Desk Copy

ECE 301
Fall Semester, 2005
Test #2

wlg Version A

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Print (last, first)

Work the exam on your own engineering paper. Work on one side of your paper only. Attach your work to the back of this exam sheet and staple in the top left hand corner. Each problem counts 25%.

(1) You are given the following RC circuit.

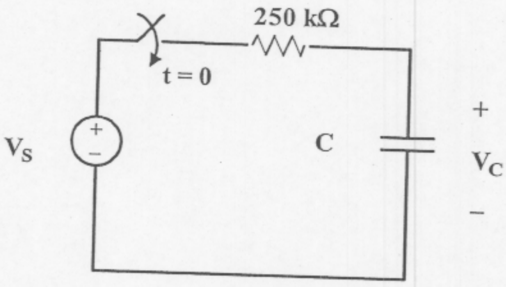

Figure 1(a): RC Circuit.

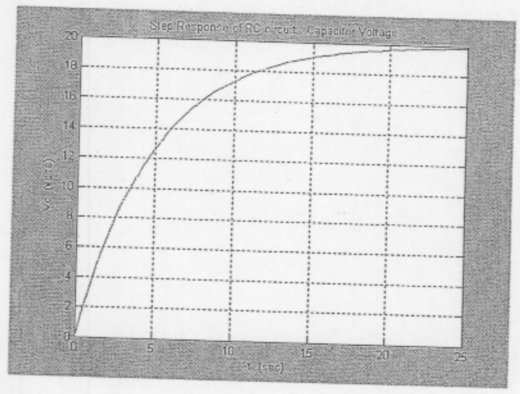

Figure 1(b): Step response of RC circuit.

(a) Determine the RC time constant.
(b) Determine V_S.
(c) Determine C.

(2) You are given the series RLC circuit of Figure 2.

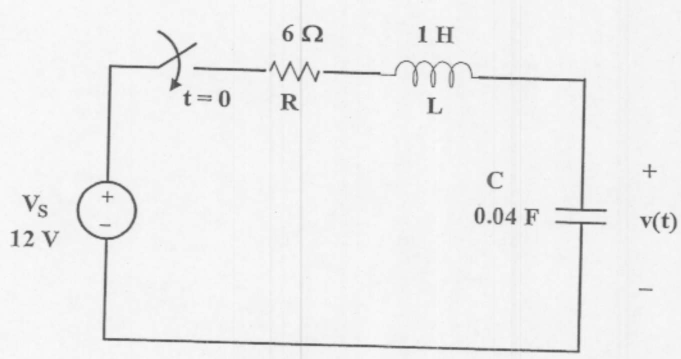

Figure 2: Circuit for problem 2.

Continued on next page.

(2) Continued:

(a) Give the differential equation that will allow you to solve for $v_c(t)$. Do not solve.
(b) Give the characteristic equation for the circuit. Use numbers.
(c) Give the values of ω_n (undamped natural resonant frequency) and ξ damping coefficient.
(d) State which of the following is true:
   (i) circuit is overdamped,
   (ii) circuit is underdamped,
   (iii) circuit is critically damped.
   State why.
(e) How much energy is stored in the electric field of the capacitor in steady state?
(f) How much energy is stored in the magnetic field of the inductor in steady state?

(3) You are given the circuit of Figure 3. (a) Find $i_0(t)$ for $t>0$. (b) Sketch the waveform for $-1 \le i_o(t) < \infty$.

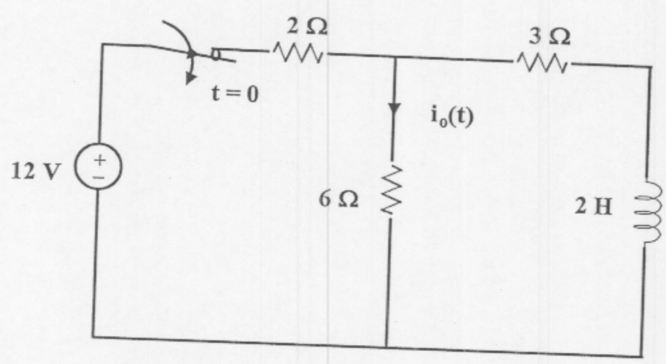

Figure 3: Circuit for problem 3.

(4) You are given the circuit of Figure 4. Find $v_o(t)$ for $t \ge 0$.

