ECE 301 Desk Copy
Fall Semester 2005
HW # 4

wlg Due: October 11, 2005

Name mean
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations.** **Be sure to show how you got your answers.** Each problem counts 10 points.

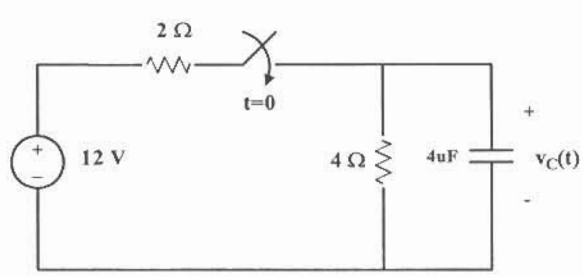
(1) A circuit is described by the following differential equation.

$$4\frac{dv(t)}{dt} + v(t) = 10$$

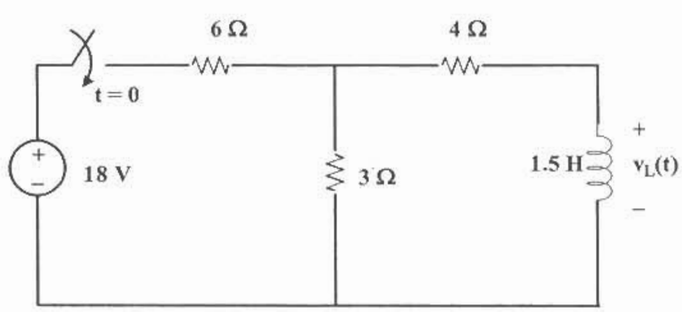
(a) What is the time constant of the circuit?
(b) What is $v(\infty)$?
(c) If v(0) = 2, find v(t) for $t \geq 0$.

Ans: On your own.

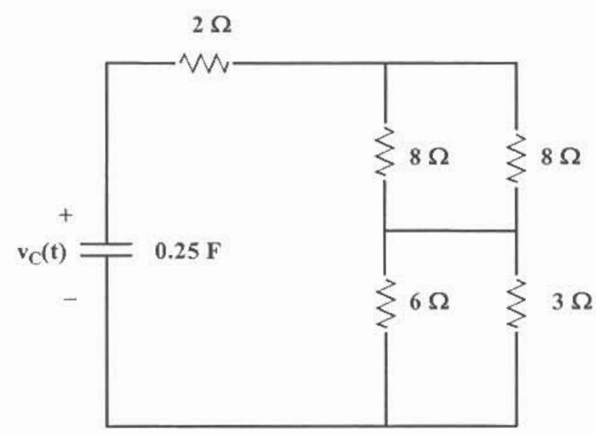
(2) You are given the RC circuit shown below. The switch has been open for a very long time. Find $v_C(t)$ for $t \geq 0$. Use the differential equation technique to solve this problem.
Ans: $8(1 - e^{-0.1875t})$ V



(3) Determine $v_L(t)$ for the circuit below. Use the differential equation technique.
Ans: $6e^{-4t}$ V

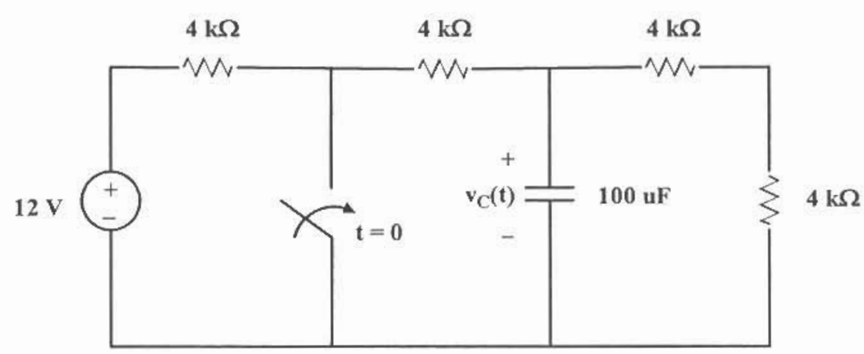


(4) You are given the circuit shown below. v(0) = 20 V. Find v(t) ≥ 0.
Ans: On your own.



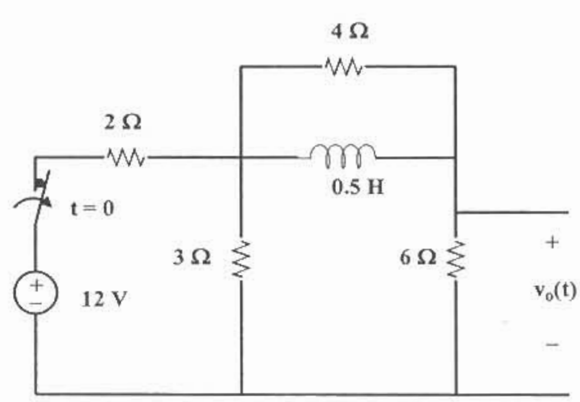
(5) You are given the circuit shown below. Use the differential equation technique to find $v_C(t)$.

