Desk Copy

ECE 301
Fall Semester, 2005
HW #6

wlg
Due: Nov 29

Name wlg
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 15 points.

(1) You are given the circuit of Figure 1.
  (a) If the phasor current $\mathbf{I}_{ad} = 0.5\angle 0$ A, find the phasor current $\mathbf{I}_{ca}$. Ans: $0.236\angle{-45^\circ}$ A.
  (b) Find the phasor source voltage, $\mathbf{V}_s$. $1.57\angle{-58^\circ}$ V.

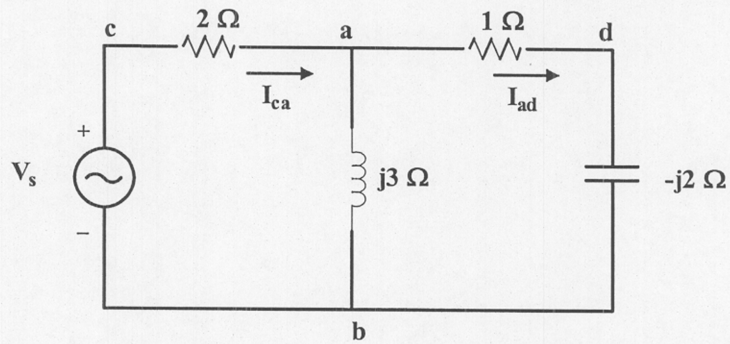


Figure 1: Circuit for problems 1 and 2.

(2) Find the impedance seen by the source voltage (Z_{cb}) of Figure 1. Ans: Verify by using $\mathbf{V}_s$ and $\mathbf{I}_{ca}$ of problem 1.

(3) Determine Y_{eq} for the circuit of Figure 3. Ans: $0.521\angle 24.9^\circ$ S

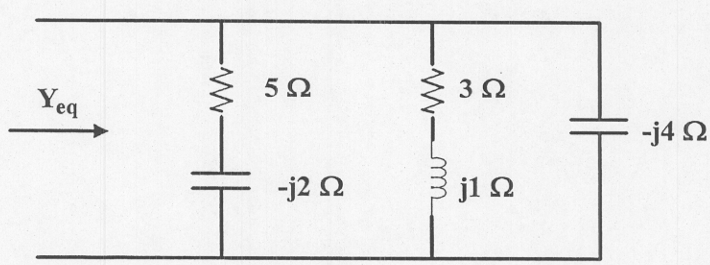


Figure 3: Circuit for problem 3.

(4) Given the circuit of Figure 4. Use mesh analysis to find the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$.
Ans: $\mathbf{I}_1 = 0.27\angle 15.8$ A, $\mathbf{I}_2 = 2.09\angle 152.9$ A.

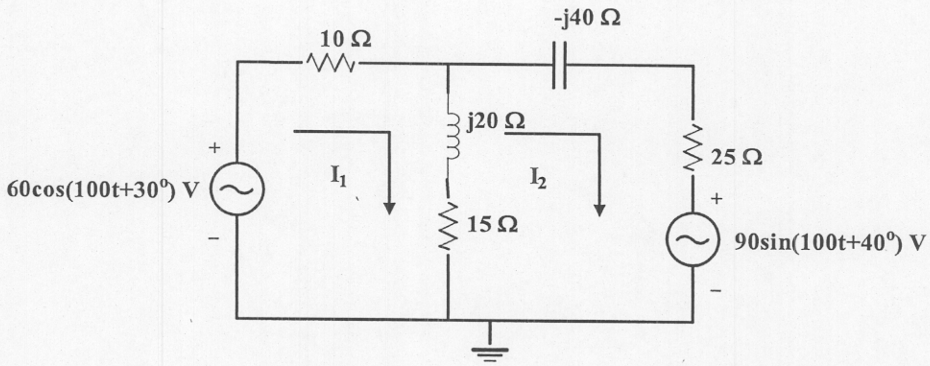


Figure 4: Circuit for problem 4.

