ECE 301
Fall Semester, 2005
HW #5

Desk Copy

wlg
Due: Nov 8

Name green
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations.** **Be sure to show how you got your answers.** Each problem counts 10 points.

(1) You are given the op-amp circuit of Figure 1. It is desired that V_o be the average of the input signals. That is,

$$V_o = \frac{-[V_1 + V_2 + V_3]}{3}$$

Assume $R_1 = R_2 = R_3 = 12\ k\Omega$. Find the value of R_{FB}. On your own.

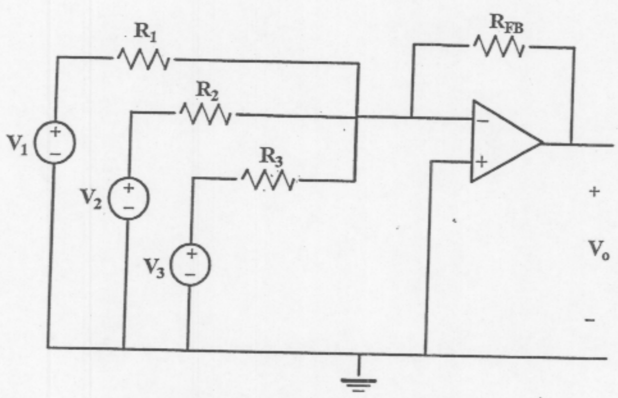


Figure 1: Circuit for problem 1.

(2) You are given the following op amp circuit of Figure 2. Solve for V_o in terms of V_{in}.
Ans: $V_o = -8V_{in}$.

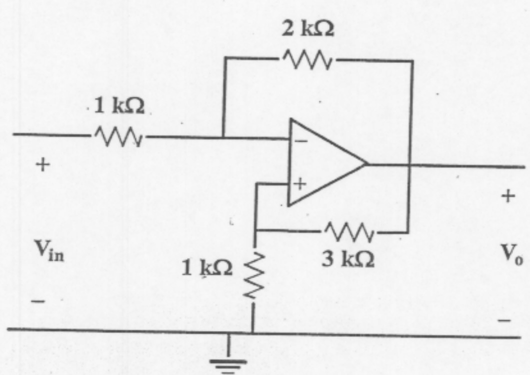


Figure 2: Circuit for problem 2.

(3) You are given the op amp circuit of Figure 3. Solve for V_o in terms of V_{in}. Ans: $V_o = 16.8V_{in}$.

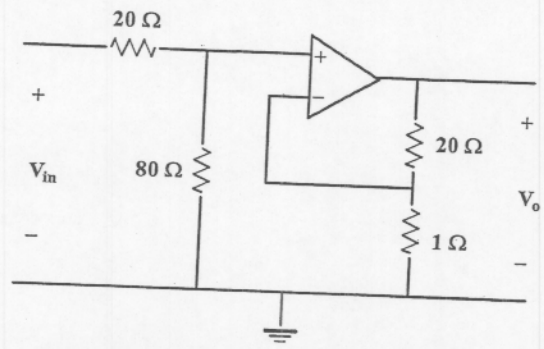


Figure 3: Op amp circuit for problem 3.

(4) You are given the op amp circuit of Figure 4. Solve for V_o in terms of V_{in}. Ans: $V_o = V_{in}/3$.

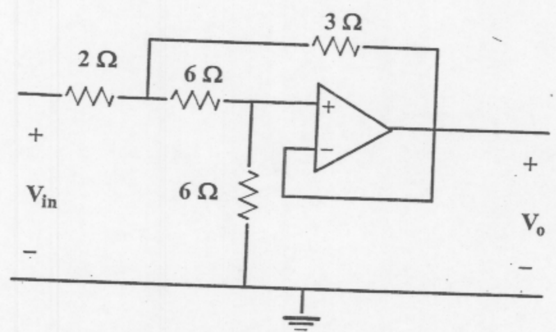


Figure 4: Op amp circuit for problem 4.