Ans: $v_c(t) = 6e^{-3.75t}\ V$



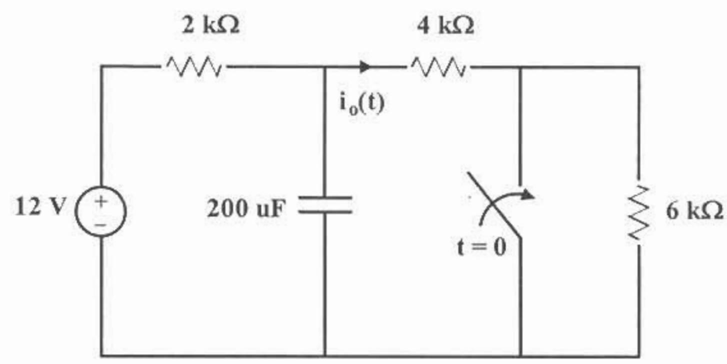
(6) You are given the circuit below. Find $v_o(t)$ for t ≥ 0. Use the step-by-step technique.

Ans: $v_o(t) = 1.85e^{-5.5t}\ V$



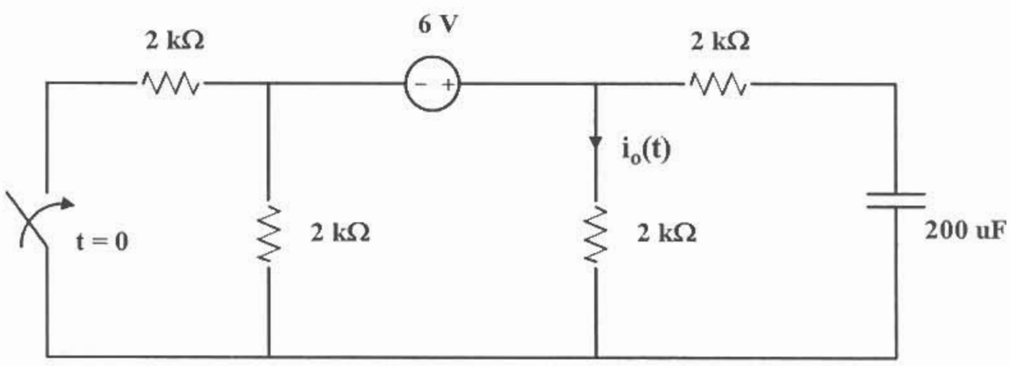
(7) The switch in the following circuit has been open for a very long time. It is closed at t = 0. Use the step-by-step technique to find $i_o(t)$ for $t \geq 0$.

Ans: $i_o(t) = (2 + 0.5e^{-3.75t})\ mA$



(8) The switch in the following circuit has been open for a very long time. It is closed at t = 0. Use the step-by-step technique to find $i_o(t)$ for $t \geq 0$.

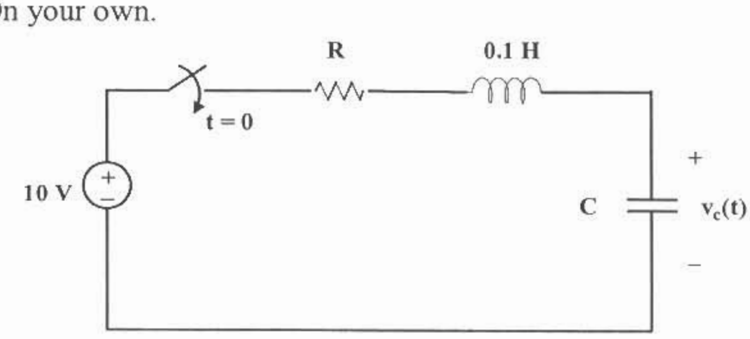
Ans: $i_o(t) = [2 - 0.125e^{-1.875t}]\ mA$



(9) Consider the series RLC circuit shown below. Assume all initial conditions (t<0) are zero. The switch has been open for a very long time and is closed at t = 0.

(a) Develop a 2nd order differential equation that will allow you to solve for $v_c(t)$.
(b) Determine the 2nd order characteristic equation in terms of R, L, and C.
(c) Determine R and C so that $\omega_n = 5$ rad/sec and $\xi = 0.5$.

Ans: On your own.



(10) You are given the parallel RLC circuit below that is driven by a 2 A current source. Prior to t = 0, all initial conditions are zero. At t = 0 the switch is closed.

