Desk Copy

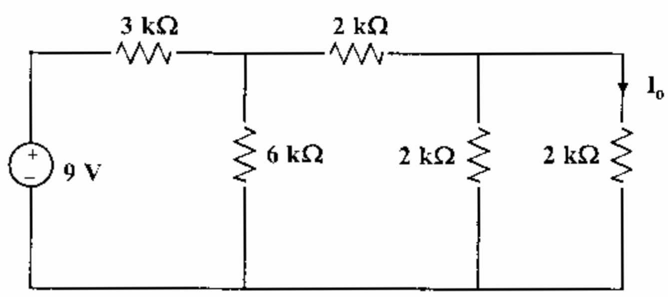
ECE 301
Fall Semester 2005
HW # 2

wlg Due: Sept 20

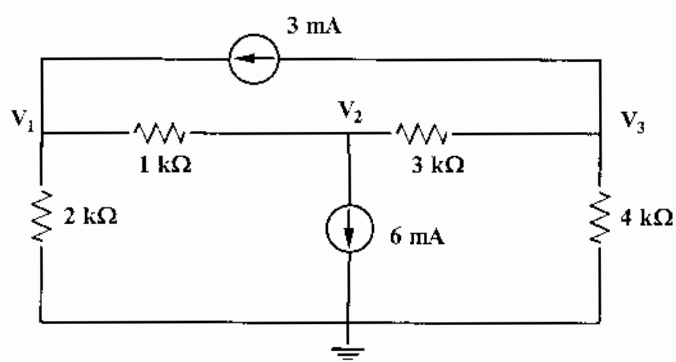
Name wlg
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. <u>Be sure to show how you got your answers.</u>** Each problem counts 10 points.

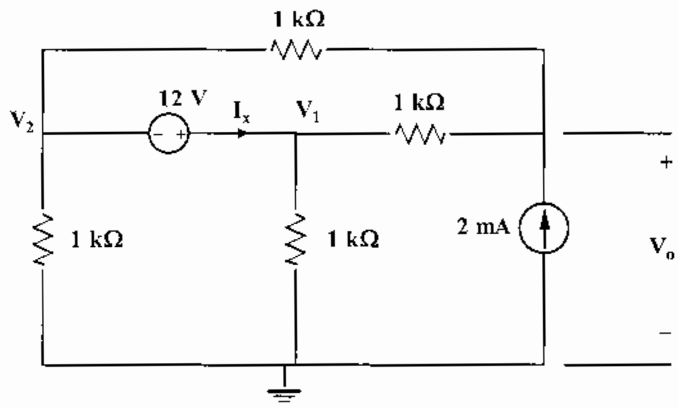
(1) Find I_o by using nodal analysis. Ans $I_o = 0.6$ mA



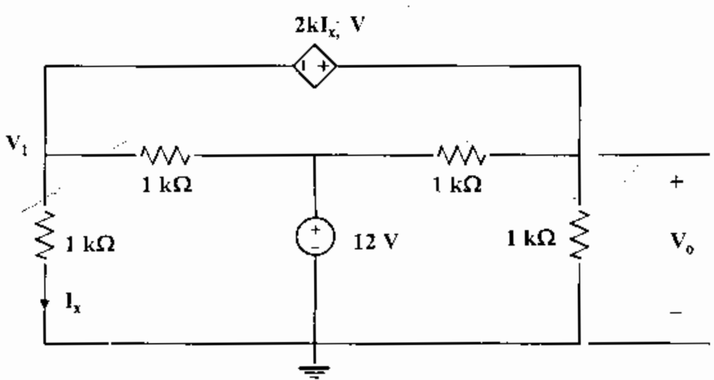
(2) Determine the node voltage equations for V_1, V_2, and V_3 for the following circuit. Use MATLAB to solve for these voltages. Ans $V_1 = -6$ V, $V_2 = -12$ V, $V_3 = -12$ V



(3) Use nodal analysis to solve for V_o and I_x. $V_o = 2$ V, $I_x = 12$ mA



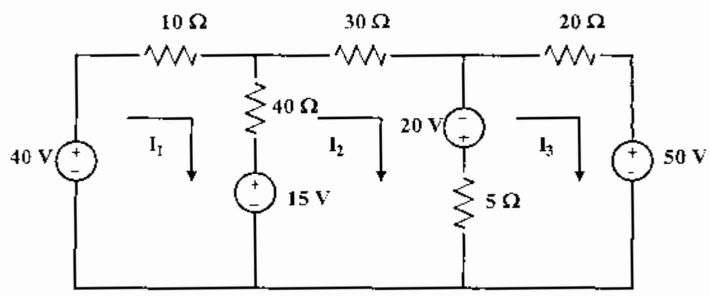
(4) Use nodal analysis to solve for V_o and I_x in the following circuit. $V_o = 9$ V, $I_x = 3$ mA



