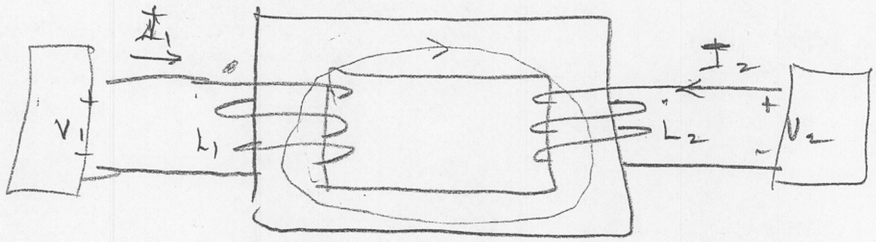
#10

Transformers

The linear transformer

Core of materials such as air, plastic, wood. (non metal)

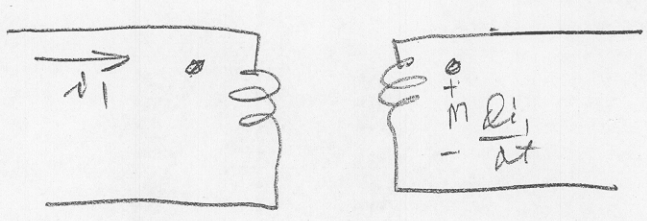


$$V_1 = L_1 \frac{di_1}{dt} \pm L_{12} \frac{di_2}{dt}$$

$$V_2 = \pm L_{21} \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

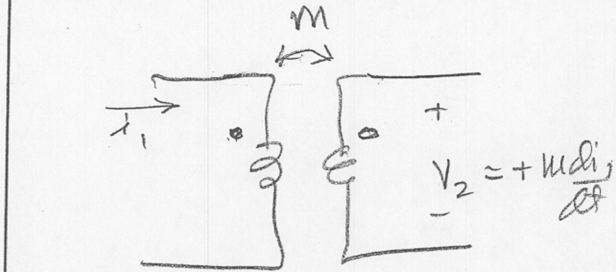
$L_{12} = L_{21} = M$ mutual inductance, H

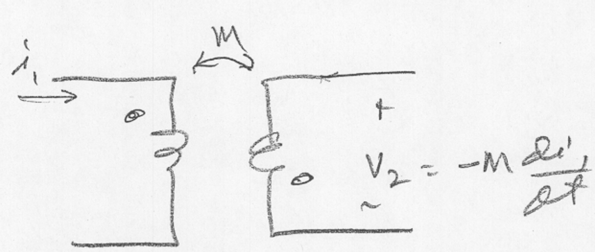
About (•) dots

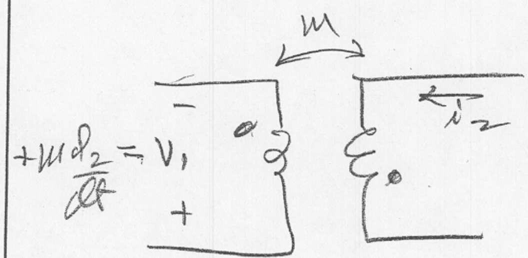


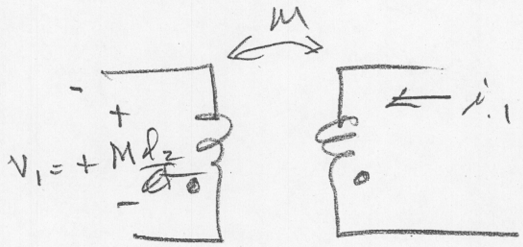
If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the 2nd coil.

