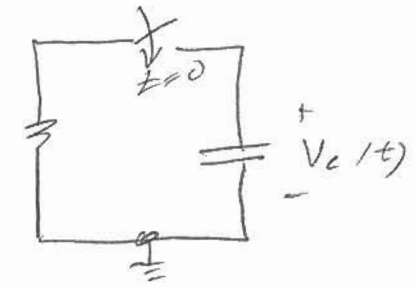
ECE 301
Transients Part II

<u>RC & RL transients</u>

The force free RC ckt.



$V_c(0) = V_c$

$$\frac{V_c}{R} + C\frac{dV_c}{dt} = 0$$

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = 0$$

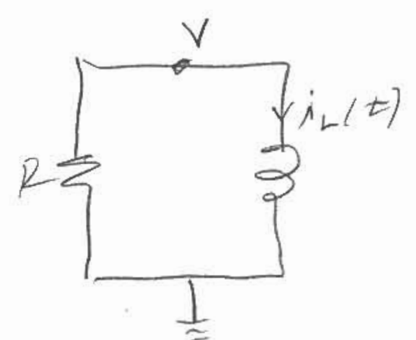
$$s + \frac{1}{RC} = 0 \quad \text{ch. eq}$$

$$V_c(t) = Ke^{-\frac{t}{RC}} = Ke^{-\frac{t}{\tau}}$$

$\tau = RC$
$K = V_c$

$$\boxed{V_c(t) = V_c e^{-\frac{t}{RC}}}$$

The force free RL ckt



$i_L(0) = I_L$

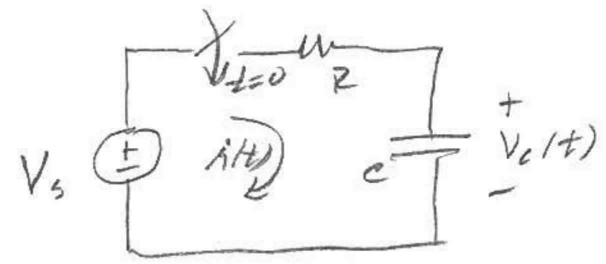
$$\frac{V}{R} + i_L = 0 \qquad V = L\frac{di_L}{dt}$$

$$\frac{L}{R}\frac{di_L}{dt} + i_L = 0$$

$$\frac{di_L}{dt} + \frac{i_L}{L/R} = 0 \qquad \tau = \frac{L}{R}$$

$$\boxed{i_L(t) = I_L e^{-\frac{Rt}{L}}}$$

FORCED RC ckt



$$V_s = Ri + V_c \qquad i = C\frac{dV_c}{dt}$$

$$RC\frac{dV_c}{dt} + V_c = V_s$$

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_s}{RC}$$

$$V_c = V_{pc} + V_{nc}$$

$$V_{pc} = K$$

$$\frac{K}{RC} = \frac{V_s}{RC}$$

$$K = V_s = V_c(\infty)$$

$$V_C(t) = V_s + K_N e^{-\frac{t}{RC}}$$

$$V_C(0) = 0 = V_C(0^+) = V_s + K_N$$

$$K_N = V_C(0^+) - V_s = V_C(0^+) - V_s(\infty)$$

$$\boxed{V_C(t) = V_s - V_s e^{-\frac{t}{\tau}}}$$

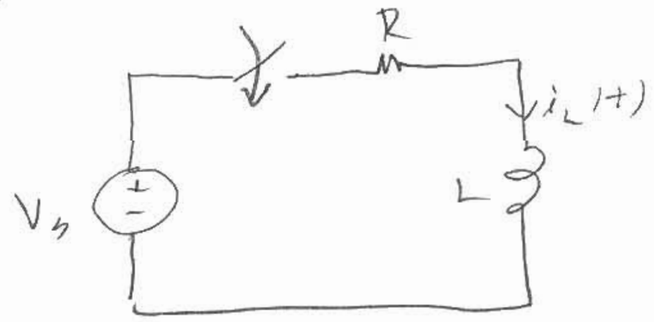
Also

$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)] e^{-\frac{t}{\tau}}$$

$\tau = RC$, $V_C(\infty) = V_s$, This case $V_C(0) = 0$

$$\boxed{V_C(t) = V_s - V_s e^{-\frac{t}{RC}}}$$

For the forced RL ckt



$$V_s = R i_L + L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} + \frac{i_L}{L/R} = \frac{V_s}{L}$$

differential equation tech.