(5) You are given the op amp configuration of Figure 5 that solves a first order differential equation. Determine the differential equation.

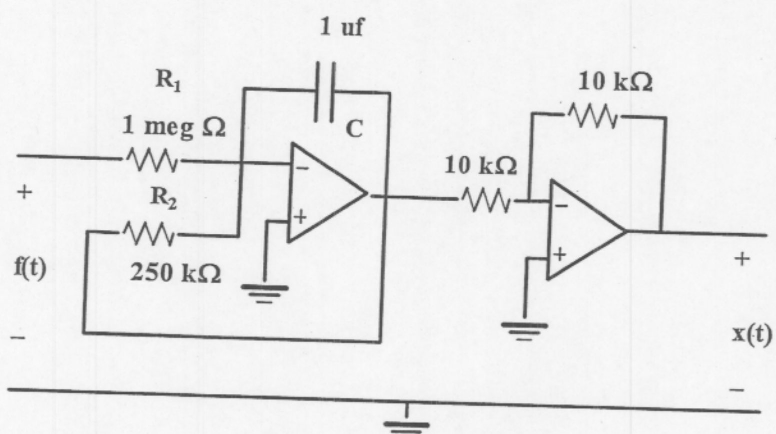


Figure 5: Op amp circuit for problem 5.

(6) The circuit of problem (5) can be used as a low pass filter. If $R_1 = R_2 = 1\ k\Omega$ and $C = 0.1\ \mu F$ give the filter transfer function. What is the low frequency cut-off point in rad/sec?

(7) Use the freqs function of MATLAB to plot the frequency response of the above filter.

(8) You are given the op amp circuit of Figure 8. This configuration can be used for a high-pass filter.

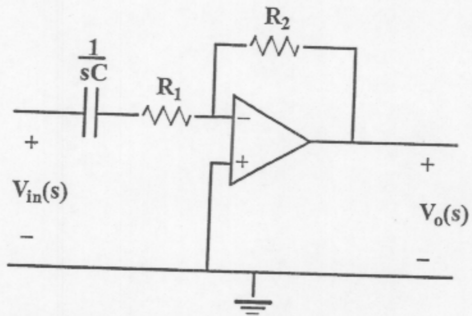


Figure 8: Op amp circuit for problem 8.

Show that the transfer function between the output and input of the configuration is,

$$\frac{V_o(s)}{V_{in}(s)} = \frac{-sR_2C_1}{(1+sR_1C_1)}$$

(a) Assume $C_1 = 1\ \mu F$. Determine the value of R_1 so that the cut-off frequency of the filter is $\omega = 2000$ rad/sec.