(5) Work problem 3.15 in the text. Ans v = 8.89 V

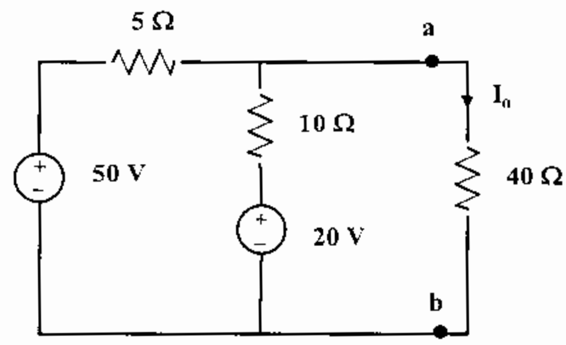
(6) Work problem 3.19 from the text. Ans I = -0.163I, A

(7) You are given the following circuit. Use mesh analysis to find I_1, I_2 and I_3. Write out the equations and then place them in matrix form. $I_1 = 1.28$ A, $I_2 = 0.976$ A, $I_3 = -2.60$ A

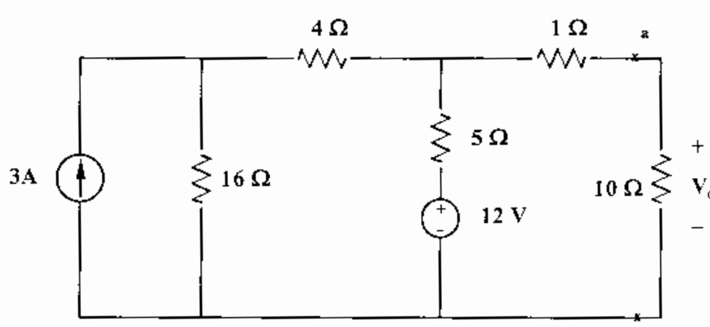


(8) You are given the circuit shown below. Ans: $I_o = 12/13$ A

(a) Use source transformation to determine the current I_o.
(b) Use superposition to determine I_o.
(c) Find the Thevenin equivalent circuit to the left of terminals a-b and then find I_o.



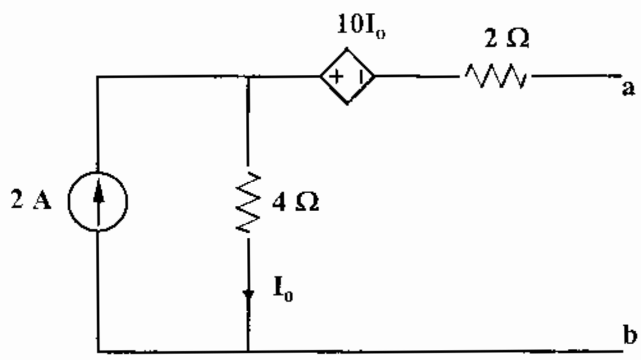
(9) Find the Thevenin equivalent circuit for the network to the left of terminals "a" and "b". Give your values for V_{TH} and R_{TH}. Use the Thevenin circuit to find V_o. Ans: $R_{TH}= 5\ \Omega$, $V_{TH}=19.2$ V, $V_o=12.8$ V.



(10) Replace the 10 Ω resistor of Problem (9) with R_x. Find R_x for maximum power transfer. Give the value of the power delivered to R_x. Ans: $P_{Rx} = 18.43$ W

(Extra) This problem is not required. Work for extra credit (10 points).

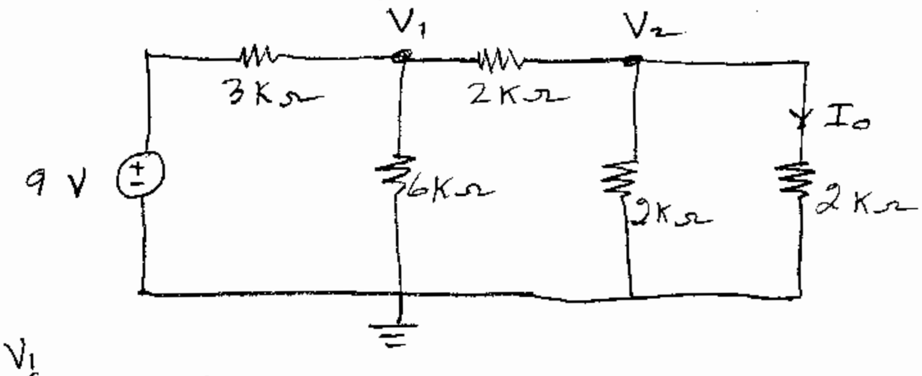
Find the Norton equivalent circuit for the following network. In doing this problem, find the open circuit voltage at terminals a-b. This is called V_{oc}. It is also V_{TH}. Then find I_{sc}, which is I short circuit. This is the current that flows from a to b when a short is placed between a and b. $R_{TH} = V_{oc}/I_{sc}$, The Norton current source is the I_{sc} that you obtained. Ans: $R_{TH} = -4\ \Omega$, $I_N = 3$ A.



On this problem, do not find R_{TH} by applying a source at a-b and calculating $R_{TH} = V_s/1$, where 1 represents a 1 amp source applied at a-b. You may do the problem like this for fun if you wish. But I want you to find R_{TH} from V_{oc}/I_{sc}.

WQ1

(1) Find I_o in the circuit below by using nodal analysis.