Some examples







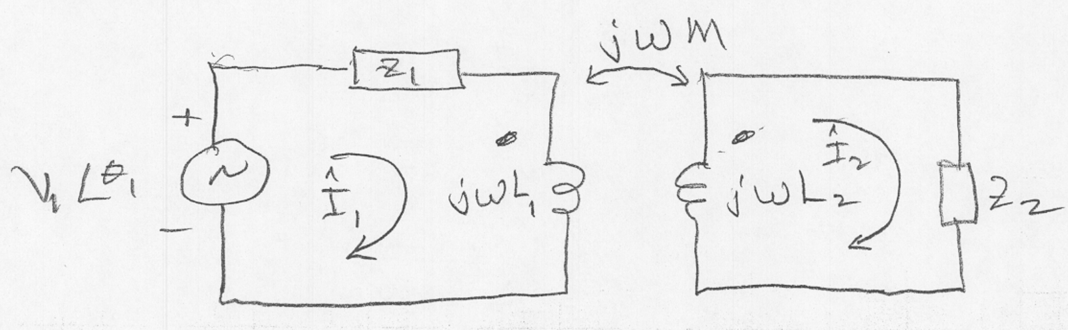


Recall from Laplace transforms

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s)$$

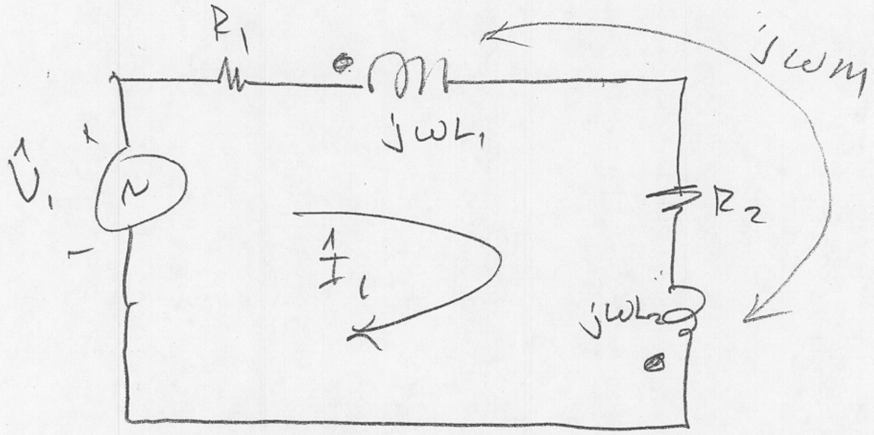
or $\mathcal{L}\left[\frac{d}{dt}\right] \longrightarrow s \longrightarrow j\omega$

So in the A.C. circuit of a transformer



$$\begin{cases} (j\omega L_1 + Z_1)\hat{I}_1 - j\omega M \hat{I}_2 = V_1\angle\theta_1 \\ -j\omega M \hat{I}_1 + (Z_2 + j\omega L_2) I_2 = 0 \end{cases}$$

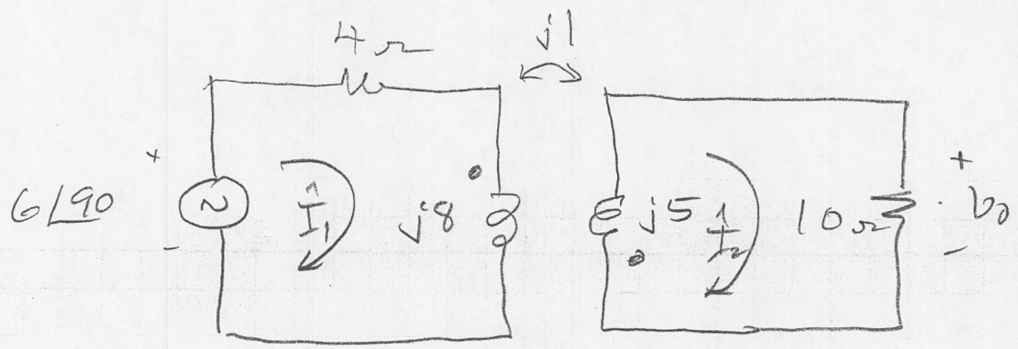
There are cases such as below.



$$(R_1 + R_2 + j\omega L_1 + j\omega L_2)\hat{I}_1 - j\omega M I_1 - j\omega M I_1 = V_1$$

$$(R_1 + R_2 + j\omega L_1 + j\omega L_2 - j2\omega M) I_1 = \hat{V}_1$$

Example



Find I_1 and I_2 and V_0

$$(4+j8)I_1 + jI_2 = 6\angle 90$$

$$jI_1 + (10+j5)I_2 = 0$$

$$I_1 = .5409 + j0.306 \text{ A}$$

$$I_2 = 0.06\angle -89.43^\circ \text{ A}$$

$$V_0 = 10 I_2 = 0.6\angle -89.43^\circ \text{ V}$$

It can be shown that energy stored in the transformer is given by

$$W = \tfrac{1}{2}L_1 i_1^2 + \tfrac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

plus sign used if both currents enter the dots or both currents enter the undots.