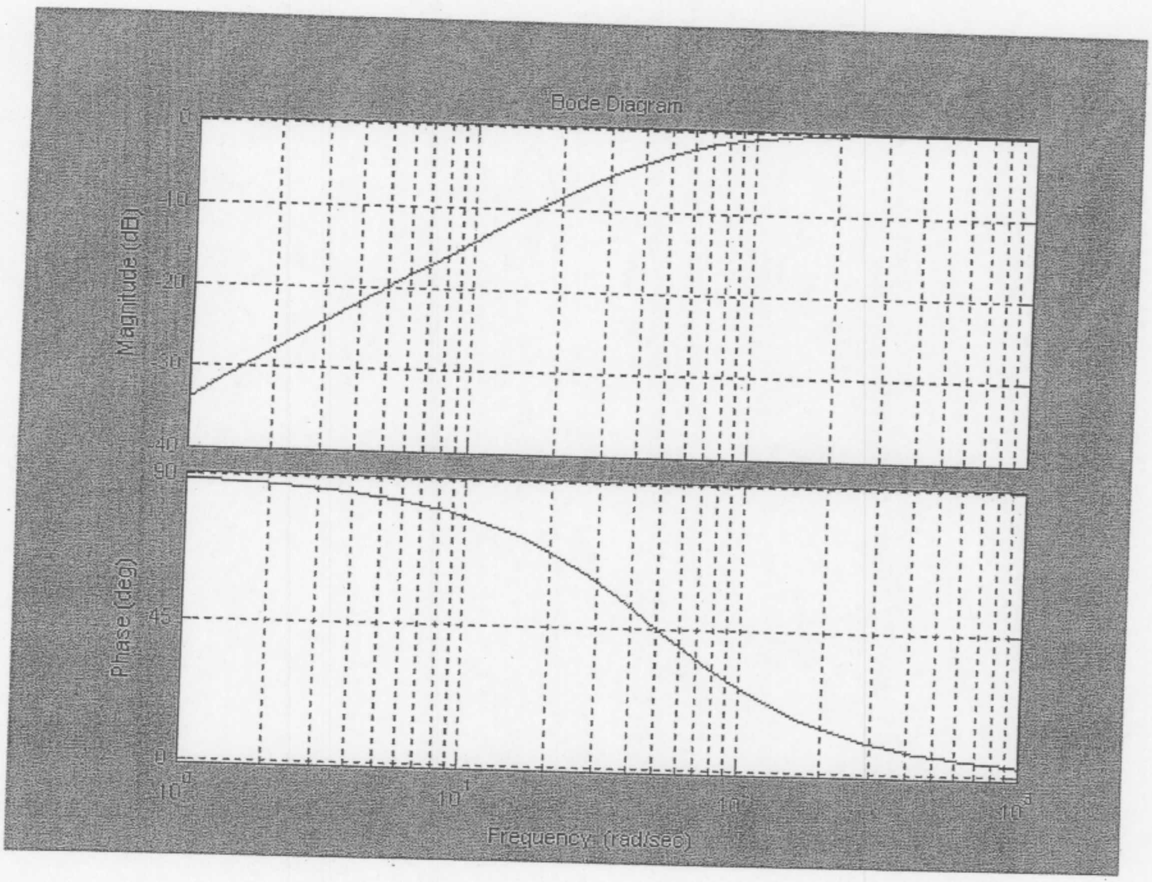
(b) Determine the value of R_2 so that the gain of the filter as ω approaches infinity is 1.

(c) Use the bode statement of MATLAB to find the bode plot for the filter.

Programs are given on the following pages to illustrate how to obtain bode plots and freqs (linear frequency response) plots.

```
% program to show  students how to use the bode statement to obtain a bode plot.
% This program is written for G(s) = 0.02s/(1+0.02s)
% This is a frist order high pass filter, On office computer: October 25,2005, wlg,
% Program name is:  simple_bode2.m

num = [0.02 0];
den = [0.02 1];
bode(num,den)                % This is the bode statement
grid
```