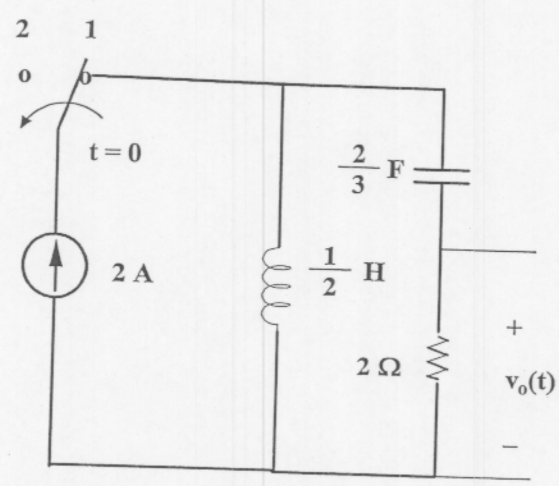

Figure 4: Circuit for problem 4.

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(1) You are given the circuit below with the step response shown to the right.

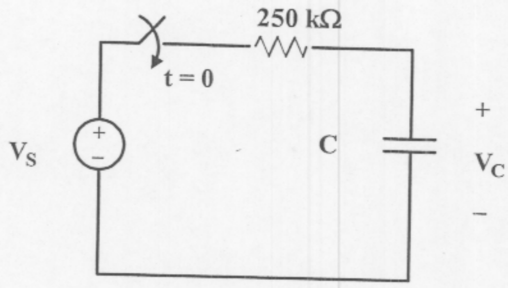

Figure 1(a): RC Circuit.

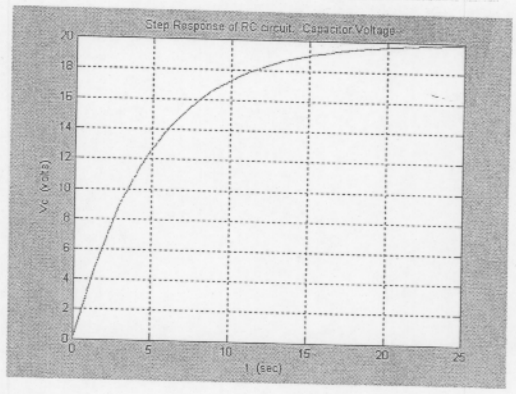

Figure 1(b): Step response of RC circuit.

(a) Determine the RC time constant.
(b) Determine V_S.
(c) Determine C.

From the diagram, we look for 0.632×20 which is ≈ 12.6 V. This occurs at $\tau = t = 5$ sec

(a) $\boxed{RC = \tau = 5 \text{ sec}}$

(b) By inspection of the circuit and the plot, $\boxed{V_S = 20 \text{ V.}}$

(c) Now $RC = 5$. We know $R = 250\ k\Omega$

$$C = \frac{5}{250K} = 20 \times 10^{-6}$$

$$\boxed{C = 20\ \mu F}$$

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(2) You are given the following circuit

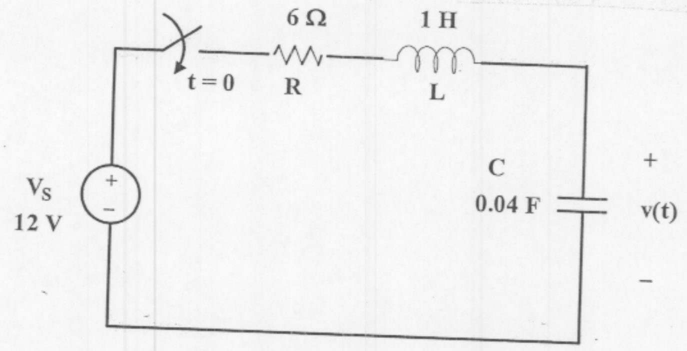


Figure 2: Circuit for problem 2.

(a) Give the de that will allow you to solve for $V_c(t)$.

$$Ri + L\frac{di}{dt} + V_c(t) = V_s$$

$$i = C\frac{dV_c}{dt}$$

$$RC\frac{dV_c}{dt} + LC\frac{d^2V_c}{dt^2} + V_c(t) = V_s$$

$$\boxed{\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c(t)}{LC} = \frac{V_s}{LC}}$$

with numbers

$$\boxed{\frac{d^2V_c}{dt^2} + 6\frac{dV_c}{dt} + 25V_c(t) = 25 \times 12}$$

(b) The characteristic equation is

$$\boxed{s^2 + 6s + 25 = 0}$$

(2) CONT.

