Best Copy

ECE 301
Fall Semester, 2005
Test #2

wlg Version B

Name green
Print (last, first)

Work the exam on your own engineering paper. Work on one side of your paper only. Attach your work to the back of this exam sheet and staple in the top left hand corner. Each problem counts 25%.

(1) You are given the following RL circuit.

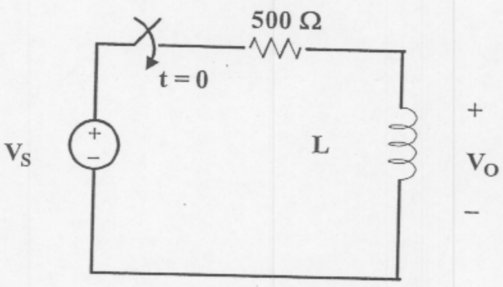


Figure 1(a): RL Circuit.

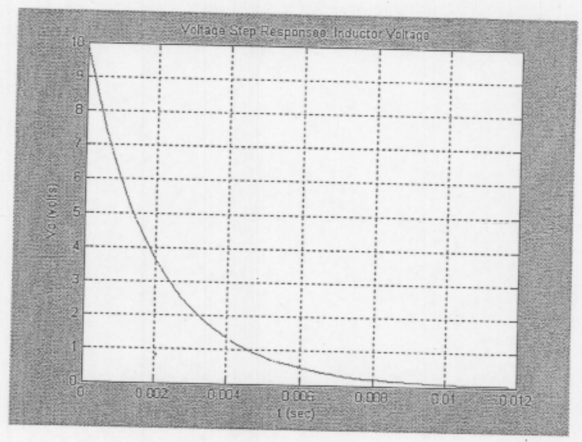


Figure 1(b): Step Response of RL circuit.

(a) Determine the L/R time constant.
(b) Determine V_S.
(c) Determine L

(2) You are given the parallel RLC circuit of Figure 2.

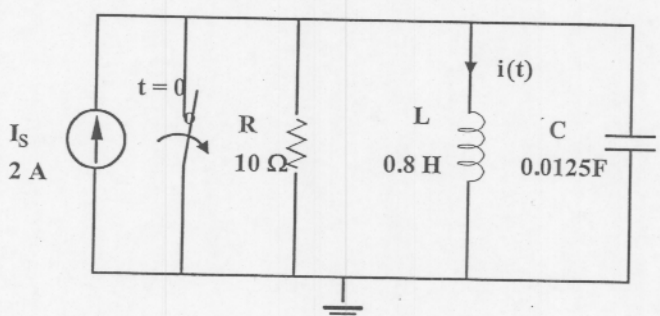


Figure 2: Circuit for problem 2.

Continued on next page.

(2) Continued:

(a) Give the differential equation that will allow you to solve for i(t). Do not solve.
(b) Give the characteristic equation of the circuit. Use numerical values.
(c) Give the values of ξ (damping coefficient) and ω_n (undamped natural resonant frequency).
(d) State which of the following is true:
   (i) circuit is overdamped,
   (ii) circuit is underdamped,
   (iii) circuit is critically damped.

(e) How much energy is stored in the magnetic field of the inductor in steady state?
(f) How much energy is stored in the electric field of the capacitor in steady state?

(3) You are given the circuit of Figure 3. Find $v_c(t)$ for $t \geq 0$. Sketch the waveform of $i_o t)$, $-1\text{sec} \leq t \leq \infty$.

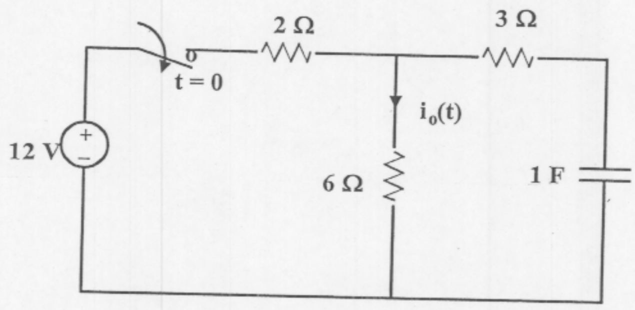


Figure 3: Circuit for problem 3.

(4) You are given the circuit of Figure 4. The switch is moved from 1 to 2 at t = 0. Find $v_o(t)$, $t \geq 0$.