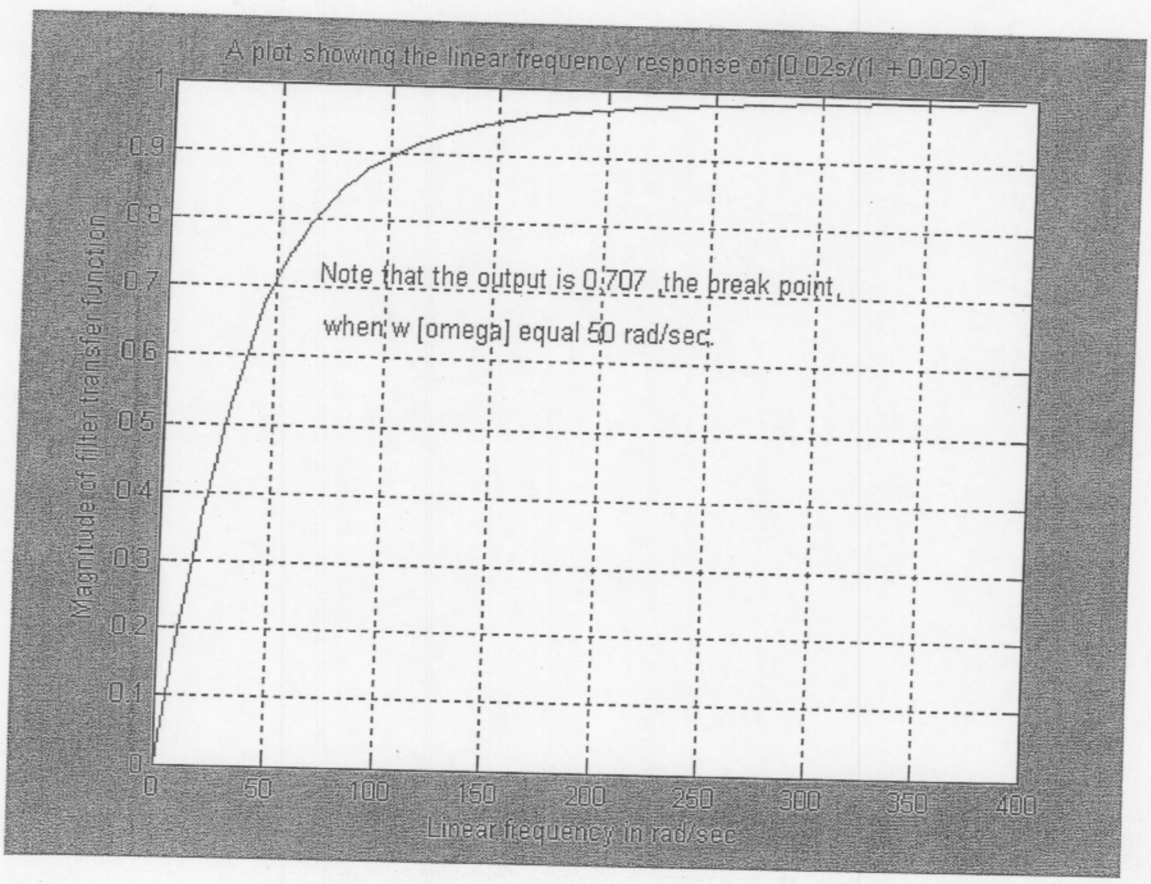


The Bode Plot

```
% program to show  students how to use the bode statement to obtain a linear frequency response.
% This program is written for G(s) = 0.02s/(1+0.02s)
% This is a frist order high pass filter.
% On office computer: October 25,2005, wlg. Program name is:  simple_linear_freq.m

num = [0.02 0];
den = [0.02 1];
w = 0.1:5:400;

G = freqs(num,den,w);   % This is the freqs statement
magG = abs(G);
plot(w,magG)
ylabel('Magnitude of filter transfer function')
xlabel('Linear frequency in rad/sec')
title('A plot showing the linear frequency response of [0.02s/(1 + 0.02s)]')
gtext('Note that the output is 0.707 ,the break point,')
gtext('when w [omega] equal 50 rad/sec.')
grid
```



The Linear Frequency Response

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HW #5
Fall, 2005

(1) You are given the op-amp circuit of Figure 1. It is desired that V_o be the average of the input signals. That is,

$$V_o = \frac{-[V_1 + V_2 + V_3]}{3}$$

Assume $R_1 = R_2 = R_3 = 12\ k\Omega$. Find the value of R_{FB}. On your own.

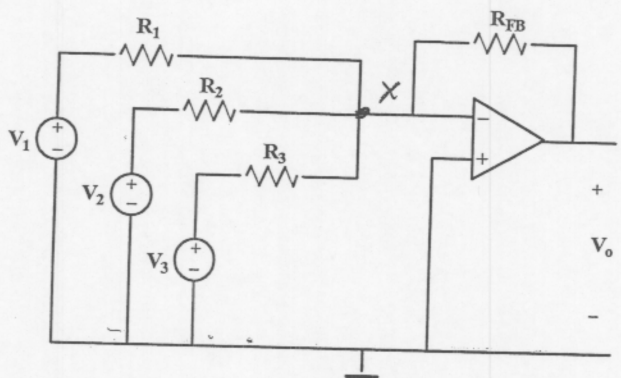


Figure 1: Circuit for problem 1.