Degree of Coupling

By using the energy equation that is

$$\tfrac{1}{2}L_1 i_1^2 + \tfrac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0$$

We can get

$$\sqrt{L_1 L_2} - M > 0$$

$$M \leq \sqrt{L_1 L_2}$$

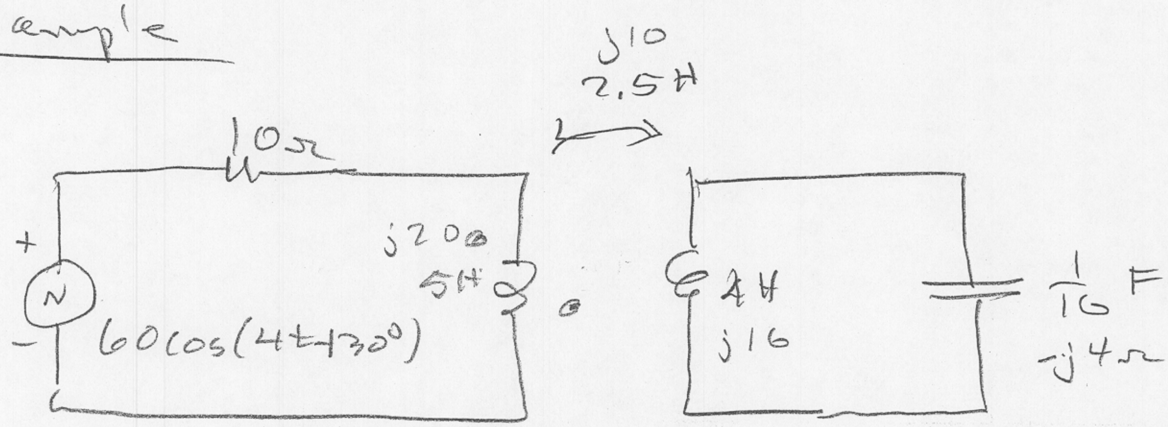
We define the coefficient of coupling, K, as

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

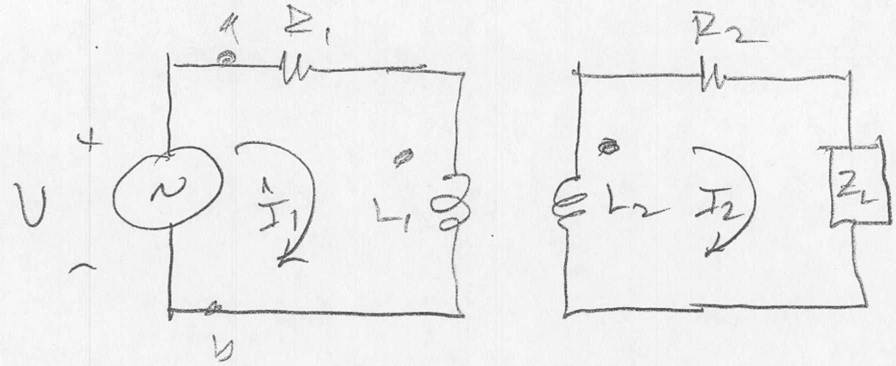
Sometimes we are given K and L_1, L_2 and must solve for M, etc.

$$0 \leq K \leq 1$$

Example



$I_1 = 60\angle 30^\circ$ A , $I_2 = 3.254\angle 160.6^\circ$ A



We want to find the impedance seen looking into a-b.