Ans:

(a) $\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{v(t)}{LC} = \frac{I_S}{LC}$

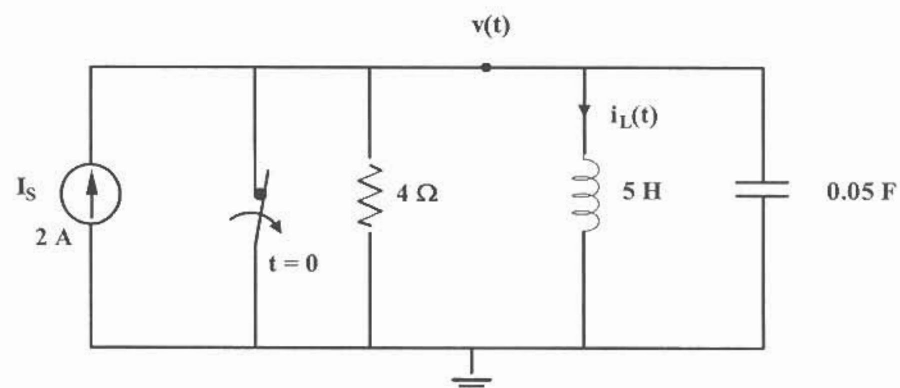
(b) $i_L(t) = [2 - 2.667e^{-t} + 0.667e^{-4t}]\ A$

(c) $\omega_n = 2\,rad/\sec$ $\qquad \zeta = 1.25$

(a) Develop a 2nd order differential equation that when solved will yield the Solution for $i_L(t)$.

(b) Solve the differential equation.

(c) Determine ζ and ω_n for the circuit.



(1) <u>Given</u>

$$4\frac{dv}{dt} + v = 10$$

OR

$$\frac{dv}{dt} + \frac{v}{4} = \frac{10}{4}$$

(a) Time constant; $\tau = 4$ sec.

(b) $v(\infty) = 10$ V

(c) $v(0) = 2$, find $v(t)$

$v(t) = v_p + v_c$

$v_p = K$

$\frac{K}{4} = \frac{10}{4}$

$K = 10$

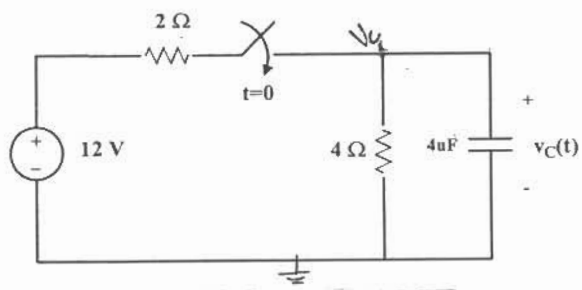
$v(t) = 10 + K_c e^{-0.25t}$

$v(0) = 2 = 10 + K_c$

$K_c = -8$

$$\boxed{v(t) = 10 - 8e^{-0.25t}, \text{ V}}$$

(2) Given the circuit below, Find $V_c(t)$ using the d.e. technique.



$$\frac{V_c-12}{2} + \frac{V_c}{4} + 4\times10^{-6}\frac{dV_c}{dt} = 0$$

$$2V_c - 24 + V_c + 16\times10^{-6}\frac{dV_c}{dt} = 0$$

$$16\times10^{-6}\frac{dV_c}{dt} + 3V = 24$$

$$\boxed{\frac{dV_c}{dt} + \frac{3\times10^{6}}{16}t = 1.5\times10^{6}}$$

$$V_c = V_p + V_t$$

$$V_p = 8$$

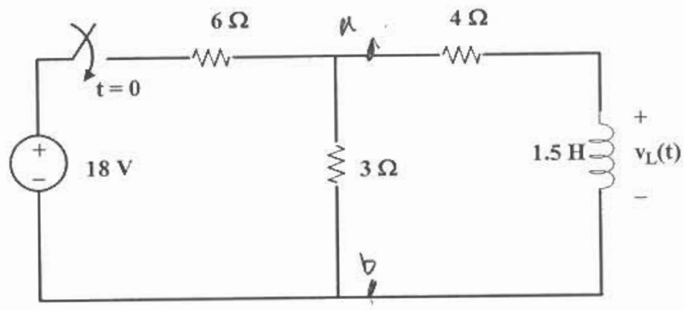
$$V_c = 8 + K_t e^{-\frac{3}{16}\times10^{6}t}$$

$$V_c(0^-) = 0 = V_c(0^+)$$

$$0 = 8 + K_t$$

$$\boxed{V_c(t) = 8 - 8e^{-\frac{3}{16}\times10^{6}t}\ \text{V}}$$

(3) Find $v_L(t)$. Use the differential equation technique.