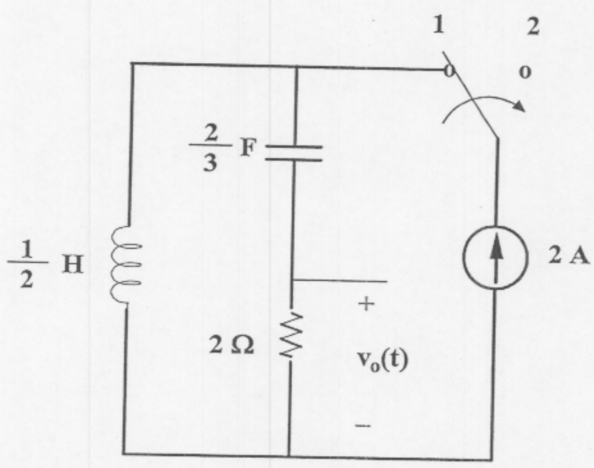


Figure 4: Circuit for problem 4.

ECE 301
Test 2

Version B

(1) Given the following RL circuit with the step response shown to the right
(a) Determine the $\frac{L}{R}$ time constant
(b) Determine V_s. (c) Determine L.

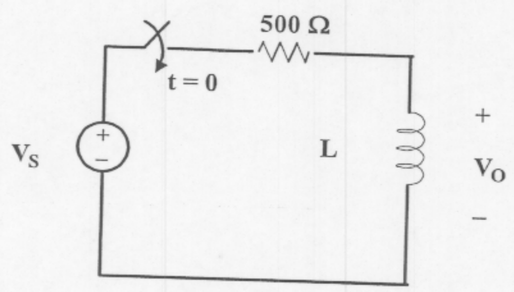


Figure 1(a): RL Circuit.

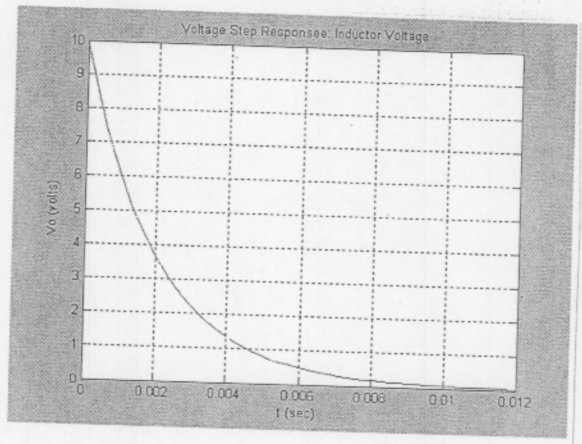


Figure 1(b): Step Response of RL circuit.

(a) Determine the L/R time constant.
(b) Determine V_S.
(c) Determine L

We look for $10(1-e^{-1}) = 10(.368) = 3.68$ A.
This occurs at $t = \tau \approx 0.002$ sec

(a) $\boxed{\frac{L}{R} = .002 \text{ sec}}$

(b) $\boxed{V_s = 10 \text{ V}}$

(c) $L = R \times .002 = 500 \times .002$

$\boxed{L = 1\,H}$

(2) You are given the following parallel RLC circuit.

(2) You are given the parallel RLC circuit of Figure 2.

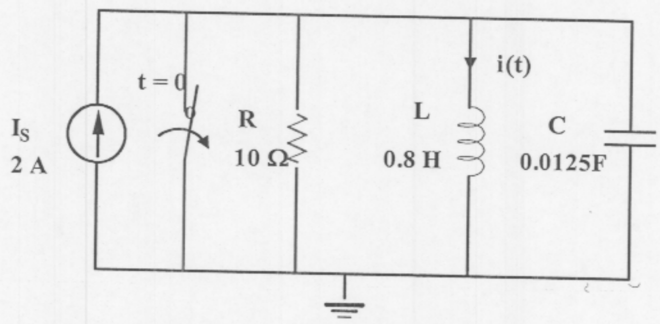


Figure 2: Circuit for problem 2.

(a) We can write, using node equations,

$$\frac{V(t)}{R} + C\frac{dV(t)}{dt} + i(t) = 0 \qquad (1)$$

but

$$V(t) = L\frac{di}{dt} \qquad (2)$$

(2) into (1)

$$\frac{L}{R}\frac{di}{dt} + LC\frac{d^2i}{dt^2} + i(t) = 0$$

or

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i(t)}{LC} = 0$$

(B)

(1) cont. (2)

The characteristic equation is

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

with numbers

$$\boxed{s^2 + 8s + 100 = 0}$$

(c) $\omega_n^2 = 100$

$$\boxed{\omega_n = 10}$$

$2\xi\omega_n = 8$

$$\xi = \frac{8}{2\times10} = \frac{8}{20} = 0.4$$

(d) (iii) underdamped, $\xi < 1$.