At point X we write

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_o}{R_{fb}} \qquad R_1 = R_2 = R_3 = 12K$$

OR

$$V_o = -[V_1 + V_2 + V_3] \times \frac{R_{fb}}{1{,}2K}$$

We want

$$V_o = -\frac{[V_1 + V_2 + V_3)}{3}$$

So we make $\boxed{R_{fb} = 4K\Omega}$ ANS

(2) You are given the following op amp circuit of Figure 2. Solve for V_o in terms of V_{in}. Ans: $V_o = -8V_{in}$.

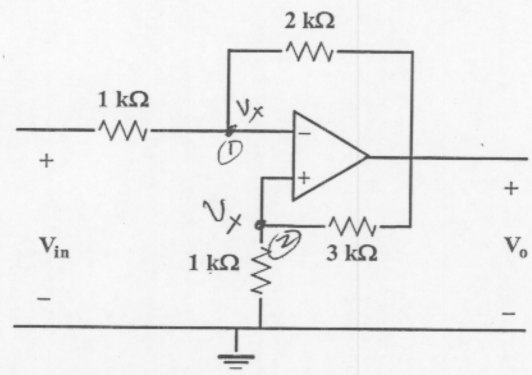


Figure 2: Circuit for problem 2.

We do not know the voltage at the point where the 1k and 3k resistor connect. We call this V_x. This reflects to the negative terminal as shown. We can write the following 2 equations.

At ①

$$\frac{V_{in} - V_x}{1K} + \frac{V_o - V_x}{2K} = 0$$

clearing gives

$$\boxed{2V_{in} + V_o = 3V_x} \qquad (1)$$

At ②

$$\frac{V_x}{1K} + \frac{V_x - V_o}{3K} = 0$$

clearing $\quad 4V_x = V_o$

OR $\quad \boxed{V_x = \frac{V_o}{4}} \qquad (2)$

(2) continued

Substituting (2) into (1) gives

$$2V_{IN} + V_o = \frac{3V_o}{4}$$

$$8V_{IN} + 4V_o = 3V_o$$

OR $\boxed{V_o = -8V_{IN}}$ Ans.

(3) You are given the op amp circuit of Figure 3. Solve for V_o in terms of V_{in}. Ans: $V_o = 16.8V_{in}$.

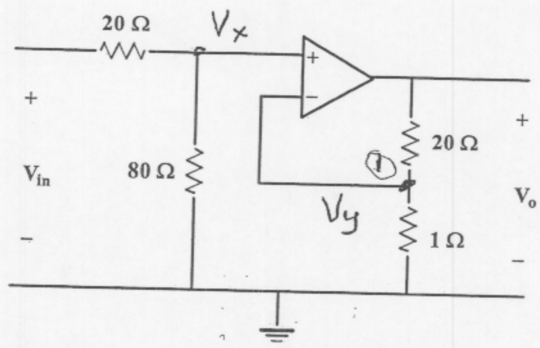


Figure 3: Op amp circuit for problem 3.

We name the voltage at the intersection of the 20 Ω and 80 Ω resistors, Vx.

Since no current enters the + terminal or negative terminal of the op amp, we can write (voltage division)

$$V_x = \frac{80 V_{in}}{20+80} = 0.8 V_{in}$$

Now, $V_y = V_x$ and we write a node at ①. This gives

$$\frac{0.8\, V_{in}}{1} + \frac{\qquad}{20} = 0$$

OR

$$16\, V_{in} + 0.8\, V_{in} - V_o = 0$$

This gives

$$\boxed{V_o = 16.8\, V_{in}}$$ Ans

(4) You are given the op amp circuit of Figure 4. Solve for V_o in terms of V_{in}. Ans: $V_o = V_{in}/3$.

