Desk Copy

ECE 301
Fall Semester, 2005
HW #7

wlg
Due: Nov 29

Name wlg
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 15 points.

(1) You are given the circuit of Figure 1. (a) Calculate the power factor of the entire circuit. (b) What is the average power supplied by the source? Ans: pf = 0.936 lagging, P = 59 W.

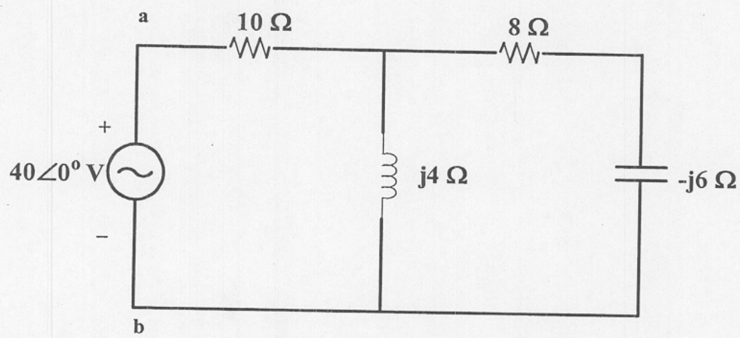


Figure 1: Circuit for problem 1.

(2) Find the RMS value of the current waveform shown in Figure 2. Ans: $I_{rms} = 2.89$ A

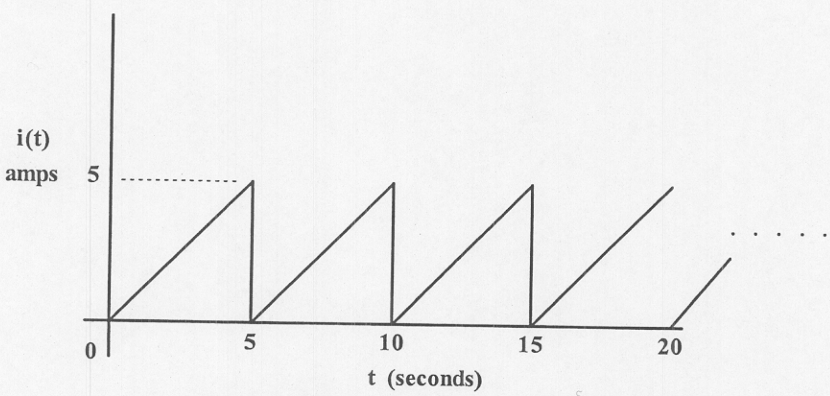


Figure 2: Waveform for problem 2.

(3) Consider the circuit of Figure 3. Find the following with respect to the source voltage.

(a) the power factor; Ans 0.9956 lagging
(b) the average power delivered by the source; Ans: $P = 15.56$ W
(c) the reactive power; $Q = 1.47$ VARs
(d) the apparent power; $S = 15.63$ VA
(e) the complex power; $\mathbf{S} = 15.56 + j1.466$ VA

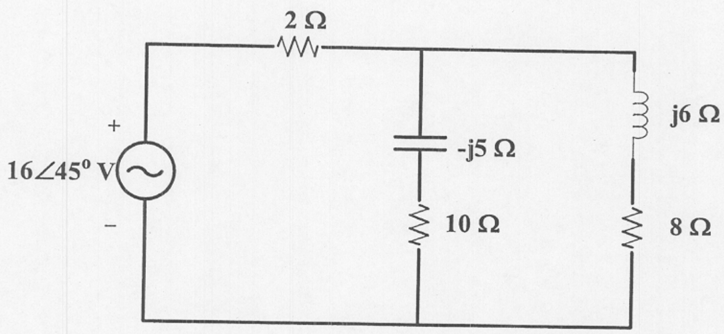


Figure 3: Circuit for problem 3.

(4) For each of the following cases, find the complex power, the average power, and the reactive power.

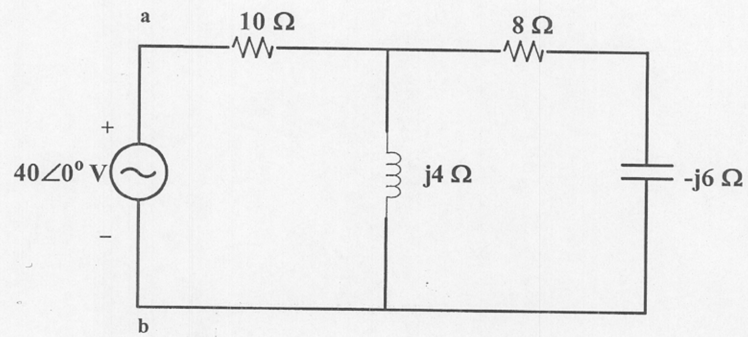
(a) $v(t) = 112\cos(wt + 10^\circ)$ V
$i(t) = 4\cos(wt - 50^\circ)$ A
Ans: $S = 224\angle 60^\circ$ VA, $P = 112$ W, $Q = 194$ VAR