(e) $W_L = \frac{1}{2} L i^2$

$i(\infty) = 2\ A$

$W_L = \frac{1}{2} \times 0.8 \times 2^2 = 1.6\ J$

$$\boxed{W_L = 1.6\ J}$$

(f) $W_C = \frac{1}{2} C V_c^2$ $\quad V_c = 0$

$$\boxed{W_C = 0}$$

3

③ cont

$$\tau = R_{eq} C = 9 \times 1 = 9 \text{ sec}$$

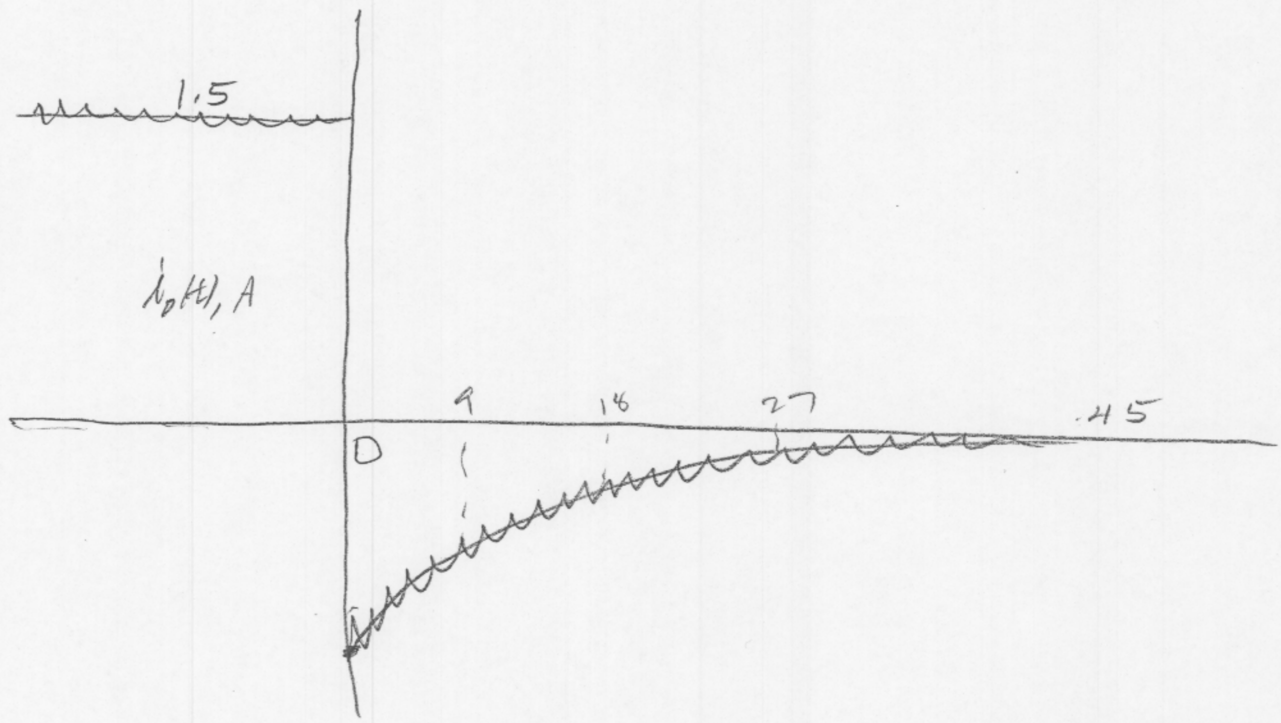
$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)] e^{-\frac{t}{\tau}}$$

$$V_c(t) = 9 e^{-\frac{t}{9}}$$

$$i_0(t) = C \frac{dV_c}{dt} = 1 \times \left(-\frac{1}{9} \times 9 e^{-\frac{t}{9}}\right)$$

$$i_0(t) = -e^{-\frac{t}{9}} \quad A \qquad t > 0$$

$$i_0(t) = 1.5 A \qquad t < 0$$



(3) You are given the circuit below. Find $V_c(t)$, $t \geq 0$. Sketch $i_o(t)$ $-1 \leq t < \infty$.