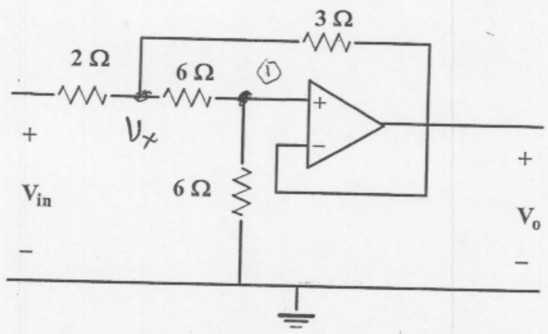


Figure 4: Op amp circuit for problem 4.

Point ① in the diagram is V_o. We name the intersection of the of the 2 Ω and 6 Ω resistor, a node voltage V_x.

At V_x we write (Σi leaving = 0)

$$\frac{V_x - V_{IN}}{2} + \frac{V_x - V_o}{6} + \frac{V_x - V_o}{3} = 0$$

Clearing gives

$$\boxed{V_x = \frac{V_{IN} + V_o}{2}} \qquad (1)$$

At ①, (Σi leaving = 0)

$$\frac{V_o - V_x}{6} + \frac{V_o}{6} = 0$$

$$\boxed{V_x = 2V_o} \qquad (2)$$

Substituting (1) into (2) gives

$$\boxed{V_o = \frac{V_{IN}}{3}} \qquad \text{Ans}$$

(5) You are given the op amp configuration of Figure 5 that solves a first order differential equation. Determine the differential equation.

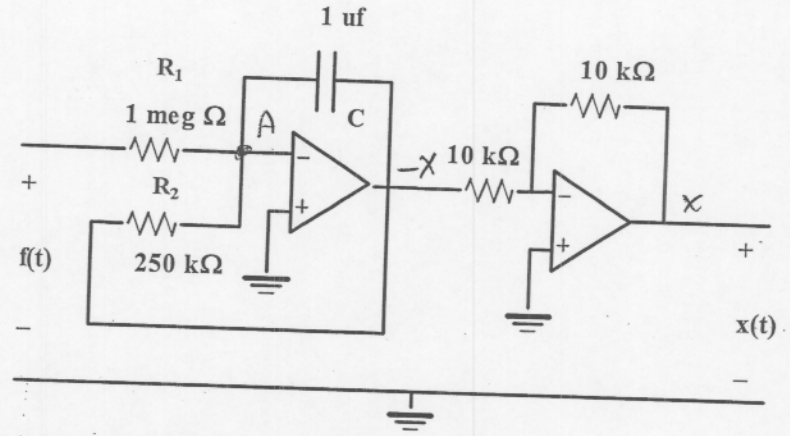


Figure 5: Op amp circuit for problem 5.

with $-x$ at the output of the integrator, the input must be $\dot{x}$. But then

$$\dot{x} = \frac{1}{R_1 C} f(t) - \frac{1}{R_2 C} x$$

If we put in numbers for R_1, R_2, C, we have

$$\dot{x} = f(t) - 4x(t)$$

so the differential equation is

$$\dot{x} + 4x(t) = f(t)$$

OR

$$\boxed{\frac{dx(t)}{dt} + 4x(t) = f(t)}$$

(6) The circuit of problem (5) can be used as a low pass filter. If $R_1 = R_2 = 1\ k\Omega$ and $C = 0.1\ \mu F$ give the filter transfer function. What is the low frequency cut-off point in rad/sec?