At V_1

$$\frac{V_1 - 9}{3K} + \frac{V_1}{6K} + \frac{V_1 - V_2}{2K} = 0, \text{ gives}$$

$$2V_1 - 18 + V_1 + 3V_1 - 3V_2 = 0$$

OR

$$\boxed{6V_1 - 3V_2 = 18} \qquad (1)$$

At V_2

$$\frac{V_2 - V_1}{2K} + \frac{V_2}{2K} + \frac{V_2}{2K} = 0$$

OR

$$\boxed{-V_1 + 3V_2 = 0} \qquad (2)$$

Solving (1) & (2) gives $V_1 = 3.6V$.

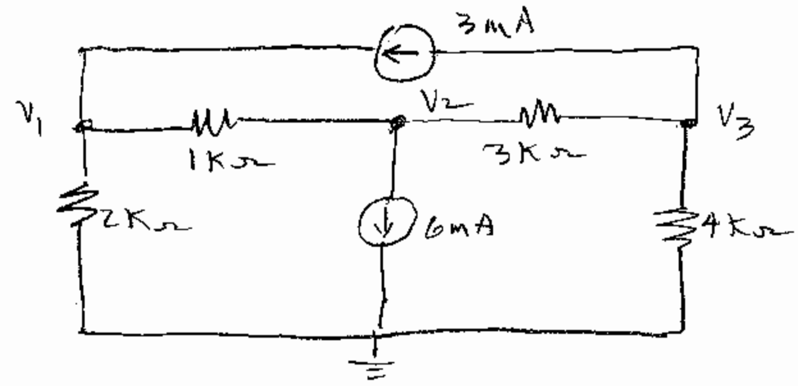
$V_2 = 1.2V$

Now,

$$I_o = \frac{V_2}{2K} = \frac{1.2}{2K}$$

$$\boxed{I_o = 0.6 mA}$$

(2) Develop the nodal equations to solve for V_1, V_2, V_3. Use MATLAB to solve for the actual values



At V_1

$$\frac{V_1}{2K} + \frac{V_1 - V_2}{1K} = 3K^{-1}$$

OR $$V_1 + 2V_1 - 2V_2 = 6$$

$$\boxed{3V_1 - 2V_2 + 0V_3 = 6}$$

At V_2

$$\frac{V_2 - V_1}{1K} + \frac{V_2 - V_3}{3K} = -6K^{-1}$$

$$3V_2 - 3V_1 + V_2 - V_3 = -18$$

$$\boxed{-3V_1 + 4V_2 - V_3 = -18}$$

At V_3

$$\frac{V_3 - V_2}{3K} + \frac{V_3}{4K} = -3K^{-1}$$

OR $$4V_3 - 4V_2 + 3V_3 = -36$$

$$\boxed{0V_1 - 4V_2 + 7V_3 = -36}$$

(2) cont.

We have

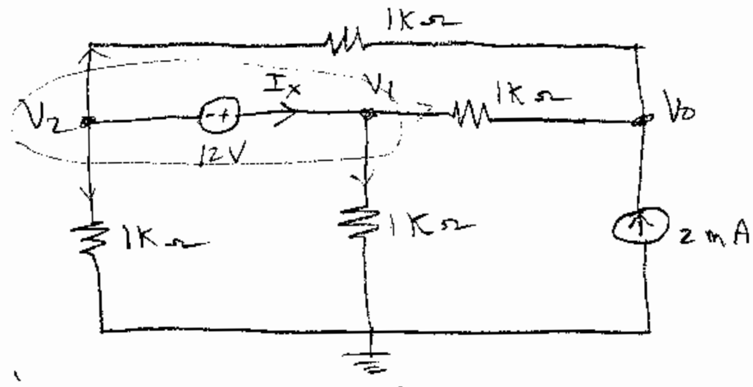
$$\begin{bmatrix} 3 & -2 & 0 \\ -3 & 4 & -1 \\ 0 & -4 & 7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -18 \\ -36 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 0 \\ -3 & 4 & -1 \\ 0 & -4 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ -18 \\ -36 \end{bmatrix}$$

Simple MATLAB program:

```
» R = [3, -2, 0;-3, 4, -1;0, -4, 7];
» E = [6,; -18; -36];
»
» V = inv(R)*E

V =

   -6.0000
  -12.0000
  -12.0000

» % V1 = -6 V, V2 = -12 V,  V3 = -12 V
»
```

(3) Use nodal analysis to solve for V_0 & I_x.



Using a supernode:

At V_2

$$\frac{V_2}{1K} + \frac{V_2 - V_0}{1K} + \frac{V_1}{1K} + \frac{V_1 - V_0}{1K} = 0$$

OR $$2V_1 + 2V_2 - 2V_0 = 0$$

OR $$\boxed{V_1 + V_2 - V_0 = 0}$$

At V_1

$$\frac{V_0 - V_1}{1K} + \frac{V_0 - V_2}{1K} = 2K^{-1}$$

OR $$\boxed{-V_1 - V_2 + 2V_0 = 2}$$

Constraint Equation;

$$V_2 + 12 - V_1 = 0$$

OR $$\boxed{-V_1 + V_2 + 0V_0 = -12}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & -1 & 2 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -12 \end{bmatrix}$$