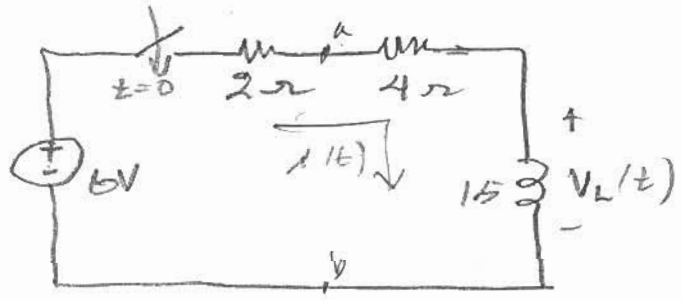


$$v_L(0^+) = \frac{18 \times 3}{9} = 6\,V$$

Using Thevenin's theorem

$$V_{TH} = \frac{18 \times 3}{9} = 6\,V \qquad (3.1)$$

$$R_{TH} = \frac{6 \times 3}{9} = 2\,\Omega$$



$$6\,i(t) + v_L(t) = 6$$

but $i(t) = \frac{1}{L}\int_0^t v\,dt + \cancel{i(0)}^{\,0}$

$$\frac{6}{1.5}\int v_L\,dt + v_L(t) = 6 \qquad (3,2)$$

(3) cont

Taking the derivative of 3.2 gives

$$4V_L(t) + \frac{dV_L}{dt} = 0$$

$$\frac{dV_L}{dt} + 4V_L = 0 \qquad \text{with } V_L(0^+) = 6\ V$$

$$V_L = Ke^{-4t}$$

$$K = V_L(0^+) = 6V$$

$$V_L = 6e^{-4t}$$

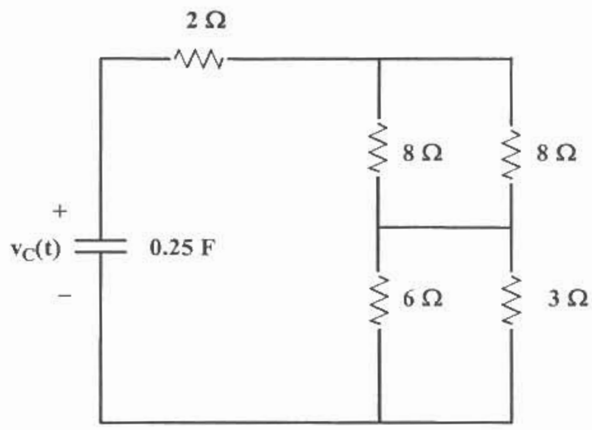
$$V_L(0) = 6, \quad V_L(\infty) = 0$$

$$V_L(t) = V_L(\infty) + (V_L(0) - V_L(\infty))e^{-\frac{t}{\tau}}$$

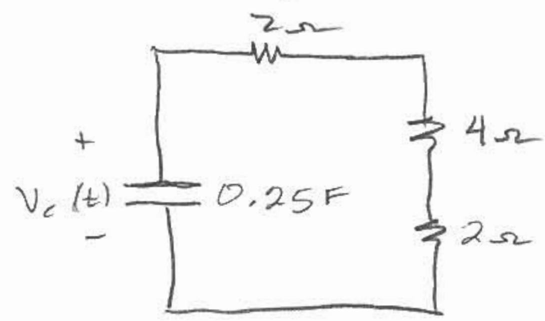
check

$$V_L(t) = 6e^{-4t}, V \qquad \text{check}$$

(4) $V(0) = 20$ V. Find $V_c(t) \geq 0$



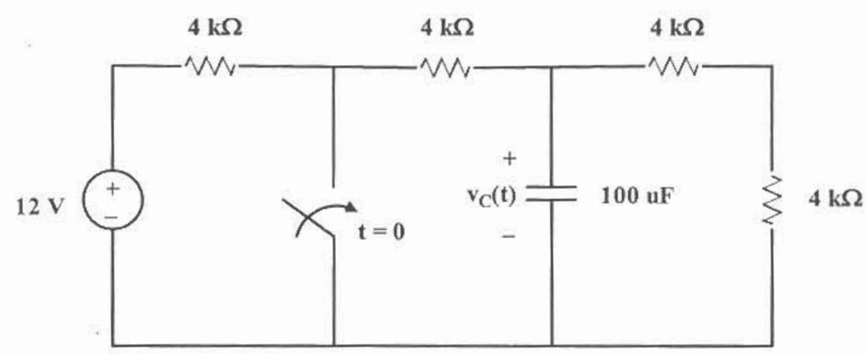
Reduces to;



$\tau = RC = 8 \times 0.25 = 2$ sec

$$V_c(t) = 20e^{-0.5t} \text{ V} \qquad t \geq 0$$

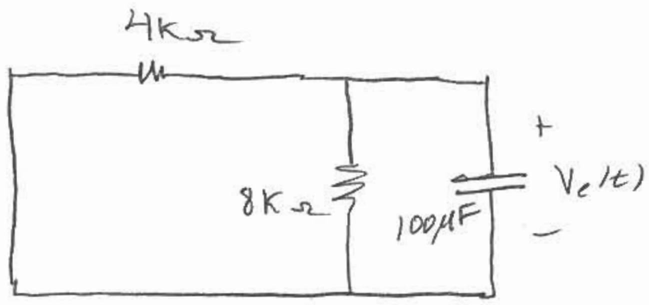
(5) Use D.E. technique. Find $V_c(t)$, $t \geq 0$