(5) You are given the circuit of Figure 6. Find V_o using nodal analysis. Ans $V_o = 4.04\angle 70.35°$ V

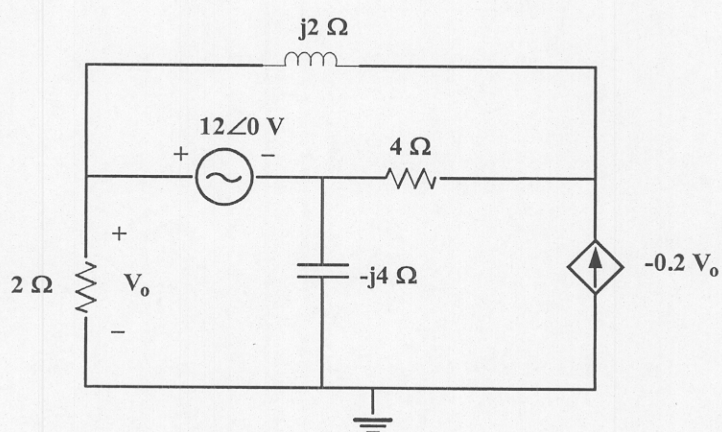


Figure 5: Circuit for problem 5.

(6) Find the Thevenin equivalent circuit to the left of a-b of Figure 6. Ans: $V_{TH} = -50\angle 30°$ V
$Z_{TH} = (10 - j20)\ \Omega$

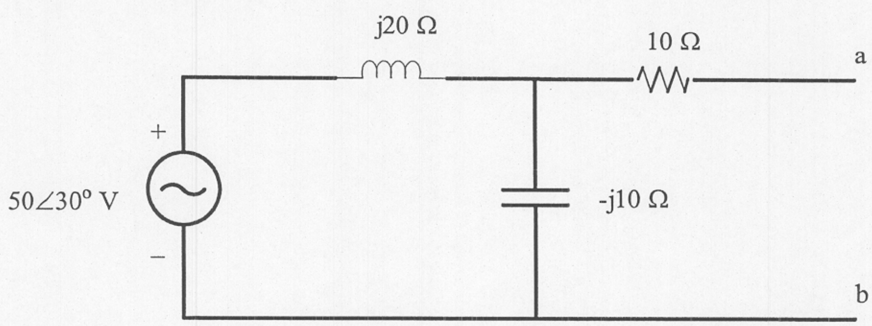
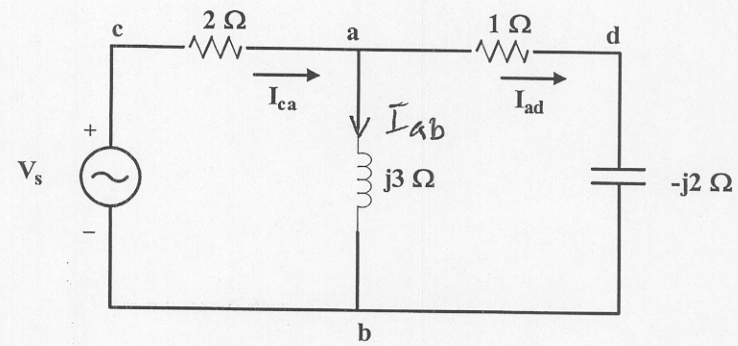


Figure 6: Circuit for problem 6.

(1) You are given the circuit of Figure 1.
   (a) If the phasor current $\mathbf{I_{ad}} = 0.5\angle 0$ A, find the phasor current $\mathbf{I_{ca}}$. Ans: $0.236\angle\text{-}45°$ A.
   (b) Find the phasor source voltage, $\mathbf{V_s}$. $1.57\angle\text{-}58°$ V.



(a)

$$\hat{V}_{ab} = I_{ad}(1-j2) = .5(1-j2) = 1.12\angle -63.4 \text{ V}$$

$$\hat{I}_{ab} = \frac{\hat{V}_{ab}}{j3} = \frac{(1.12\angle -63.4)}{(3\angle 90)} = 0.373\angle -153.4 \text{ A}$$

so,

$$\hat{I}_{ca} = \hat{I}_{ad} + \hat{I}_{ab}$$

$$= 0.5 + (0.373\angle -153.4$$

$$\boxed{\hat{I}_{ca} = 0.236\angle -45 \quad A}$$

(b)

$$\hat{V}_s = 2 \times \hat{I}_{ca} + \hat{V}_{ab}$$

$$\hat{V}_s = 2 \times (0.236\angle -45) + (1.12\angle -63.4)$$

$$\boxed{\hat{V}_s = 1.57\angle -58° \text{ V}}$$

(2) Find the impedance seen by the source voltage (Z_{ab}) of Figure 1. Ans: Verify by using $\mathbf{V_s}$ and $\mathbf{I_{ca}}$ of problem 1.