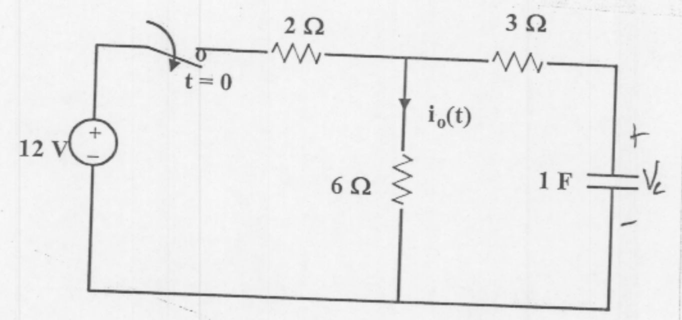
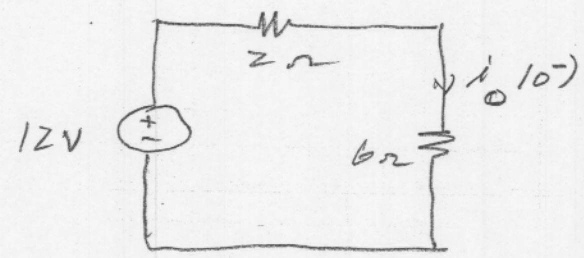


Figure 3: Circuit for problem 3.

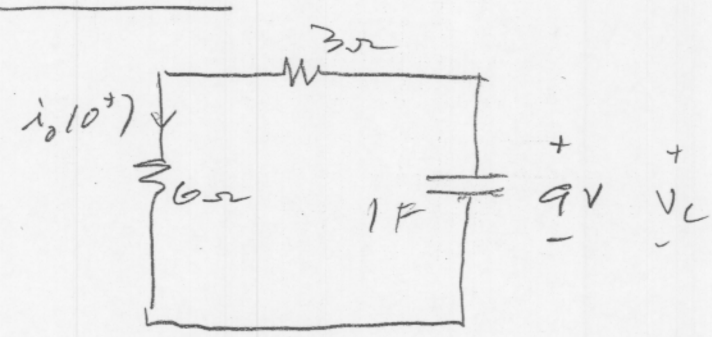
For $t < 0$, the circuit is



$$i_o(0^-) = \frac{12}{8} = \frac{3}{2} = 1.5 \text{ A}$$

$$V_c(0^-) = V_c(0^+) = 1.5 \times 6 = 9V$$

For $t > 0$



$V_c(0^+) = 9V, \quad V_c(\infty) = 0$

(3) continued

$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]e^{-\frac{t}{\tau}}$$

$$\tau = RC = 9 \text{ sec}$$

$$\boxed{V_c(t) = 9e^{-\frac{t}{9}}} \text{ V}$$

From the circuit,

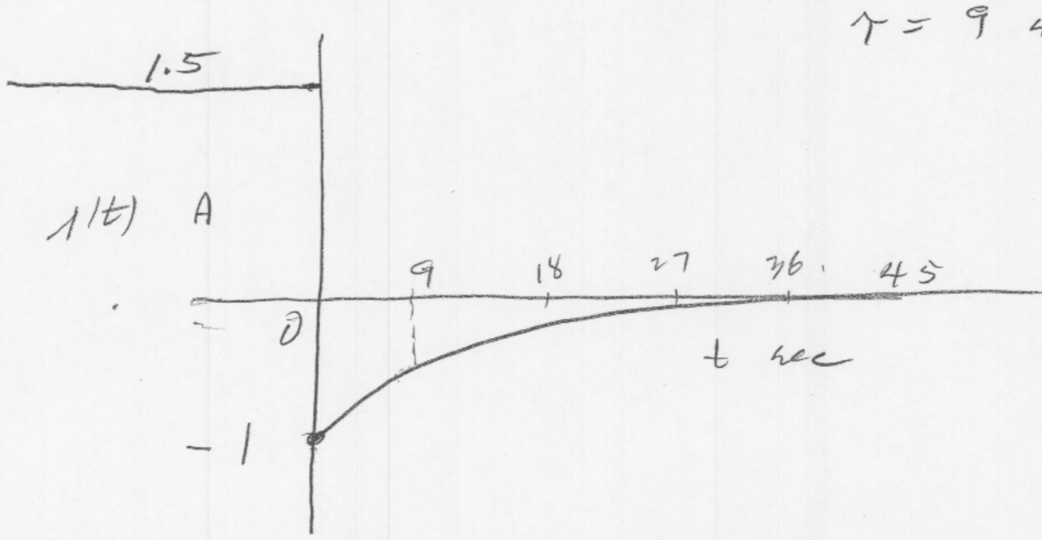
$$\boxed{i_o(0^-) = \frac{12}{8} = 1.5 \text{ A}}$$

For $t > 0$

$$i = C\frac{dV_c}{dt} = 1 \times \frac{d}{dt}\left[9e^{-\frac{t}{9}}\right]$$

$$\boxed{i(t) = -e^{-\frac{t}{9}}} \text{ A} \qquad t \geq 0$$

$$\tau = 9 \text{ sec}$$



(4) You are given the following circuit. Find $V_o(t)$, $t \geq 0$.