$t < 0$

$$V_c(0^-) = \frac{12 \times 8K}{8K + 8K} = 6\,V$$

$t > 0$



$$R_{eq} = \frac{(8K)(4K)}{8K + 4K} = \frac{8}{3}K\Omega$$

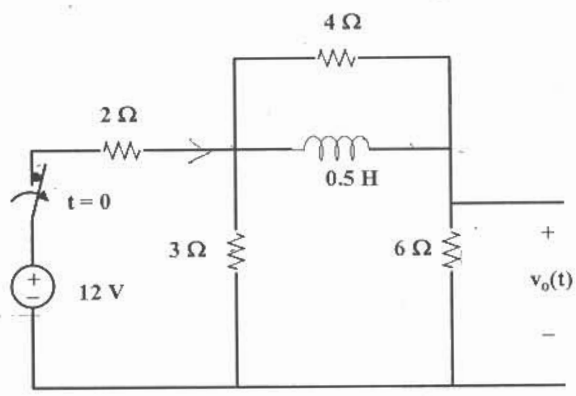
$$\tau = (R_{eq})(C) = \frac{8}{3} \times 10^3 \times 0.1 \times 10^{-3} = 3.75$$

Known solution,

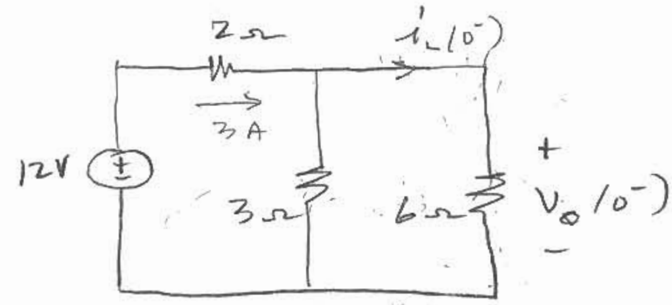
$$V_c(t) = V(0)\, e^{-\frac{t}{\tau}}$$

$$\boxed{V_c(t) = 6\, e^{-3.75t}\ V} \qquad t \geq 0$$

⑥ FIND $V_o(t)$, $t > 0$. Use step-by-step technique.



$t < 0$



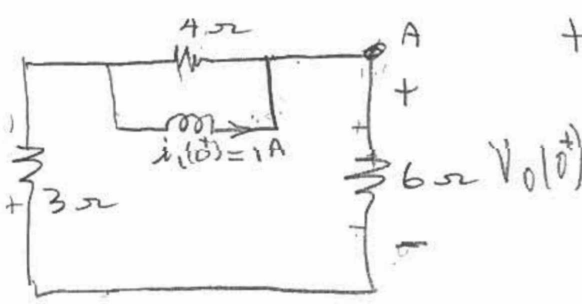
$6 \parallel 3 = 2\,\Omega$

$V_o(0^-) = 6\,V$

$i_L(0^-) = \frac{6}{6} = 1\,A = i_L(0^+)$

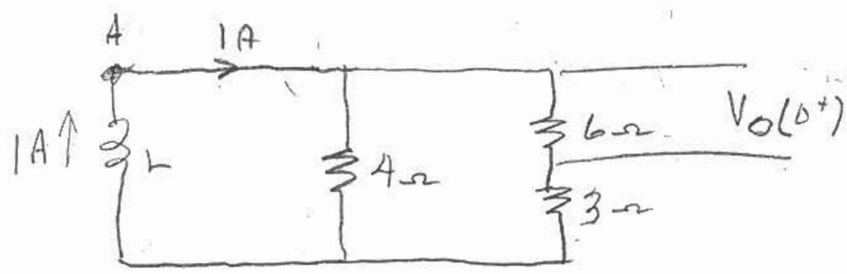
Note: $V_C(0^+)$ not generally equal to $V_C(0^-)$

$t > 0$



NEED $V_L(0^+)$

To find $V_L(0^+)$ consider;



(b)

Using current splitting:

$$V_o(0^+) = \left(\frac{1 \times 4}{4+6+3}\right) \times 6 = \frac{24}{13} = 1.85V$$

step-by-step

$$V_o(t) = V_o(\infty) + (V_o(0^+) - V(\infty))e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R_{eq}}$$

$$R_{eq} = 4 \parallel 9 = \frac{36}{13}$$

$$\tau = \frac{L}{R} = \frac{.5}{36} \times 13 = \frac{13}{72}$$

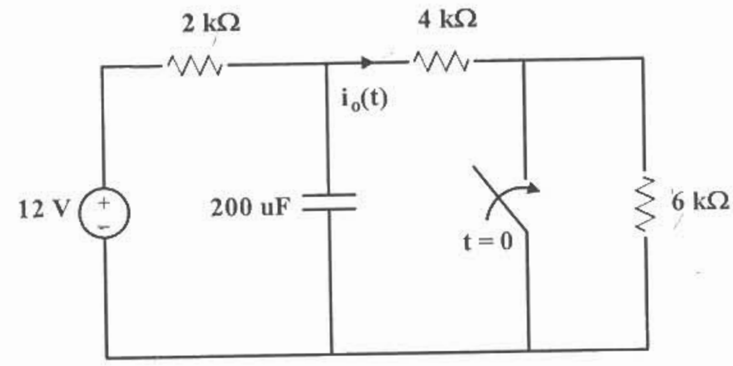
$$\frac{1}{\tau} = \frac{72}{13} = 5.5$$