$$Z_{ab} = \frac{(3\angle 90) \times (1 - j2)}{j3 + 1 - j2} = 4.5 - j1.5$$

then

$$Z_{cb} = 2 + Z_{ab} = 2 + 4.5 - j1.5$$

$$\boxed{Z_{cb} = 6.5 - j1.5 = 6.67\angle -13^\circ \ \Omega}$$

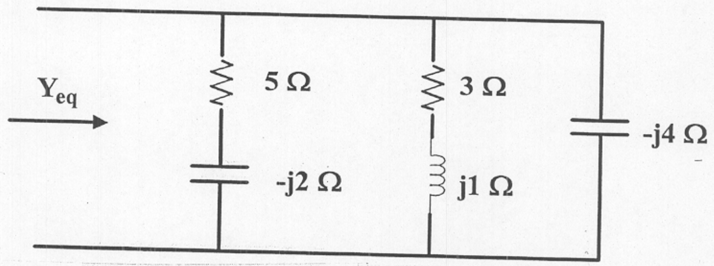
To verify

$$Z_{cb} = \frac{\hat{V}_s}{I_{ca}}$$

$$Z_{cb} = \frac{1.57\angle -58}{0.236\angle -45}$$

$$\boxed{Z_{cb} = 6.65\angle -13^\circ \ \Omega \qquad \text{check}}$$

(3) Determine Y_{eq} for the circuit of Figure 3. Ans: $0.521\angle 24.9^\circ$ S



$$Y_{eq} = \left(\frac{1}{-j4}\right) + \frac{1}{(3+j)} + \frac{1}{(5-j2)}$$

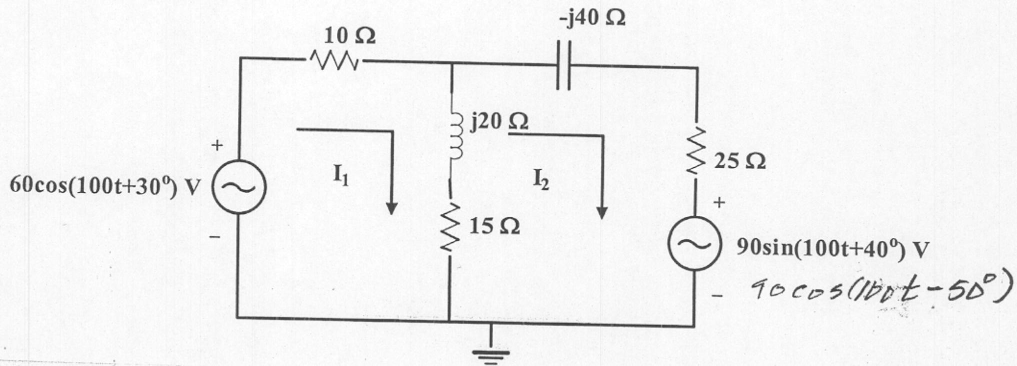
$$Y_{eq} = j0.25 + 0.3 - j0.1 + .172 + j0.069$$

$$Y_{eq} \simeq (.3 + .172) + j(.25 - .1 + 0.069)$$

$$Y_{eq} = 0.472 + j0.219$$

$$\boxed{Y_{eq} = 0.52 \angle 24.9^\circ \quad S}$$

(4) Given the circuit of Figure 4. Use mesh analysis to find the phasor currents $\mathbf{I_1}$ and $\mathbf{I_2}$.
Ans: $\mathbf{I_1} = 0.27\angle 15.8$ A, $\mathbf{I_2} = 2.09\angle 152.9$ A.



$$10\hat{I}_1 + (15+j20)(\hat{I}_1-\hat{I}_2) - 60\angle 30 = 0$$

$$\boxed{(25+j20)\hat{I}_1 + (-15-j20)\hat{I}_2 = 60\angle 30}$$

$$(15+j20)(\hat{I}_2-\hat{I}_1) + (-j40)I_2 + 25 I_2 + 90\angle -50$$

$$(-15-j20)\hat{I}_1 + (40-j20)\hat{I}_2 = -90\angle -50$$

$$\begin{bmatrix} 25+j20 & -15-j20 \\ -15-j20 & 40-j20 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 60\angle 30 \\ -90\angle -50 \end{bmatrix}$$

$90\angle 130$

$\hat{I}_1 = 0.27\angle 15.8^\circ$ A

$\hat{I}_2 = 2.09\angle 152.9^\circ$ A

(5) You are given the circuit of Figure 6. Find V_o using nodal analysis. Ans $V_o = 4.04\angle 70.35^\circ$ V