$V_1 = 7V$, $V_2 = -5V$, $\boxed{V_0 = 2V}$

$$I_x = \frac{V_1}{1K} + \frac{V_1 - V_0}{1K} = (7 + 7 - 2)K\Omega^{-1}A.$$

$$\boxed{I_x = 12\ mA}$$

(3) Alternate method

$$\frac{V_2}{1K} + \frac{V_2 - V_0}{1K} + I_x = 0$$

or $\boxed{-V_0 + 0V_1 + 2V_2 + 1000 I_x = 0}$

$$\frac{V_1}{1K} + \frac{V_1 - V_0}{1K} - I_x = 0$$

$$\boxed{-V_0 + 2V_1 + 0V_2 - 1000 I_x = 0}$$

$$\frac{V_0 - V_1}{1K} + \frac{V_0 - V_2}{1K} = 2K^{-1}$$

$$\boxed{2V_0 - V_1 - V_2 + 0 I_x = 2}$$

constraint Equation

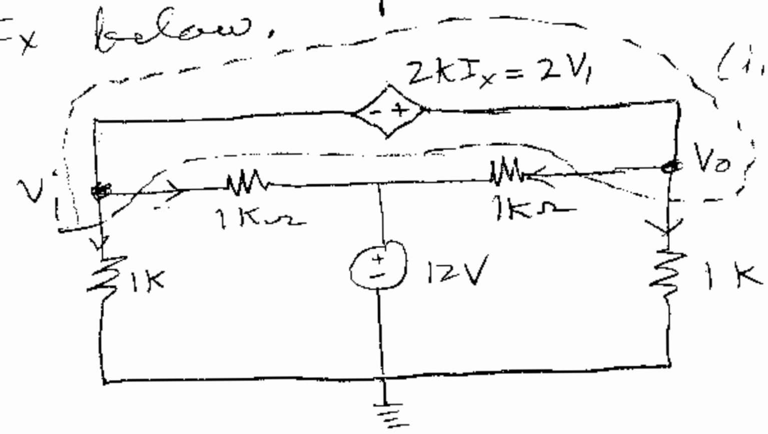
$$V_2 + 12 - V_1 = 0$$

$$\boxed{0V_0 - V_1 + V_2 + 0 I_x = -12}$$

$$\begin{bmatrix} -1 & 0 & 2 & 1000 \\ -1 & 2 & 0 & -1000 \\ 2 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ -12 \end{bmatrix}$$

$\boxed{V_0 = 2V}$, $V_1 = 7.0\,V$, $V_2 = -5V$, $\boxed{I_x = 12\,mA}$

(4) Use nodal analysis to solve for V_o and I_x below.



(i.e. $I_x = \frac{V_1}{1K}$)

$$\frac{V_1}{1K} + \frac{V_1 - 12}{1K} + \frac{V_o - 12}{1K} + \frac{V_o}{1K} = 0$$

OR $\quad 2V_o + 2V_1 = 24$

OR $\quad \boxed{V_o + V_1 = 12}$

Constraint Eq

$$V_1 + 2V_1 - V_o = 0$$

OR $\quad \boxed{-V_o + 3V_1 = 0}$

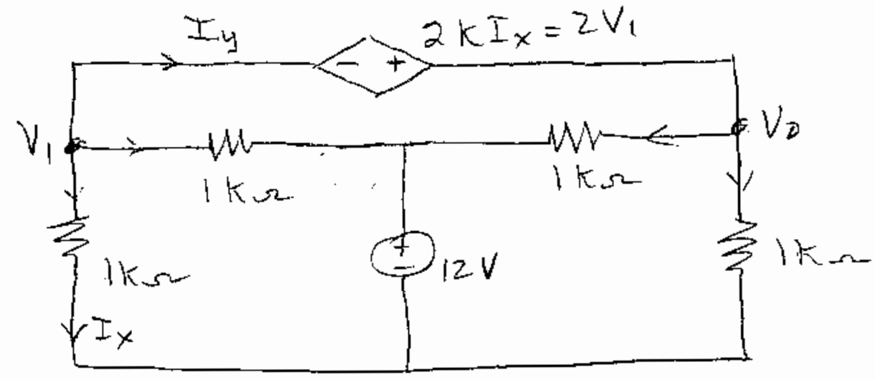
$$\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