(c) $s^2+6s+25=0$

$$\boxed{\omega_n = \sqrt{25} = 5}$$

$$2\xi\omega_n = 6$$

$$\boxed{\xi = \frac{3}{\omega_n} = \frac{3}{5} = 0.6}$$

(d) (ii) $\boxed{\text{underdamped, } \xi<1.}$

(e) $W_c = \frac{1}{2}CV_c(\infty)$

$$V_c(\infty) = 12V$$

$$W_c = \frac{0.04}{2}(12)^2 = 0.02 \times 144$$

$$\boxed{W_c = 2.88 \text{ Joules}}$$

(f)

$$W_L = \frac{1}{2}L\,i(\infty)$$

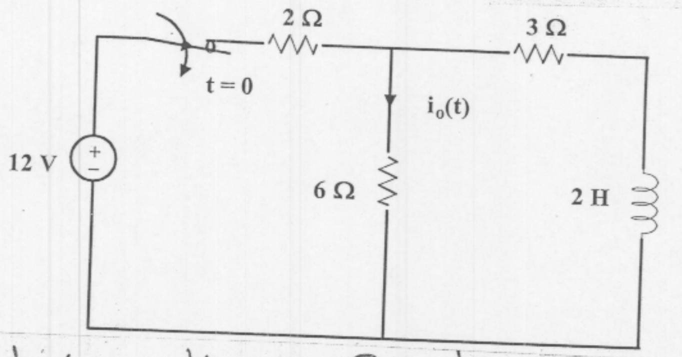
$i(\infty) = 0$ (capacitor acts like an open ckt)

$$\therefore \boxed{W_L = 0}$$

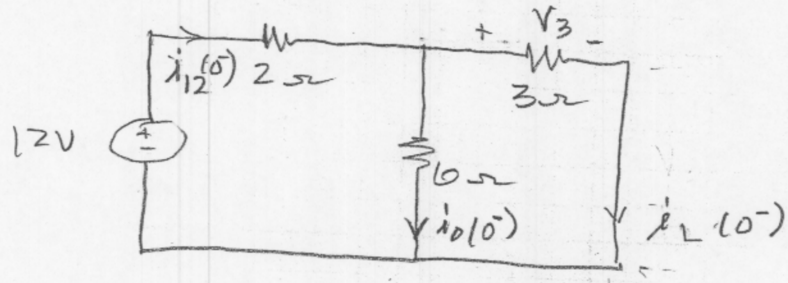
(3) You are given the following circuit.

(a) Find $i_o(t)$ for $t > 0$

(b) Sketch the waveform for $t <$ $i_o(t)$ $< \infty$



For $t < 0$, the inductor acts like a short circuit, we have



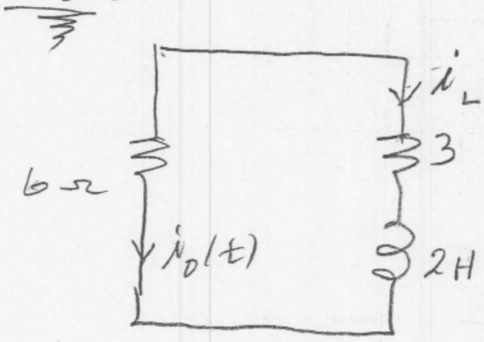
Current from the 12V source is

$$i_{12} = \frac{12}{4} = 3\ A$$

$$V_3 = (3A)(2\Omega) = 6V$$

$$i_L(0^-) = \frac{6V}{3\Omega} = 2\ A \qquad i_o(0^-) = 1\ A$$

For $t > 0$



$i_L(0^+) = i_L(0^-) = 3\ A$

(note)

$i_o(0^+) = -3$

$i_o(\infty) = 0$

(3) CONTINUE

$$\boxed{\tau = \frac{L}{R_{th}} = \frac{2}{9} \text{ sec}}$$

$$i_o(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-\frac{t}{\tau}}$$

$$\boxed{i_o(t) = -2e^{-4.5t} \text{ A}} \qquad t \geq 0$$

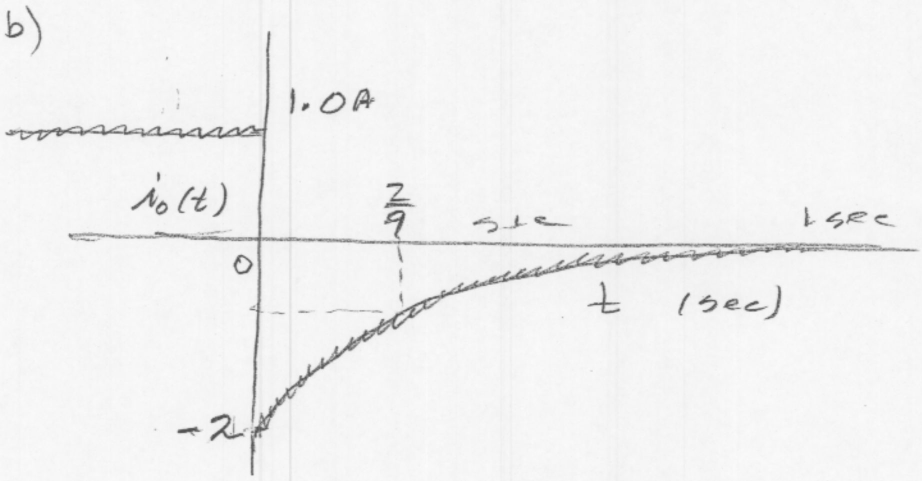
FOR $t < 0$

$$i_o(t) = \frac{9}{6} = \frac{3}{2} \text{ A}$$

$$\boxed{i(t) = \frac{3}{2} \text{ A}, \quad t < 0}$$

$$\boxed{i(t) = -2e^{-4.5t} \quad t \geq 0}$$

(b)



(4) You are given the following circuit. Find $V_o(t)$, $t \geq 0$.