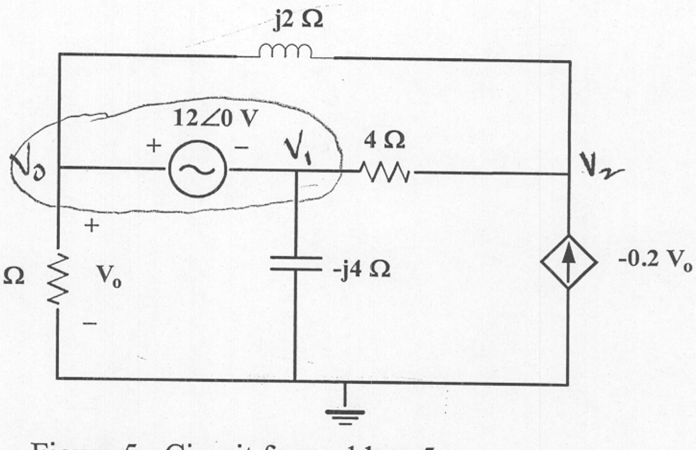


Figure 5: Circuit for problem 5.

$$\frac{V_o}{2} + \frac{V_o - V_2}{j2} + \frac{V_1}{-j4} + \frac{V_1 - V_2}{4} = 0$$

$$0.5V_o - j0.5V_o + j0.5V_2 + j0.25V_1 + 0.25V_1 - 0.25V_2$$

$$\boxed{(0.5 - j0.5)V_o + (0.25 + j0.25)V_1 + (-0.25 + j0.5)V_2 = 0}$$

$$\frac{V_2 - V_1}{4} + \frac{V_2 - V_o}{j2} + 0.2V_o = 0$$

$$0.25V_2 - 0.25V_1 - j0.5V_2 + j0.5V_o + 0.2V_o = 0$$

$$\boxed{(0.2 + j0.5)V_o - 0.25V_1 + (0.25 - j0.5)V_2 = 0}$$

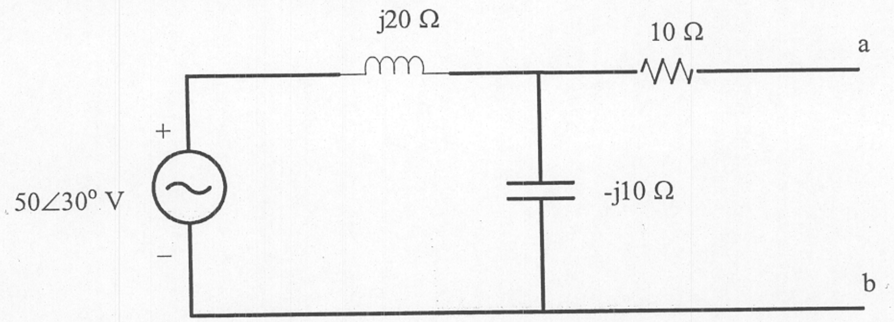
$$V_o - 12 - V_1 = 0$$

$$\boxed{V_o - V_1 + 0V_2 = 12}$$

$$\begin{bmatrix} 0.5 - j0.5 & 0.25 + j0.25 & -0.25 + j0.5 \\ 0.2 + j0.5 & -0.25 & 0.25 - j0.5 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$\boxed{V_o = 4.04\angle 70.35^\circ \text{ V}}$$

(6) Find the Thevenin equivalent circuit to the left of a-b of Figure 6. Ans: $V_{TH} = -50\angle 30°$ V
$Z_{TH} = (10 - j20)\ \Omega$

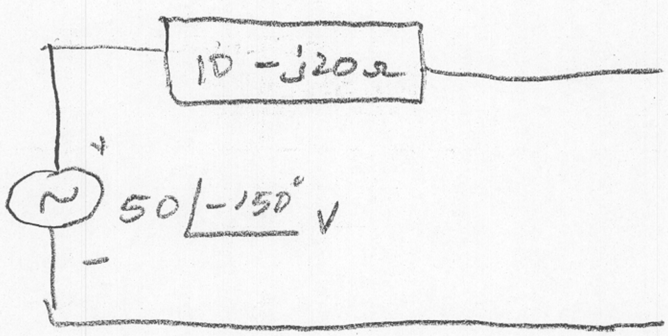


$$Z_{TH} = \frac{(10\angle -90)(20\angle 90)}{-j10 + j20} + 10\ \Omega$$

$$Z_{TH} = (-j20\ \Omega + 10)\ \Omega = \underline{(10 - j20)\ \Omega}$$

$$V_{TH} = \frac{(50\angle 30)(10\angle -90)}{j20 - j10} = \frac{500\angle -60}{10\angle 90}$$

$$V_{TH} = 50\angle -150°\ V = -50\angle 30°\ V$$



Thevenin Circuit