$\boxed{V_o = 9V}$, $V_1 = 3V$

$$I_x = \frac{V_1}{1K} = \frac{3}{1K}$$

$$\boxed{I_x = 3mA}$$

(4) Alternate solution



At V_1

$$\frac{V_1}{1K} + \frac{V_1 - 12}{1K} + I_y = 0$$

$$\boxed{0V_o + 2V_1 + 1000 I_y = 12}$$

At V_o

$$\frac{V_o - 12}{1K} + \frac{V_o}{1K} - I_y = 0$$

$$\boxed{2V_o + 0V_1 - 1000 I_y = 12}$$

Constraint

$$V_1 + 2V_1 - V_o = 0$$

$$\boxed{-V_o + 3V_1 + 0 I_y = 0}$$

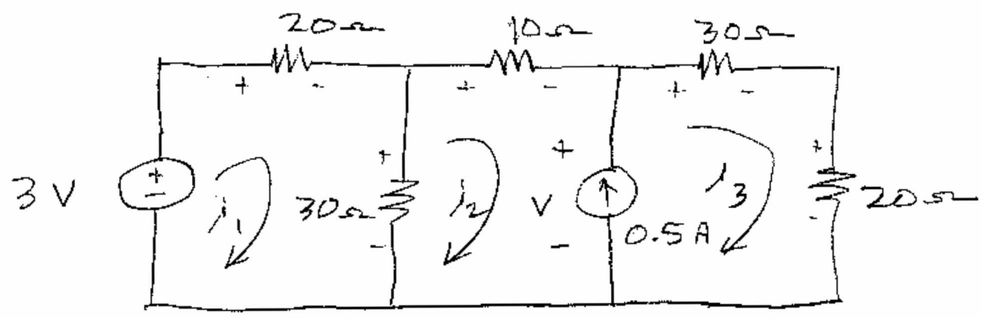
$$\begin{bmatrix} 0 & 2 & 1000 \\ 2 & 0 & -1000 \\ -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \\ I_y \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 0 \end{bmatrix}$$

$V_o = 9V$, $V_1 = 3V$, $I_y = 6mA$

$$\boxed{I_x = \frac{V_1}{1K} = \frac{3}{1000} = 3mA}$$

(5) Problem 3.15 in the text,

Use mesh analysis to solve for the voltage V in the following ckt,



$$-3 + 20 i_1 + 30(i_1 - i_2) = 0$$

$$\boxed{50 i_1 - 30 i_2 + 0 i_3 = 3}$$

$$-30(i_1 - i_2) + 10 i_2 + 30 i_3 + 20 i_3 = 0$$

$$\boxed{-30 i_1 + 40 i_2 + 50 i_3 = 0}$$

$$i_3 - i_2 = 0.5$$

$$\boxed{0 i_1 - i_2 + i_3 = 0.5}$$

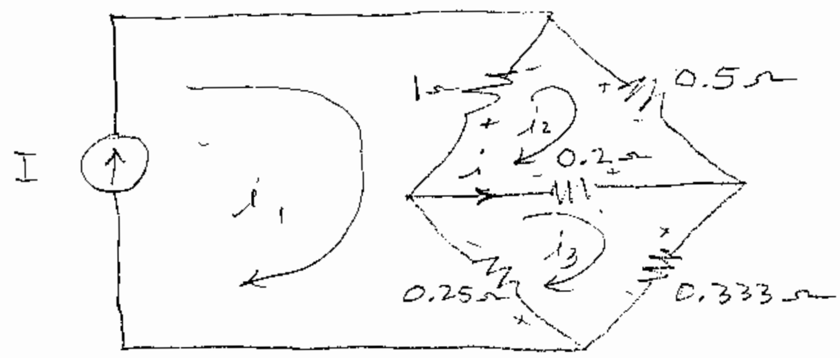
$$\begin{bmatrix} 50 & -30 & 0 \\ -30 & 40 & 50 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0.5 \end{bmatrix}$$

$i_3 = 0.17778$ A

$V = 50 i_3$

$$\boxed{V = 8.89 \text{ V}}$$

(6) problem 3.19 in the text.



Find i using mesh analysis.
Assign i_1, i_2, i_3 as shown.
Note that
$$i_1 = I.$$

Around mesh #2

$$1(i_2 - I) + 0.5 i_2 + 0.2(i_2 - i_3) = 0$$

$$\boxed{1.7 i_2 - 0.2 i_3 = I}$$

Around mesh #3

$$0.25(i_3 - I) - 0.2(i_2 - i_3) + 0.333 i_3 = 0$$

$$-0.2 i_2 + 0.783 i_3 = 0.25 I$$

$$\begin{bmatrix} 1.7 & -0.2 \\ -0.2 & 0.783 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} I \\ 0.25 I \end{bmatrix}$$

(6) continued

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.7 & -0.2 \\ -0.2 & .783 \end{bmatrix}^{-1} \begin{bmatrix} I \\ .25I \end{bmatrix}$$

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.6065 & 0.1549 \\ 0.1549 & 1.3167 \end{bmatrix} \begin{bmatrix} I \\ .25I \end{bmatrix}$$ FROM MATLAB

$$i_2 = 0.6065I + .1549 \times .25I$$

$$i_2 = 0.6452\ I$$

$$i_3 = 0.1549I + 1.3167 \times .25I$$

$$i_3 = 0.4841\ I$$

$$i = i_3 - i_2$$

$$i = 0.4841I - 0.6452I$$

$$i = -0.1611\ I$$

(b) Solution using MATLAB Symbolic program.

Start with

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.7 & -0.2 \\ -0.2 & 0.783 \end{bmatrix}^{-1} \begin{bmatrix} I \\ 0.25I \end{bmatrix}$$

We write

$$A = [1.7, -0.2; -0.2, 0.783];$$

$$B = sym('I; 0.25*I')$$

$$I = \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = symmul(inv(A), B)$$

A program for this is shown below.