$$V = (R_1 + j\omega L_1) I_1 - j\omega M \hat{I}_2$$

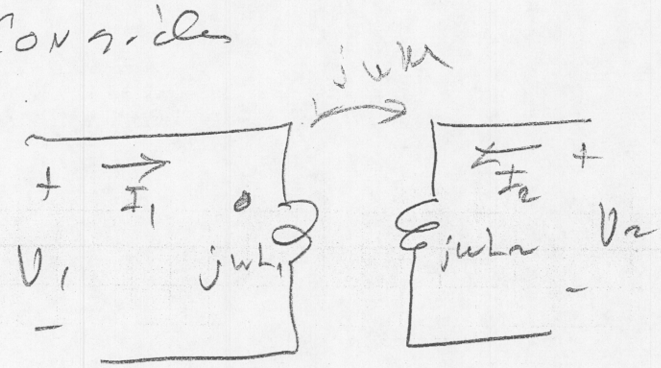
$$0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L) I_2$$

$$\boxed{Z = \frac{V}{I_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + Z_L + j\omega L_2}}$$

Can be useful

$L = \frac{N^2 \mu A}{l}$ The Ideal transformer

Consider



$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

solve for I_1 from (1)

$$I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1}$$

$$V_2 = \frac{j\omega M (V_1 - j\omega M I_2)}{j\omega L_1} + j\omega L_2 I_2$$

$$V_2 = j\omega L_2 I_2 + \frac{M V_1}{L_1} - \frac{j\omega M^2}{L_1} I_2$$

$$M = \sqrt{L_1 L_2}$$

perfect coupling

$$V_2 = \cancel{j\omega L_2 I_2} + \frac{M V_1}{L_1} - \cancel{j\omega \frac{L_1 L_2}{L_1} I_2}$$

$$V_2 = + \frac{\sqrt{L_1 L_2}\, V_1}{L_1} = \sqrt{\frac{L_2}{L_1}}\, V_1$$

$$V_2 = \frac{N_2}{N_1} V_1$$

If we assume ideal transformer

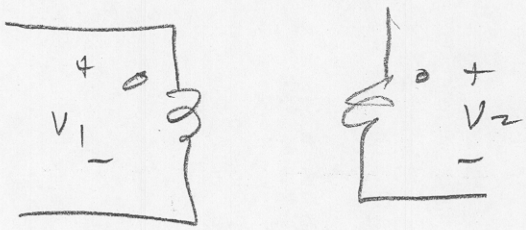
- L_1, L_2, $M \to \infty$
- coupling $k = 1$
- primary and secondary $R_1 = R_2 = 0$ (lossless)

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$V_1 i_1 = V_2 i_2$$

$$\boxed{\frac{V_2}{V_1} = \frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

<u>Sign considerations</u>



Polarities point to dots

$$\frac{V_2}{V_1} = + \frac{N_2}{N_1}$$

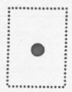
current: current enters •, produces a current that leaves the other dot,

If I_1 and I_2 <u>both</u> enter into OR both leave the dotted terminals use $-n$, otherwise use $+n$