Let $x(t) = -V_o(t)$

$f(t) = V_{IN}(t)$

The differential equation of problem 5 becomes

$$-\frac{dV_o(t)}{dt} - \frac{1}{R_2 C} V_o(t) = \frac{V_{IN}(t)}{R_1 C}$$

If we put in numbers

$$\frac{dV_o}{dt} + 10{,}000\, V_o(t) = -10{,}000\, V_{IN}(t) \qquad (1)$$

Take the Laplace transform of (1)

$$sV_o(s) + 10{,}000\, V_o(s) = -10{,}000\, V_{IN}(s)$$

$$\boxed{\frac{V_o(s)}{V_{IN}(s)} = \frac{-10{,}000}{s + 10{,}000} = \frac{-1}{1 + s/10{,}000}}$$

The low frequency cutoff is at

$$\boxed{\omega = 10{,}000 \text{ rad/sec}}$$

(7) Use the freqs function of MATLAB to plot the frequency response of the above filter.

We want to plot

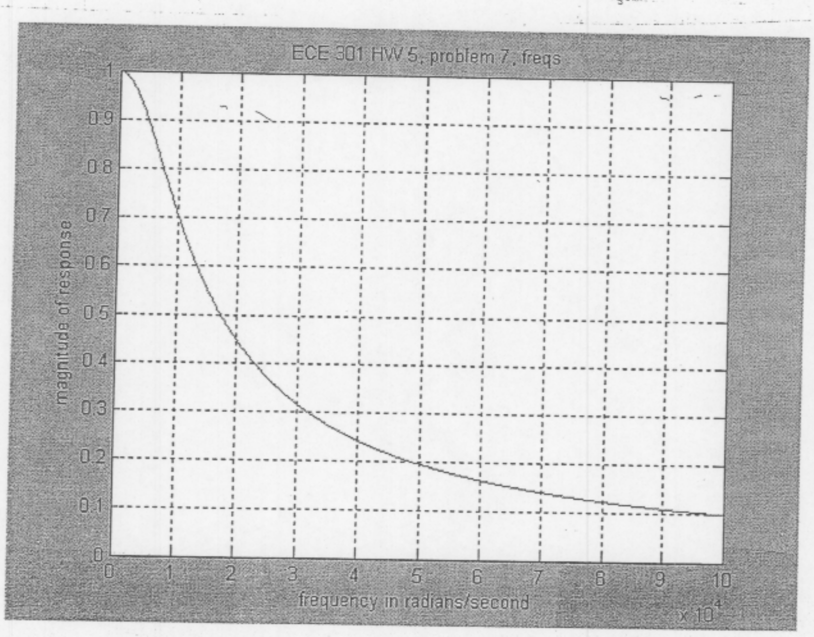
$$\frac{V_o(s)}{V_{in}(s)} = \frac{-10{,}000}{s+10{,}000}$$

A Program and the output is shown below.

```
%   Program for HW 5 ECE 301 Fall 2005
%   Simple freqs.   Program name:   freq_resp.m

w = 0:100:100000;
num = [-10000];
den = [1 10000];
G = freqs(num,den,w);
magG = abs(G);

plot(w,magG)
grid
ylabel('magnitude of response')
xlabel('frequency in radians/second')
title('ECE 301 HW 5, problem 7, freqs')
```



(8) You are given the op amp circuit of Figure 8. This configuration can be used for a high-pass filter.

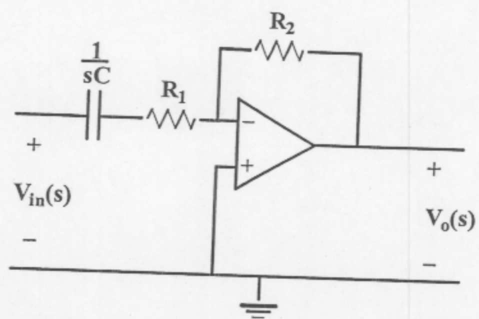


Figure 8: Op amp circuit for problem 8.