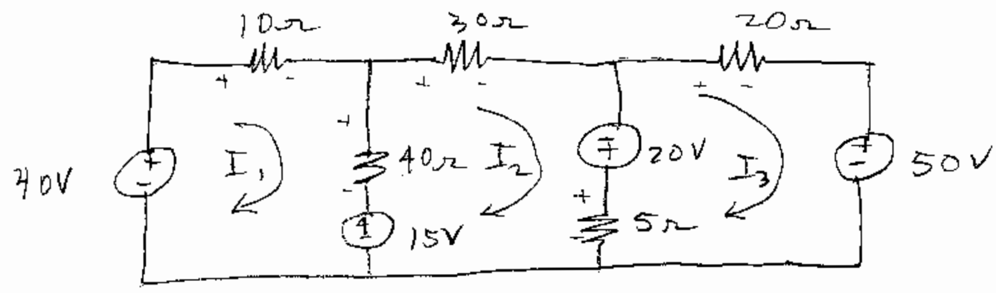
```
% Program for solving a matrix equation by using the
% symbolic tool kit of MATLAB.
% We are given;
% [i2;i3] =   inv[1.7, -.2;-.2,0.783]*[I;0.24*I]
% We solve this below
% Program on office computer, W. Green
% Written September 21, 2005
% Program name:  symul.m

A = [1.7, -0.2; -0.2, 0.783]
B = sym('[I; 0.25*I]')

% I = [i2;i3];

I = symmul(inv(A),B)
```

(6) continued

Program output;

```
>>
>>
>> symul

A =

    1.7000   -0.2000
   -0.2000    0.7830


B =

[       I]
[ 0.25*I]


I =

[ .6451862752691503369220044922934O*I]
[ .4840833397877786383703818449384*I]

>>
```

(7) Use mesh analysis to find I_1, I_2 and I_3.



$$-40 + 10I_1 + (I_1 - I_2)40 + 15 = 0$$

$$\boxed{50I_1 - 40I_2 + 0I_3 = 25}$$

$$-15 - 40(I_1 - I_2) + 30I_2 - 20 + 5(I_2 - I_3) = 0$$

$$\boxed{-40I_1 + 75I_2 - 5I_3 = 35}$$

$$-5(I_2 - I_3) + 20 + 20I_3 + 50 = 0$$

$$\boxed{0I_1 - 5I_2 + 25I_3 = -70}$$

$$\begin{bmatrix} 50 & -40 & 0 \\ -40 & 75 & -5 \\ 0 & -5 & 25 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 35 \\ -70 \end{bmatrix}$$

$I_1 = 1.28A$, $I_2 = 0.976A$, $I_3 = -2.605A$

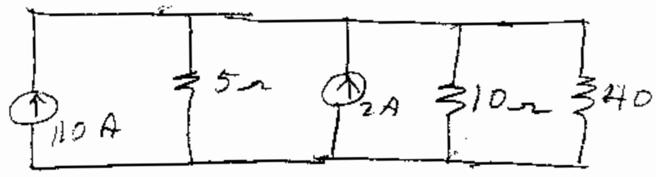
(8) For the circuit below use

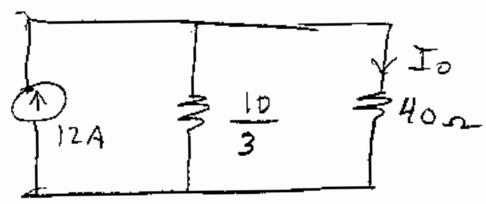
(a) source transformation,

(b) superposition

(c) Thevenin

(a)

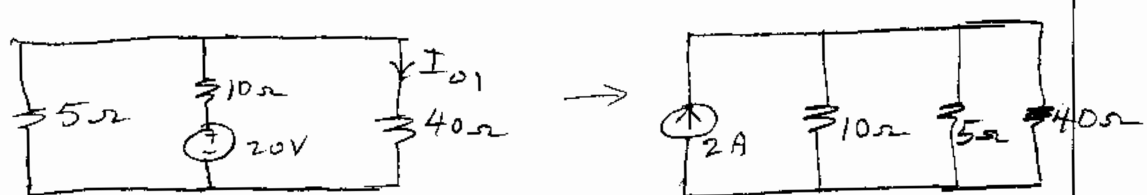




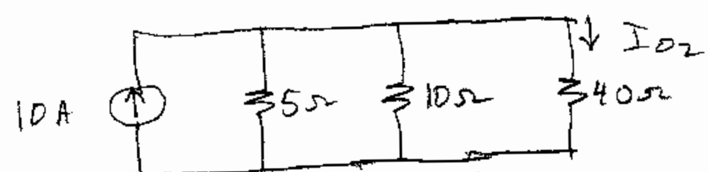
$$I_o = \frac{12 \times \frac{10}{3}}{40 + \frac{10}{3}} = \frac{120}{130}$$

