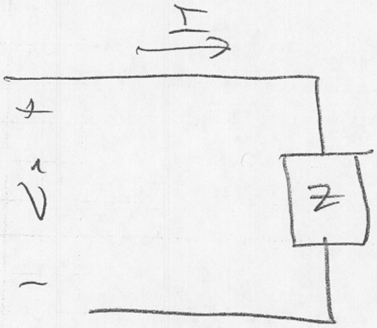
Define apparent power

$$\boxed{S = \frac{V_m I_m}{2} = V_{rms} I_{rms}} \quad \text{units VA}$$

We define p.f. $= \frac{P}{S} = \cos(\theta_V - \theta_I)$

If the current leads the voltage we have a leading (capacitive) p.f. current lags, lagging p.f.

Complex Power



The complex power supplied to the load $\hat{Z}$ is

$$\hat{S} = \frac{\hat{V}\hat{I}^*}{2} = \underline{\underline{\hat{V}_{rms} I_{rms}^*}} \qquad (A)$$

$$\hat{S} = V_{rms} I_{rms} \angle \theta_V - \theta_I$$

$$\hat{S} = V_{rms} I_{rms} \cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)$$

$$\hat{S} = P + jQ$$

Express in terms of $\hat{Z}$

$$\hat{Z} = \frac{\hat{V}}{\hat{I}} = \frac{\hat{V}_{rm}}{\hat{I}_{rm}} = \frac{V_{rm}}{I_{rms}} \angle \theta_V - \theta_I$$

$$V_{rms} = \hat{Z} I_{rms}$$

Back to (A)

$$\hat{S} = I_{rms} I_{rms}^* Z = |I_{rms}|^2 Z$$

$$\hat{S} = |I_{rms}|^2 (R + jX)$$

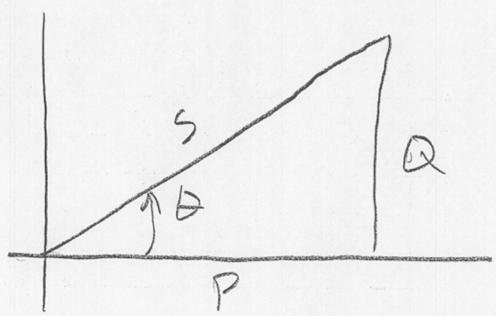
$$\hat{S} = |I_{rms}|^2 R + j|I_{rms}|^2 X$$

Also

$$V_{rms} = I_{rms} Z$$

$$I^*_{rms} = \frac{V^*_{rms}}{Z^*}$$

$$\hat{S} = \frac{\hat{V}_{rms} V^*_{rms}}{Z^*} = \frac{|V_{rms}|^2}{Z^*}$$

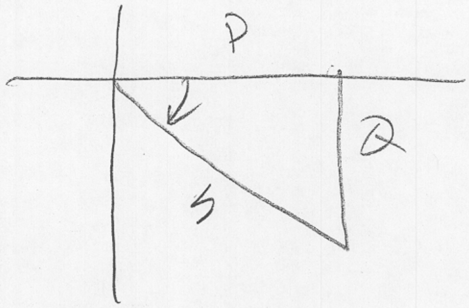


lagging p.f

Since $\hat{S} = |I_{rms}|^2 \hat{Z}$

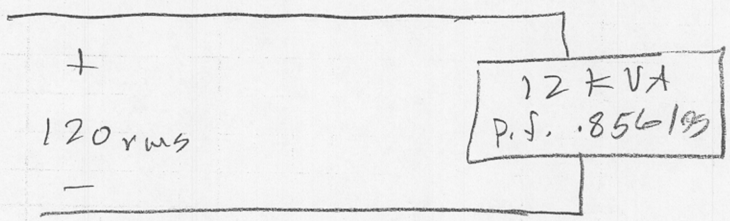
angle of $\angle Z = \angle S$

If Z is a capacitive load, I leads V, θ is negative.



leading P.f.

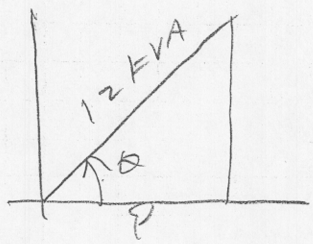
Example



(a) Find the average reactive power to the load

(b) the peak current

(c) the load impedance



$\cos\theta = .856$

$\theta = 31.13^\circ$

$P = 12K \times 0.856 = 10.272 \text{ KW}$

$Q = 12K \sin\theta = 12K \sin 31.13^\circ = 6.204 \text{ KVAR}$

$\hat{S} = P + jQ = (10.272 + j\,6.204)\text{ KVA}$

$\hat{S} = \hat{V}_{rms} I^*_{rms}$

$$I^*_{rms} = \frac{\hat{S}}{\hat{V}_{rms}} = \frac{(10.272 + j\,6.204)\,\text{kVA}}{120\angle 0}$$

$$I^*_{rms} = 100\angle 31.13^\circ$$

$$I_{rms} = 100\angle -31.13$$

$$I_m = \sqrt{2}\,|I_{rms}| = 141.4 \text{ A}$$

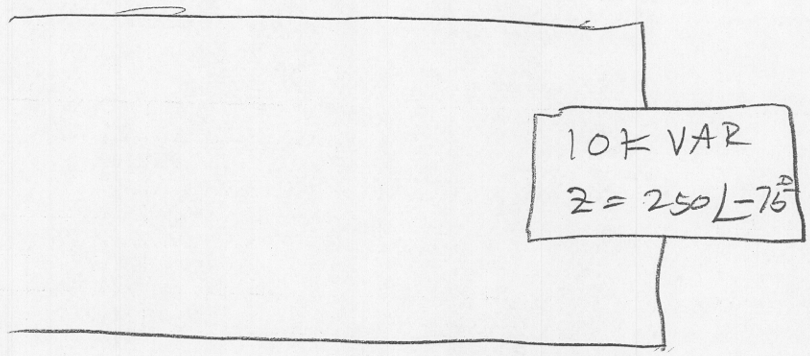
(c)

$$Z = \frac{\hat{V}_{rms}}{\hat{I}_{rms}} = \frac{120\angle 0}{100\angle -31.13}$$

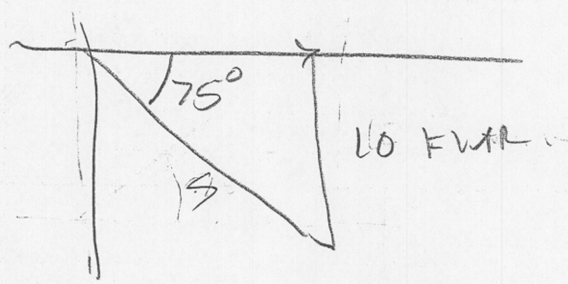
$$Z = 1.2\angle 31.13\ \Omega$$

inductive load

Example



$\angle Z$ = p.f. angle



p.f. = $\cos 75° = .2588$ leading

$$\sin 75° = \frac{10}{S}$$

$$S = \frac{10\ \text{KVAR}}{\sin 75°} = 10.35\ \text{KVA}$$

$$\frac{V_m^2}{2|Z|} = |S|$$

$$V_m^2 = 2|S||Z| = 2 \times 250 \times 10.35 \times 10^3$$

$\sqrt{5}$ $$\boxed{V_m = 1 \times 10^3 \sqrt{5.175} = 2.275\ \text{kV}}$$