$$V_o(\infty) = 0$$

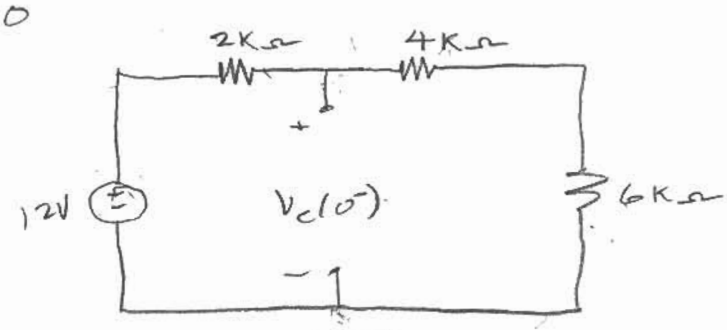
$$V_o(t) = V_o(0^+)e^{-\frac{t}{\tau}}$$

$$V_o(t) = 1.85e^{-5.5t} V, \quad t \geq 0$$

(7) Use step-by-step to find $i_o(t)$

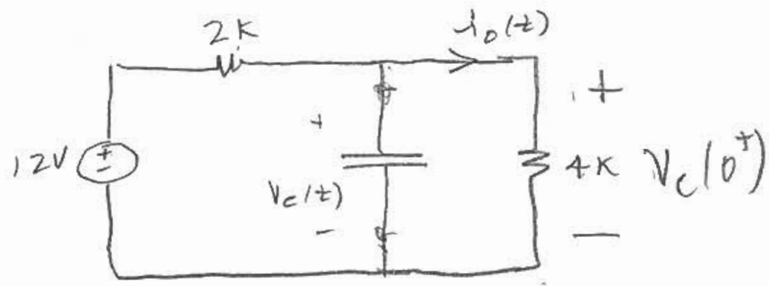


$t < 0$



$$\boxed{V_c(0^-) = V_c(0^+)} = \frac{12 \times 10K}{10K + 2K} = \underline{10\,V}$$

$t > 0$



$W_c = \frac{1}{2} C V^2$

$$\underline{i_o(0^+)} = \frac{V_c(0^+)}{4K} = \frac{10}{4K} = \underline{2.5\,mA}$$

$$\underline{i_o(\infty)} = \frac{12}{6K} = \underline{2\,mA}$$

$$R_q = \frac{(4K) \times (2K)}{4K + 2K} = \frac{8K}{6} = \frac{4}{3}K$$

(7) cont.

$$R_{eq}C = \left(\frac{4}{3}\,K\right) \times 200\times10^{-6}\ sec$$

$$= \frac{800}{3}\times10^{-3} = 0.267\ sec$$

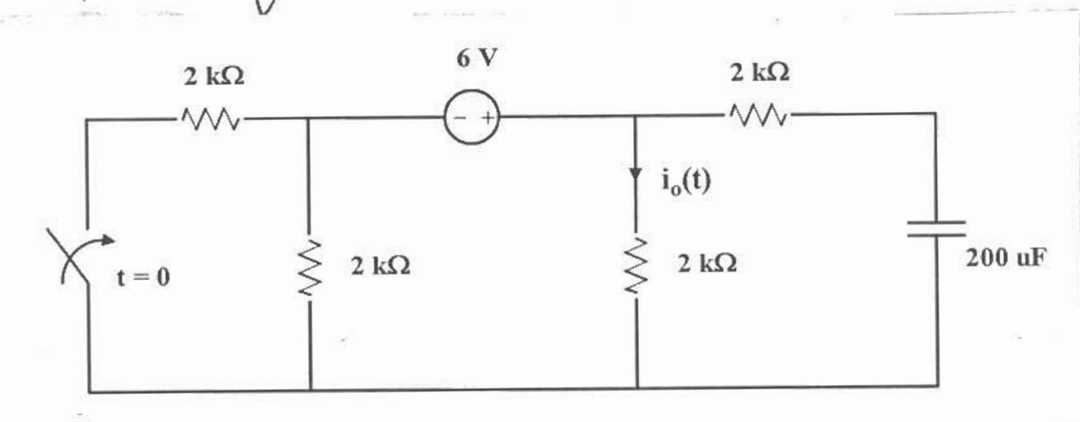
$$\frac{1}{\tau} = 3.75\ sec^{-1}$$

$$i_o(t) = i_o(\infty) + \left(i_o(0^+) - i_o(\infty)\right)e^{-\frac{t}{\tau}}$$