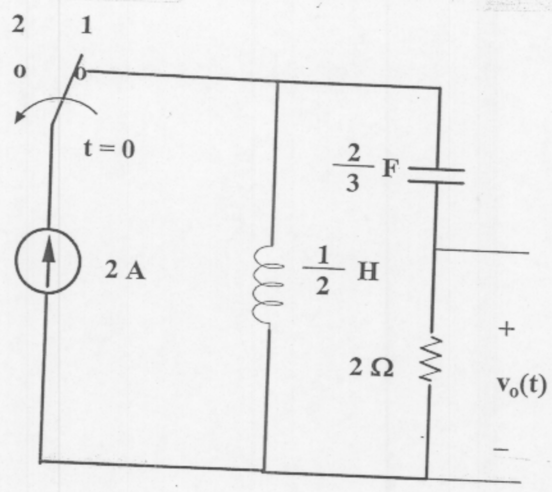
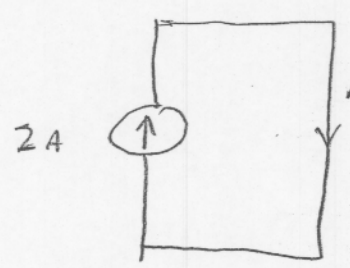


Figure 4: Circuit for problem 4.

FOR $t < 0$: Inductor acts like short; capacitor acts like open. We have

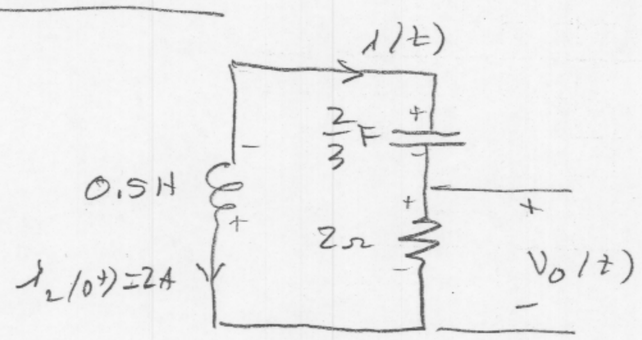


$i_L(0^-) = 2A = i_L(0^+)$

Circuit 1

$V_o(0^-) = 0 \; V$; $i_L(0^+) = 2A$

FOR $t > 0$



Circuit 2

$\frac{R}{1/2 \times 2/3}$

$L\frac{di}{dt} = +4$ $\frac{di}{dt} = +8$

(4) Cont.

Around ckt 2 in the clockwise direction we write

$$L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + \cancel{V(0^+)}^{\,0} + i(0^+)R = 0 \quad (1)$$

At $t = 0^+$

$$\frac{di(0^+)}{dt} = -\frac{i(0^+)R}{L}$$

but $i(0^+) = -i_L(0^+) = -2\,A$

$$\boxed{\frac{di(0^+)}{dt} = \frac{-(-2\times 2)}{.5} = 8\ A/sec}$$

So we have for initial conditions,

$\boxed{i(0^+) = -2\,A}$ $\boxed{\frac{di(0^+)}{dt} = 8\ A/sec}$

Take the derivative of (1);

$$L\frac{d^2i}{dt^2} + \frac{i(t)}{C} + R\frac{di}{dt} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i(t)}{LC} = 0$$

Characteristic equation is

$$s^2 + \frac{R}{2}s + \frac{1}{LC} = 0$$

with numbers

$$s^2 + 4s + 3 = 0$$

$$(s+1)(s+3) = 0 \qquad \text{overdamped}$$

(7) CONTINUED

$$i(t) = Ae^{-3t} + Be^{-t} \quad A$$

$$i(0) = \boxed{-2 = A + B}$$

$$\frac{di}{dt} = -3Ae^{-3t} - Be^{-t} \quad A$$

at $t = 0^+$

$$8 = -3A - B$$

so

$$\begin{bmatrix} 1 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

$$A = -3, \quad B = 1$$

$$i(t) = \left[-3e^{-3t} + e^{-t}\right] A$$

$$V_0(t) = 2 \times i(t) \quad V$$

$$\boxed{V_0(t) = 2\left[e^{-t} - 3e^{-3t}\right] V}$$