(b) $v(t) = 160\cos(377t)$ V
$i(t) = 4\cos(377t + 45^\circ)$ A
Ans: $S = 320\angle -45^\circ$ VA, $P = 226.3$ W, $Q = -226.3$ VAR or $Q = 226.3$ VAR leading

(5) A source delivers 50 kVA to a load with a power factor of 65% lagging. Find the load's average and reactive power. $P = 32.5$ kW, $Q = 38$ kVAR.

(6) A 40 kW induction motor, with a lagging power factor of 0.76 is supplied by a 120 V_{rms} 60 Hz sinusoidal voltage source. Find the capacitor needed, in parallel with the motor, to raise the power factor to 0.9 lagging. Ans: $C = 2.734$ mF.

(1) You are given the circuit of Figure 1. (a) Calculate the power factor of the entire circuit.
(b) What is the average power supplied by the source? Ans: pf = 0.936 lagging, P = 59 W.



(a)
Find Zab:

$$Z_{ab} = 10 + \frac{(4\angle 90)(8-j6)}{8-j2}$$

$$Z_{ab} = 12.7\angle 20.6\ \Omega$$

Power Factor angle is the angle of Z.

$$p.f. = \cos(20.6)\ \text{lagging}$$

p.f. = 0.936 lagging

(b)
The complex power is

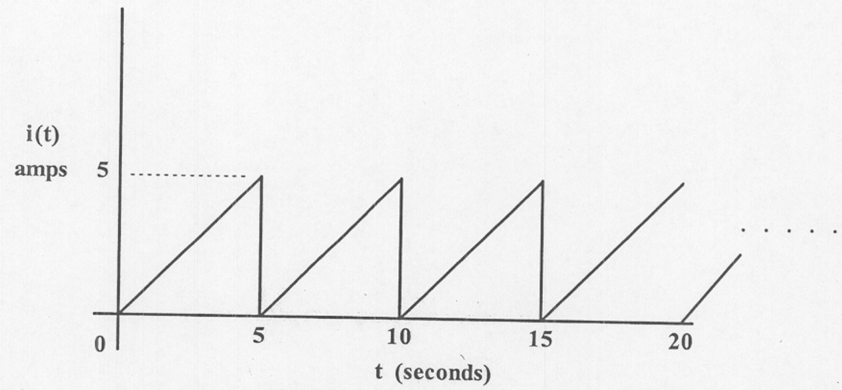
$$\hat{S} = \frac{|V_s|^2}{2Z^*} = \frac{40^2}{(2\times 12.7\angle -20.6)} = 63\angle 20.6^\circ\ VA$$

$$\hat{S} = (59 + j\,22.2)\ VA = P + jQ$$

P = 59 W

#2

(2) Find the RMS value of the current waveform shown in Figure 2. Ans: $I_{rms} = 2.89$ A



$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2 dt}$$

over the range $0 < t \leq 5$,

$$i(t) = t$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T t^2 dt} = \sqrt{\frac{1}{T}\frac{t^3}{3}\Big|_0^T}$$

$$I_{rms} = \sqrt{\frac{1}{T}\frac{T^3}{3}} = \frac{T}{\sqrt{3}} = \frac{5}{\sqrt{3}}$$

$$\boxed{I_{rms} = 2.89 \text{ A}}$$

(3) Consider the circuit of Figure 3. Find the following with respect to the source voltage.

(a) the power factor; Ans 0.9956 lagging
(b) the average power delivered by the source; Ans: P = 15.56 W
(c) the reactive power; Q = 1.47 VARs
(d) the apparent power; S = 15.63 VA
(e) the complex power; **S** = 15.56 + j1.466 VA

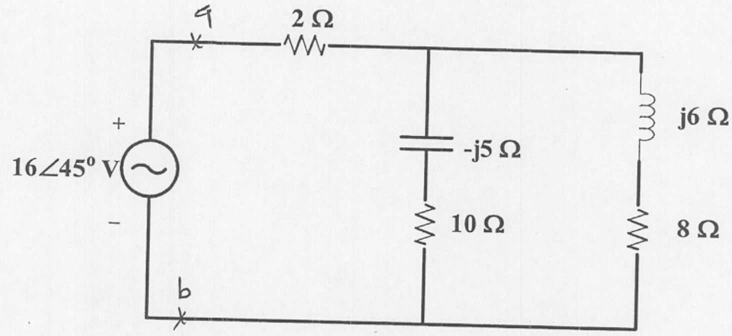


Figure 3: Circuit for problem 3.