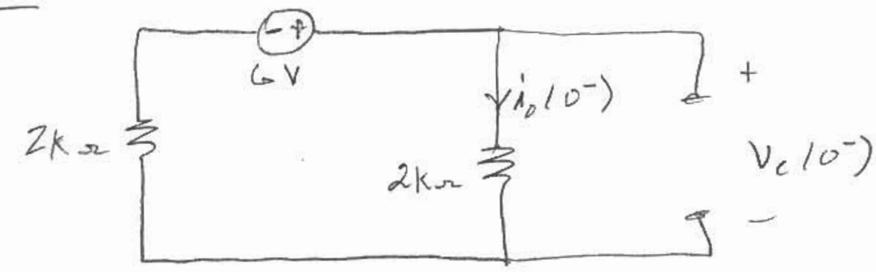
$$i_o(t) = 2.5\ mA + (2.5 - 2.0\ mA)\,e^{-3.75t}$$

$$i_o(t) = \boxed{\left[2.5 + 0.5\,e^{-3.75t}\right] mA}$$

(8) Find $i_o(t)$ using step-by-step technique



$\underline{t<0}$

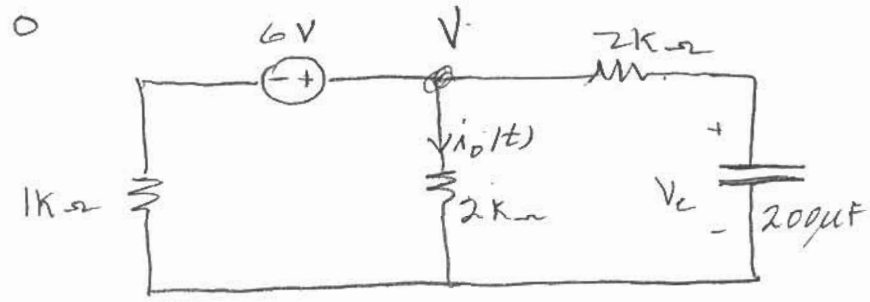


$i_o(0^-) = \frac{6}{4}$ mA

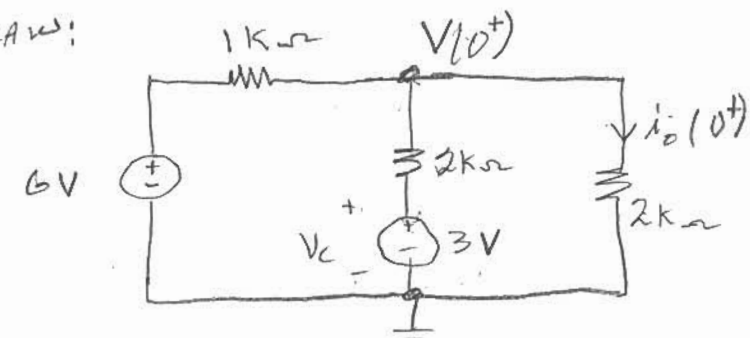
$i_o(0^-) = 1.5$ mA

$$V_C(0^-) = \frac{6 \times 2k}{2k+2k} = 3V = V_c(0^+)$$

$t>0$



Re draw:



(8) cont.

Nodal analysis

$$\frac{V-6}{1K} + \frac{V-3}{2K} + \frac{V}{2K} = 0$$

$$2V - 12 + V - 3 + V = 0$$

$$4V = 15$$

$$V = \frac{15}{4}$$

$$i_o(0^+) = \frac{(15/4)}{2K} = 1.875 \text{ mA}$$

$$i_o(\infty) = \frac{6}{3K} = 2 \text{ mA}$$

$$R_{eq} = 2K + \frac{2K \times 1K}{2K + 1K} = \left(2 + \frac{2}{3}\right)K = \left(\frac{8}{3}\right)K\Omega$$

$$R_{eq}C = \frac{8}{3} \times 10^{3} \times 0.2 \times 10^{-3} = 0.533 \text{ sec}$$

$$\frac{1}{R_{eq}C} = 1.875 \text{ sec}^{-1}$$

$$i_o(t) = i_o(\infty) + [i_o(0^+) - i_o(\infty)]\, e^{-\frac{t}{\tau}}$$

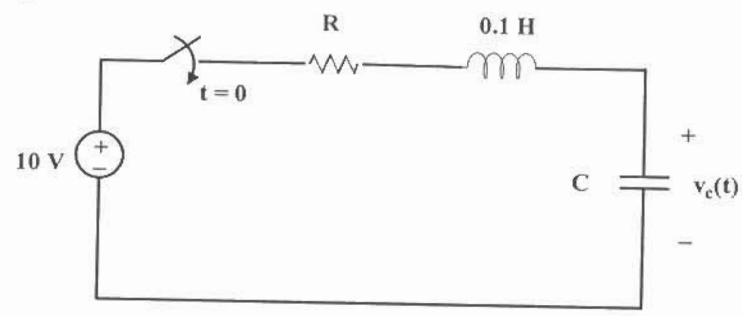
$$i_o(t) = 2 + [1.875 - 2]\, e^{-1.875t} \text{ mA}$$

$$\boxed{i_o(t) = \left[2 - 0.125\, e^{-1.875t}\right] \text{mA}}$$

(9) Consider the series RLC circuit shown below. Assume all initial conditions (t<0) are zero. The switch has been open for a very long time and is closed at t = 0.

(a) Develop a 2[nd] order differential equation that will allow you to solve for $v_c(t)$.
(b) Determine the 2[nd] order characteristic equation in terms of R, L, and C.
(c) Determine R and C so that $\omega_n = 5$ rad/sec and $\xi = 0.5$.