$i_L = i_{PL} + i_{NL}$

$i_{PL} = K$

$\frac{RK}{L} = \frac{V_s}{L}$

$K = \frac{V_s}{R} = i_L(\infty)$

$i_L(t) = \frac{V_s}{R} + K_N e^{-\frac{Rt}{L}}$

$i_L(0^+) = 0$ (current can't change instantaneously through the inductor)

$0 = \frac{V_s}{R} + K_N$

$K_N = -\frac{V_s}{R}$

$i_L(t) = \frac{V_s}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

step-by-step tech.

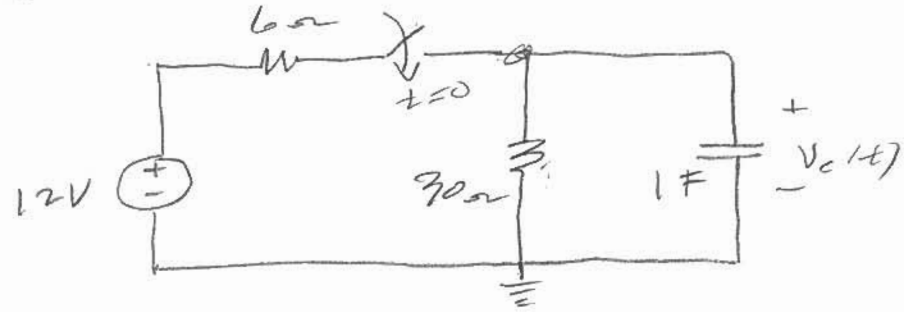
Also

$$\boxed{i_L(t) = i_L(\infty) + \left(i_L(0^+) - i_L(\infty)\right) e^{-\frac{t}{\tau}}}$$

$i_L(\infty) = \frac{V_s}{R}$, $i_L(0^+) = 0$ (here)

Example

GIVEN



All IC's are zero.

(a) solve for $V_c(t)$ using D.E. technique,

(b) solve for $V_c(t)$ using step-by-step technique

$t<0$; $V_c(0^-) = V_c(0^+) = 0$

(a) using nodal analysis:

$$\frac{V_c - 12}{6} + \frac{V_c}{30} + \frac{dV_c}{dt} = 0$$

$$5V_c - 60 + V_c + 30\frac{dV_c}{dt} = 0$$

$$30\frac{dV_c}{dt} + 6V_c = 60$$

$$\frac{dV_c}{dt} + \frac{V_c}{5} = 2$$

$$V_c = V_{cp} + V_{cN}$$

$$V_{cp} = K_p$$

$$\frac{K_p}{5} = 2$$

$$K_p = 10$$

$$V_c(t) = 10 + K_H e^{-0.2t}$$

$$0 = 10 + K_H$$

$$\boxed{V_c(t) = 10 - 10e^{-0.2t}}$$

(b) Step-by-step

$$V_c(0^+) = 0, \quad V_c(\infty) = \frac{12 \times 30}{36}$$

$$V_c(\infty) = 10 \text{ V}$$

$$R_{eq} = \frac{30 \times 6}{30 + 6} = \frac{180}{36} = 5$$

$$R_{eq} C = \tau = 5$$

$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)] e^{-\frac{t}{\tau}}$$

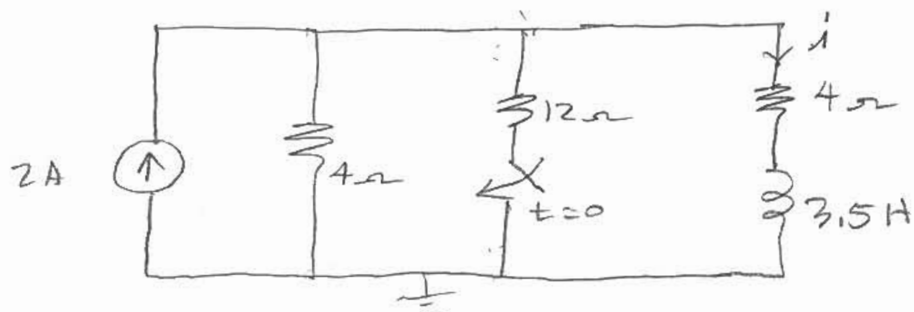
$$\boxed{V_c(t) = 10 - 10e^{-0.2t}}$$

Example 2

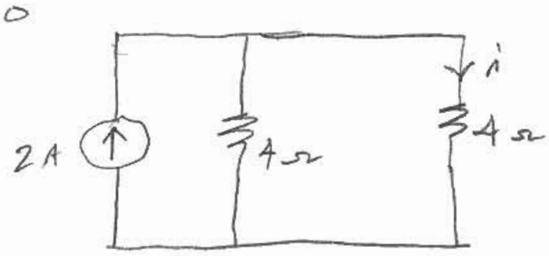
For the circuit below, obtain the expression for $i(t)$ using

(a) d.e. technique

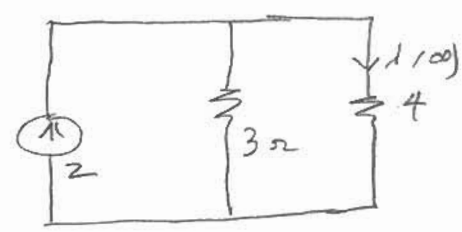
(b) step-by-step technique



$t < 0$



$t = \infty$



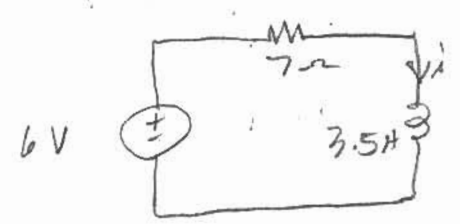
$i(0^-) = 1\,A = i(0^+)$

$\frac{4 \times 12}{4+16} = 3\,\Omega$

$i(\infty) = \frac{2 \times 3}{7} = \frac{6}{7}\,A$

(a) d.e. approach

Using Thevenin



$$7i + 3.5\frac{di}{dt} = 6$$

$$\frac{di}{dt} + 2i = \frac{6}{3.5}$$

$i = i_p + i_N$

$i_p = k_p$

$2k_p = \frac{6}{3.5}$

$k_p = \frac{3}{3.5} = 0.857$

$i = \frac{3}{3.5} + k_N e^{-2t} = 1$

$k_N = 1 - \frac{3}{3.5} = \frac{.5}{3.5}$

$$\boxed{i(t) = \frac{3}{3.5} + \frac{0.5}{3.5} e^{-2t}}$$

(5) Step by step

$i(\infty) = \frac{2 \times \frac{1}{4}}{\frac{1}{12} + \frac{1}{4} + \frac{1}{4}} = \frac{6}{1 + 3 + 3} = \frac{6}{7}$