(a) To find the power factor, find Zab.

$$Z_{ab} = 2 + \frac{(10-j5)(8+j6)}{10-j5+8+j6}$$

$$Z_{ab} = (8.154 + j0.769) = 8.19 \angle 5.39\ \Omega$$

$$\text{P.f.} = \cos(5.39°)\text{ lag} = 0.9956 \text{ lagging}$$

(b) Find $\hat{S}$

$$\hat{S} = \frac{|\hat{V}|^2}{2Z^*} = \frac{16^2}{2 \times 8.19 \angle -5.39}$$

$$\hat{S} = (15.56 + j1.47)\text{ VA}$$

$$P = 15.56\text{ W}$$

(c) $Q = \text{Im}[\hat{S}] = 1.47$ VARs

(d) $S = |15.56 + j1.47| = |15.67 \angle 5.38| = 15.67$ VA

(e) $\hat{S} = 15.56 + j1.47$ VA

(4) For each of the following cases, find the complex power, the average power, and the reactive power.

(a) $v(t) = 112\cos(wt + 10^{o})$ V
$i(t) = 4\cos(wt - 50^{o})$ A

Ans: $S = 224\angle 60^{o}$ VA, P = 112 W, Q = 194 VAR

(b) $v(t) = 160\cos(377t)$ V
$i(t) = 4\cos(377t + 45^{o})$ A

Ans: $S = 320\angle -45^{o}$ VA, P = 226.3 W, Q = -226.3 VAR
or Q = 226.3 VAR leading

(a)

$$\hat{S} = \frac{V_m I_m}{2} \angle \theta_V - \theta_I = \frac{112 \times 4}{2} \angle (10+50)$$

$$\hat{S} = 224 \angle 60 \text{ VA} = 112 + j194$$

P = 112 W

Q = 194 VARs

(b)

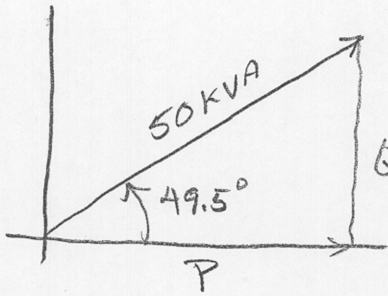
$$\hat{S} = \frac{160 \times 4}{2} \angle -45^{\circ} = 226.3 - j226.3 \text{ VA}$$

P = 226.3 W

Q = -226.3 VARs

or Q = 226.3 VARs leading

(5) A source delivers 50 kVA to a load with a power factor of 65% lagging. Find the load's average and reactive power. P = 32.5 kW, Q = 38 kVAR.



$\cos\theta = .65$

$\theta = 49.5°$

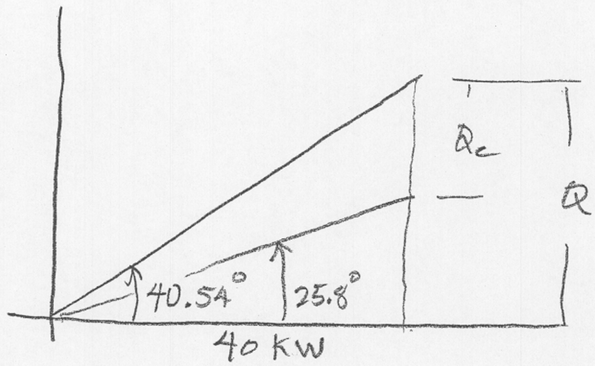
$P = 50k \cos(49.5) = 50k \times .65$

$P = 32.5 \text{ kW}$

$Q = 50k \sin 49.5°$

$Q = 38 \text{ VARs}$

(6) A 40 kW induction motor, with a lagging power factor of 0.76 is supplied by a 120 V_{rms} 60 Hz sinusoidal voltage source. Find the capacitor needed, in parallel with the motor, to raise the power factor to 0.9 lagging. Ans: C = 2.734 mF.



$\cos\theta_1 = .76$

$\theta_1 = 40.54^\circ$

$\cos\theta_2 = .9$

$\theta_2 = 25.8^\circ$

$$C^{*} = \frac{40k[\tan 40.54^\circ - \tan 25.8^\circ]}{2\pi \times 60 \times 120^2}$$

$C = 0.00274$ F

$C = 2.74$ mF

* A general equation, which can be easily derived is

$$C = \frac{P[\tan\theta_1 - \tan\theta_2]}{\omega (V_{rms})^2}$$