$$I_o = (12/13)\ A = .9231\ A$$

(b) superposition



$$I_{o1} = \frac{2\ (.025)}{.1 + .2 + .025} = 0.1538\ A$$

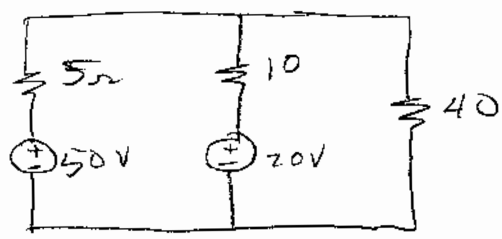


$$I_{o2} = \frac{10\ (.025)}{.2 + .1 + .025} = 0.769$$

$$I_o = I_{o1} + I_{o2} = .1538 + .769 = 0.9228$$

(8) CONTINUED

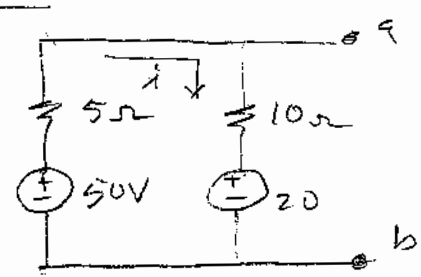
1(c) Thevenin



FOR R_{TH}

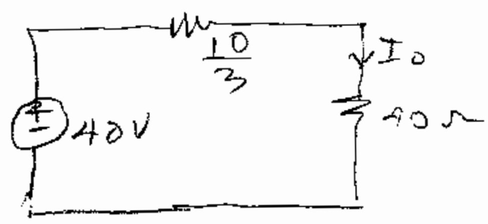
$$5 \parallel 10 = \frac{5 \times 10}{5+10} = \frac{50}{15} = \frac{10}{3}\ \Omega$$

FOR V_{TH}



$$i = \frac{30}{15} = 2\ A$$

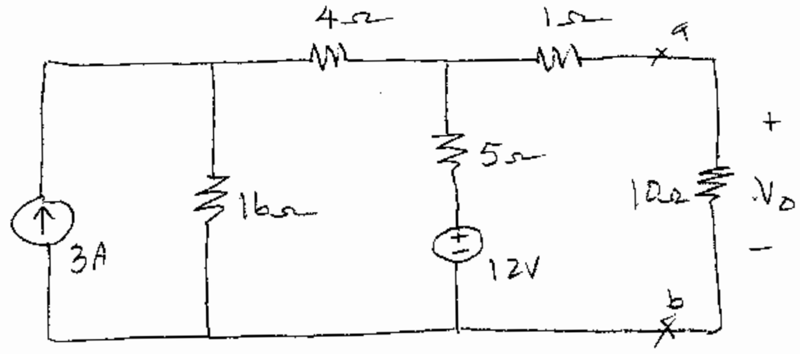
$$V_{ab} = V_{TH} = 20 + 10 \times 2 = 40V$$



$$I_0 = \frac{40}{40 + \frac{10}{3}} = \frac{120}{130} = .9231\ A$$

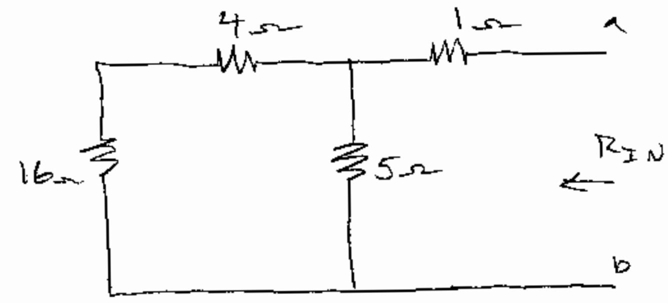
$\boxed{I_0 = 0.9231A}$ check

(9) Find the Thevenin equivalent ckt to the left of terminals a-b.



To Find R_{TH}:

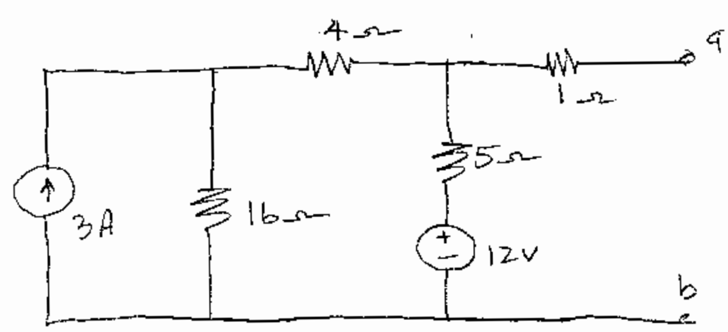
Disable the independent sources, look in a-b determine $R_{IN} = R_{TH}$.



$$5 \parallel 20 = \frac{5 \times 20}{5+20} = 4\,\Omega$$

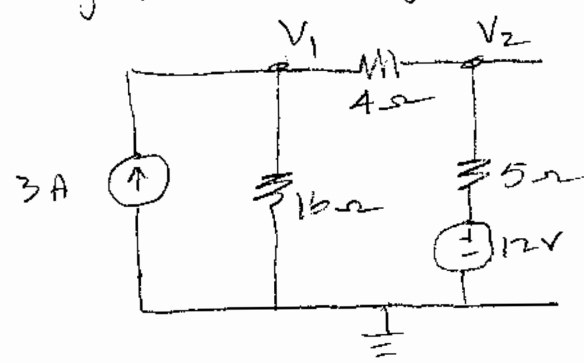
$$R_{IN} = 1 + 4 = \boxed{5\,\Omega = R_{TH}}$$