$i(\infty) = \frac{3}{3.5}$

$i(0^+) = 1\,A$

$i(t) = \frac{3}{3.5} + \left(1 - \frac{3}{3.5}\right) e^{-\frac{t}{\tau}}$

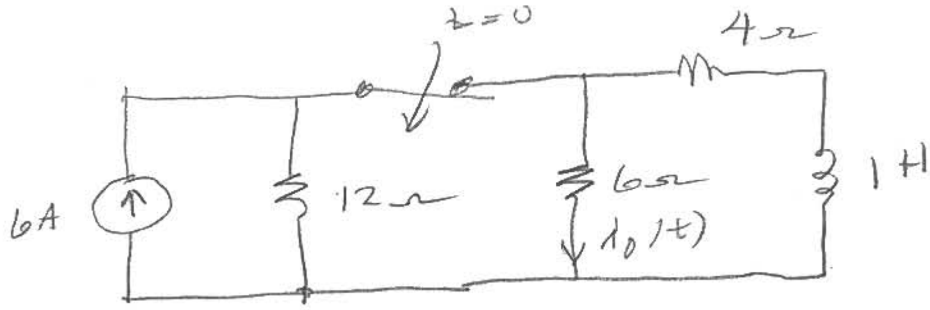
$\tau = \frac{L}{R} = \frac{3.5}{3+4} = \frac{3.5}{7} = \frac{1}{2}$

$$\boxed{i(t) = \frac{3}{3.5} + 0.5\, e^{-2t}}$$

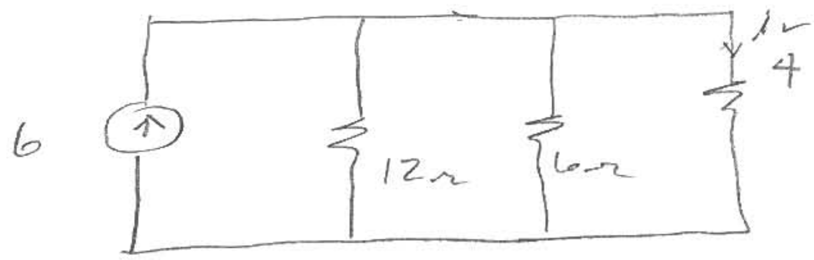
6.27

Example 3

Find $i_0(t)$



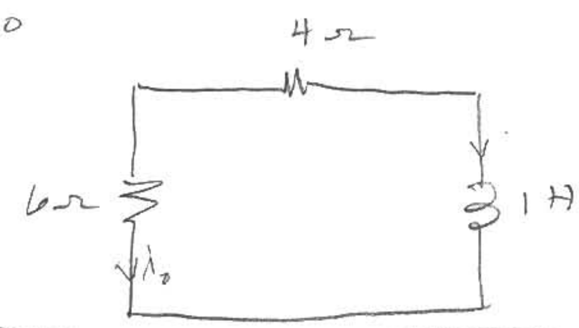
$t < 0$



$$i_L(0^-) = i_L(0^+) = \frac{6 \times \frac{1}{4}}{\frac{1}{12} + \frac{1}{6} + \frac{1}{4}}$$

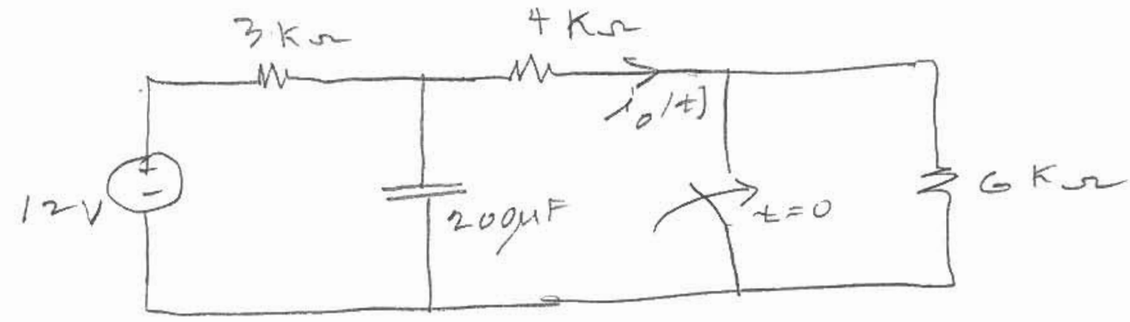
$$i_L(0^+) = \frac{18}{1 + 2 + 3} = \frac{18}{6} = 3\text{ A}$$

$t > 0$



$$\boxed{i_0 = -3e^{-10t}}$$

Example 4



Find $i_o(t)$.

On Your Own.

Ans: (Answer book) Not verified

$$i_o(t) = 2 + 0.5e^{-3.75t}, A \quad t>0$$