Ans: On your own.



(a)

$$Ri + L\frac{di}{dt} + V_c(t) = V_s \qquad (1)$$

but

$$i(t) = C\frac{dV_c}{dt} \qquad (2)$$

substitute (2) into (1)

$$\boxed{\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c(t)}{LC} = \frac{V_s}{LC}}$$

(b) By inspection of the above,

$$\boxed{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0} \qquad (3)$$

(c) $\omega_n = 5 \Rightarrow \omega_n^2 = 25$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \qquad (4)$$

(4)

Comparing coefficients of Eq (3) and (4) gives

$$\omega_n^2 = \frac{1}{LC}$$

so

$$C = \frac{1}{\omega_n^2 L} = \frac{1}{25 \times (0.1)} = 0.4\,F$$

$$\boxed{C = 0.4\,F}$$

Next,

$$2\zeta\omega_n = \frac{R}{L}$$

so,

$$R = 2\zeta\omega_n L = 2 \times 0.5 \times 5 \times 0.1$$

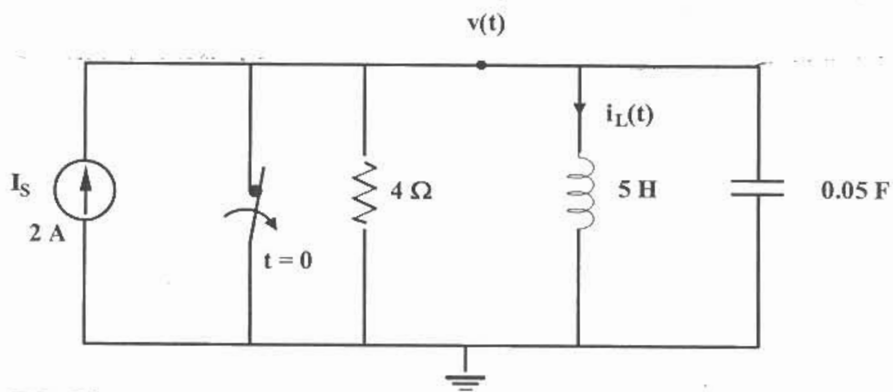
$$\boxed{R = 0.5\,\Omega}$$

(10) FOR the following circuit;

(a) Develop a 2^{nd} order differential equation that when solved will yield the Solution for $i_L(t)$.

(b) Solve the differential equation.

(c) Determine ζ and ω_n for the circuit.



(a) $$\frac{v(t)}{R} + C\frac{dv}{dt} + i_L(t) = I_s \qquad (1)$$

but

$$v(t) = L\frac{di_L}{dt} \; ; \quad \text{substitute} \qquad (2)$$

$$\frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} + i_L(t) = I_s \qquad (3)$$

$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I_s}{LC}} \qquad (4)$$

(10) cont

(b) substitute in values;

$$\frac{d^2 i_L}{dt^2} + \frac{1}{4\times.05}\frac{di_L}{dt} + \frac{1}{5\times.05} = \frac{2}{5\times.05}$$

$$\frac{d^2 i_L}{dt^2} + 5\frac{di_L}{dt} + 4 i_L = 8$$

characteristic equation is

$$s^2 + 5s + 4 = 0$$

$$(s+4)(s+1) = 0$$

$i_L(t) = i_p + i_c$

$i_p = K$

$$4K = 8$$

$$\boxed{K = 2}$$

$$i_L(t) = 2 + K_1 e^{-t} + K_2 e^{-4t} \qquad (5)$$

$i_L(0^+) = 0$ (current cannot change instantaneously through an inductor)

Go to Eq. 2;

$$V(0^-) = 0$$

But the voltage across the capacitor cannot change instantaneously so

$$V(0^+) = 0$$

(10) Then from Eq (2)

$$0 = L\,\frac{di_L(0^+)}{dt}$$

$$\frac{di_L(0^+)}{dt} = 0$$

We have the two I.C.'s

$$i_L(0^+) = 0$$

$$\frac{di_L(0^+)}{dt} = 0$$

using Eq. 5

$$i_L(t) = 2 + K_1 e^{-t} + K_2 e^{-4t}$$

$$\boxed{0 = 2 + K_1 + K_2}$$

$$\frac{di_L}{dt} = -K_1 - 4K_2 = 0$$

$$K_1 + K_2 = -2$$
$$K_1 + 4K_2 = 0$$

Giving $K_1 = -2.67$, $K_2 = 0.667$

$$\boxed{i_L(t) = 2 - 2.67e^{-t} + 0.667e^{-4t}}$$

(10)

(c) The Char. Eq. is

$$s^2 + 5s + 4 = 0$$

compare to

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

so, $\omega_n^2 = 4 \longrightarrow \boxed{\omega_n = 2}$

$$2\zeta\omega_n = 2\zeta 2 = 5$$

$$\boxed{\zeta = \frac{5}{4} = 1.25}$$