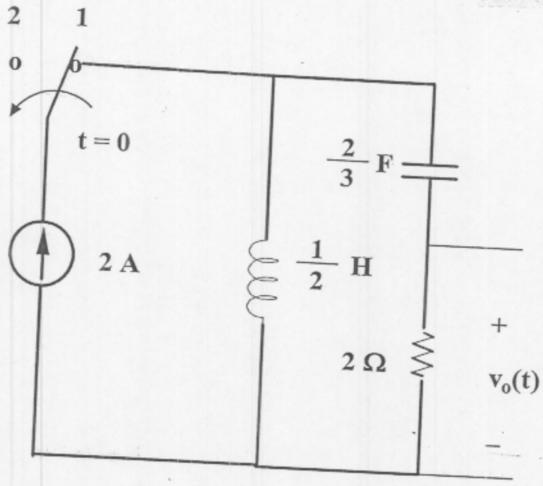
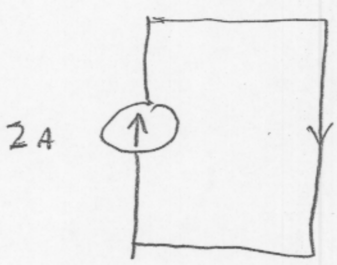


Figure 4: Circuit for problem 4.

For $t < 0$: Inductor acts like short; capacitor acts like open. We have

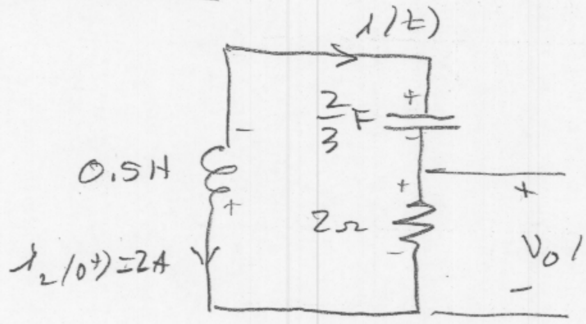


$i_L(0^-) = 2A = i_L(0^+)$

Circuit 1

$V_o(0^-) = 0 \text{ V}$; $i_L(0^+) = 2A$

For $t > 0$



Circuit 2

$\frac{R}{1/2 \times 2/3}$

$L\frac{di}{dt} = +4 \qquad \frac{di}{dt} = +8$

(4) Cont.

Around ckt 2 in the clockwise direction we write

$$L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + \cancel{V(0^+)}^{\,0} + i(0^+)R = 0 \qquad (1)$$

At $t = 0^+$

$$\frac{di(0^+)}{dt} = -\frac{i(0^+)R}{L}$$

but $i(0^+) = -i_L(0^+) = -2A$

$$\boxed{\frac{di(0^+)}{dt} = \frac{-(-2\times2)}{.5} = 8 \text{ A/sec}}$$

So we have for initial conditions,

$\boxed{i(0^+) = -2A}$ $\boxed{\frac{di(0^+)}{dt} = 8 \text{ A/sec}}$

Take the derivative of (1);

$$L\frac{d^2i}{dt^2} + \frac{i(t)}{C} + R\frac{di}{dt} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i(t)}{LC} = 0$$

Characteristic equation is

$$s^2 + \frac{R}{2}s + \frac{1}{LC} = 0$$

with numbers

$$s^2 + 4s + 3 = 0$$

$$(s+1)(s+3) = 0 \qquad \text{overdamped}$$

(7) CONTINUED

$$i(t) = Ae^{-3t} + Be^{-t} \quad A$$

$$i(0) = \boxed{-2 = A + B}$$

$$\frac{di}{dt} = -3Ae^{-3t} - Be^{-t} \quad A$$

at $t = 0^+$

$$8 = -3A - B$$

so

$$\begin{bmatrix} 1 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

$$A = -3, \quad B = 1$$

$$i(t) = \left[-3e^{-3t} + e^{-t}\right] A$$

$$V_o(t) = 2 \times i(t) \quad V$$

$$\boxed{V_o(t) = 2\left[e^{-t} - 3e^{-3t}\right] V}$$