Show that the transfer function between the output and input of the configuration is,

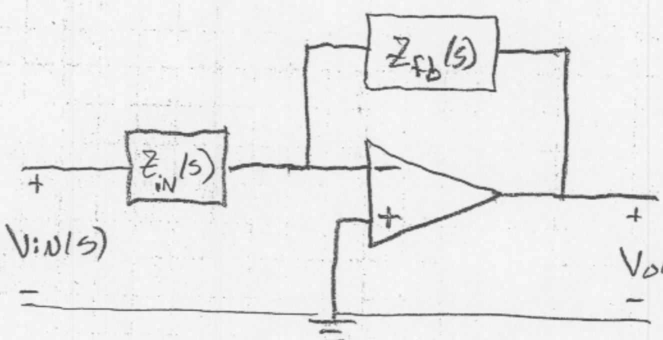
$$\frac{V_o(s)}{V_{in}(s)} = \frac{-sR_2C_1}{(1+sR_1C_1)}$$

(a) Assume $C_1 = 1\ \mu F$. Determine the value of R_1 so that the cut-off frequency of the filter is $\omega = 2000$ rad/sec.

(b) Determine the value of R_2 so that the gain of the filter as ω approaches infinity is 1.

(c) Use the bode statement of MATLAB to find the bode plot for the filter.

Recall the following basic configuration:



We have shown that,

$$\frac{V_o(s)}{V_{in}(s)} = \frac{-Z_{fb}(s)}{Z_{in}(s)} \qquad (1)$$

For our case;

$$Z_{fb} = R_2 \qquad (2)$$

(8) continued

and

$$Z_{IN} = R_1 + \frac{1}{sC_1}$$

$$Z_{IN} = \frac{1 + sR_1C_1}{sC_1} \qquad (3)$$

Now form $\frac{Z_{fb}(s)}{Z_{IN}(s)}$ using (2) and (3)

we have,

$$\frac{Z_{fb}(s)}{Z_{IN}(s)} = \frac{R_2}{\left(\frac{1 + sR_1C_1}{sC_1}\right)} = \frac{sR_2C_1}{1 + sR_1C_1} \qquad (4)$$

Substitute (4) into (1) to get,

$$\boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{-sR_2C_1}{1 + sR_1C_1}} \qquad (5)$$

(a) Let $C_1 = 1\mu F$, Determine R_1 so that the cutoff frequency is $\omega = 2000$ rad/sec.

We use

$$\frac{1}{R_1C_1} = 2000$$

or

$$R_1 = \frac{1}{2 \times 10^3 \times 1 \times 10^{-6}}$$

$$\boxed{R_1 = 500\ \Omega} \qquad (6)$$

(b)

From the transfer function we let $s = j\omega$ and then let $\omega \to \infty$.

$$\frac{V_o(j\omega)}{V_{IN}(j\omega)} = \frac{-j\omega R_2 C_1}{1 + j\omega R_1 C_1}$$

Divide the numerator and denominator by ω:

$$\frac{V_o(j\omega)}{V_{IN}(j\omega)} = \frac{-jR_2 C_1}{\frac{1}{\omega} + jR_1 C_1} \qquad (7)$$

Let $\omega \to \infty$ and we get

$$\left|\frac{V_o(j\infty)}{V_{IN}(j\infty)}\right| = \left|\frac{-\cancel{j}R_2 \cancel{C_1}}{\cancel{j}R_1 \cancel{C_1}}\right|$$

$\boxed{\text{So } R_2 = R_1}$ $\boxed{R_2 = 500\,\Omega}$ ANS

(c) The program and the output is given on the next page.

Plot:

$$\frac{-j\omega \times 0.5 \times 10^{-3}}{1 + j\omega \times 0.5 \times 10^{-3}}$$

```
%  Simple bode for HW 5 ECE 301 Fall 2005
%  This is the plot for problem 8
%  Name of program:  simple_bode5.m
%  WLG
num = [-.0005 0];
den = [.0005 1];
bode(num,den)

grid
```