Now find $V_{TH} = V_{oc}$



(1) continued

Using Nodal analysis:



$$\frac{V_1}{16} + \frac{V_1 - V_2}{4} = 3$$

$$V_1 + 4V_1 - 4V_2 = 48$$

$$\boxed{5V_1 - 4V_2 = 48}$$

$$\frac{V_2 - V_1}{4} + \frac{V_2 - 12}{5} = 0$$

$$5V_2 - 5V_1 + 4V_2 - 48 = 0$$

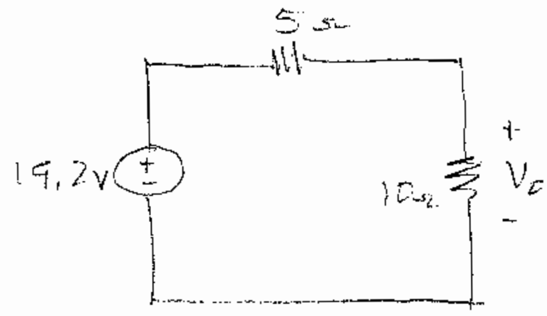
$$\boxed{-5V_1 + 9V_2 = 48}$$

$$\begin{bmatrix} 5 & -4 \\ -5 & 9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 48 \\ 48 \end{bmatrix}$$

$$V_2 = 19.2\ V$$

so $V_{TH} = 19.2\ V$

(9) continued



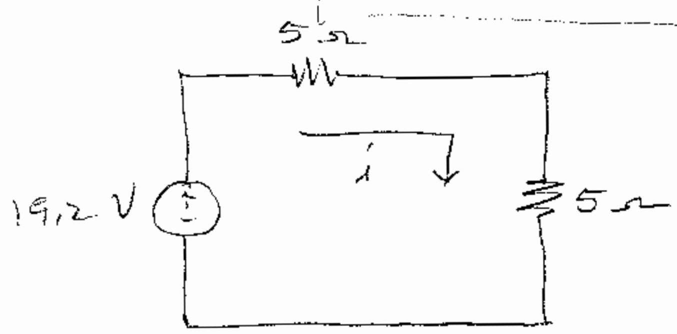
$$V_o = \frac{19.2 \times 10}{10 + 5}$$

$$\boxed{V_o = 12.8\ V}$$

(10) Replace the $10\,\Omega$ load resistor with R_x. Find R_x for maximum power transfer. Give the value of R_x and the power.

We know that

$$\boxed{R_x = R_{TH} = 5\,\Omega}$$



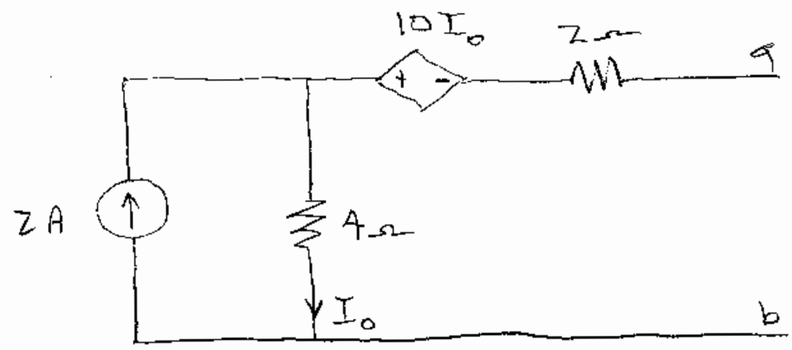
$$P_L = i^2 \times 5 = \left(\frac{19,2}{10}\right)^2 \times 5$$

$$\boxed{P_L = 18.43\ W}$$

Extra

Find the Norton equivalent ckt for the diagram below. You are required to find R_{TH} by

$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$



Find V_{oc}

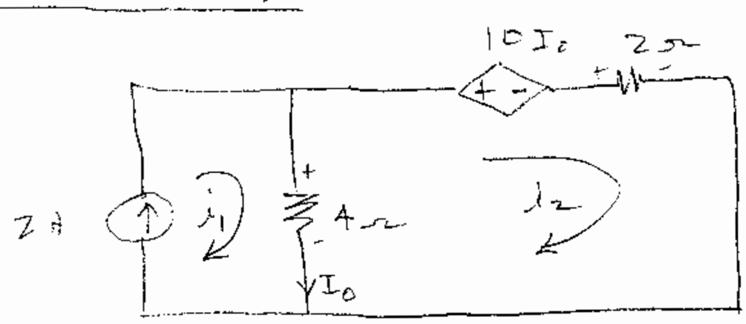
$$V_{ab} = 4 I_o - 10 I_o$$

but

$$I_o = 2 A$$

$$V_{ab} = \boxed{V_{oc} = -12 V}$$

To Find I_{sc}



Extra

$$-4I_o + 10I_o + 2i_2 = 0$$

but $I_o = i_1 - i_2 = 2 - i_2$

$$6(2 - i_2) + 2i_2 = 0$$

$$-4i_2 = -12$$

$$i_2 = 3\,A$$

$$I_{sc} = 3\,A$$

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{-12}{3} = -4\,\Omega$$