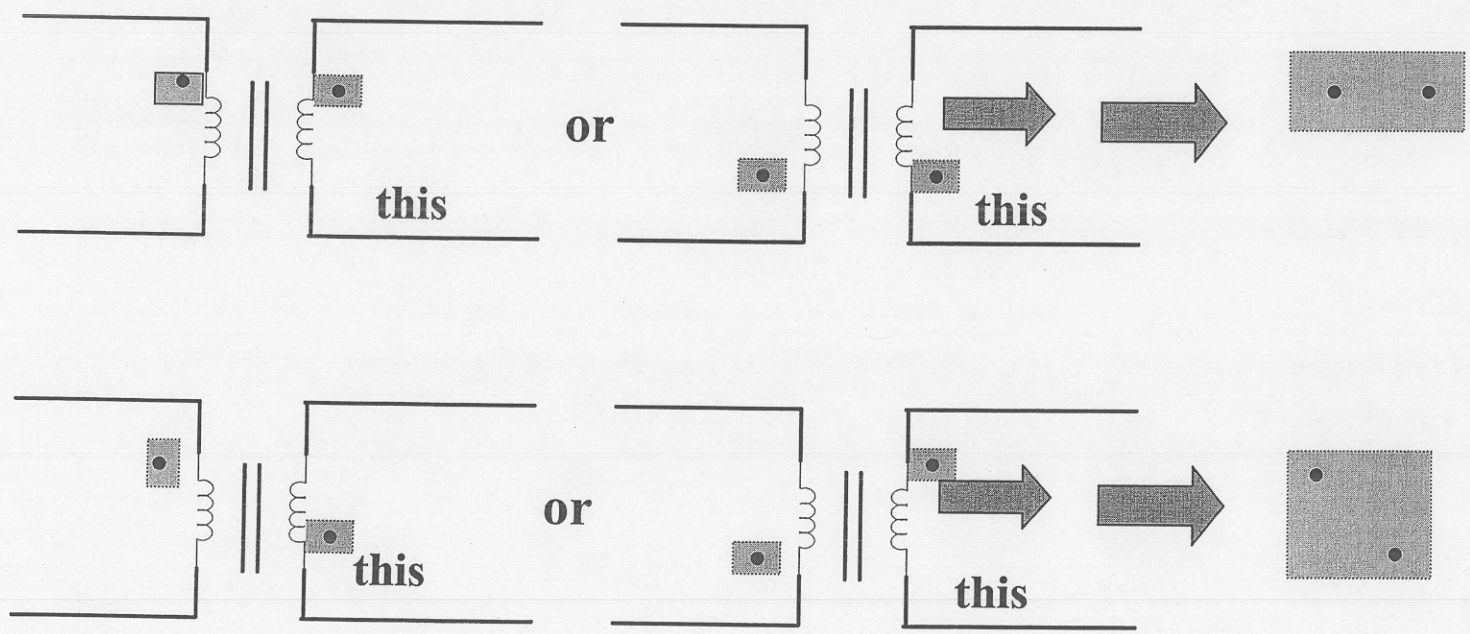
If V_1 and V_2 are both positive OR both negative at the dotted terminals use $+n$.

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{n^2}\frac{V_2}{I_2} = \frac{Z_L}{n^2}$$

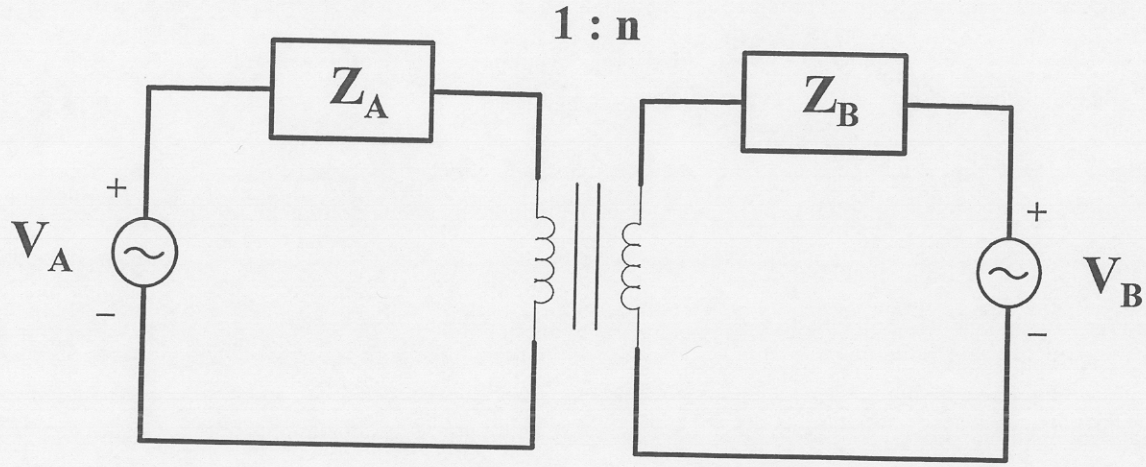
THE IDEAL TRANSFORMER

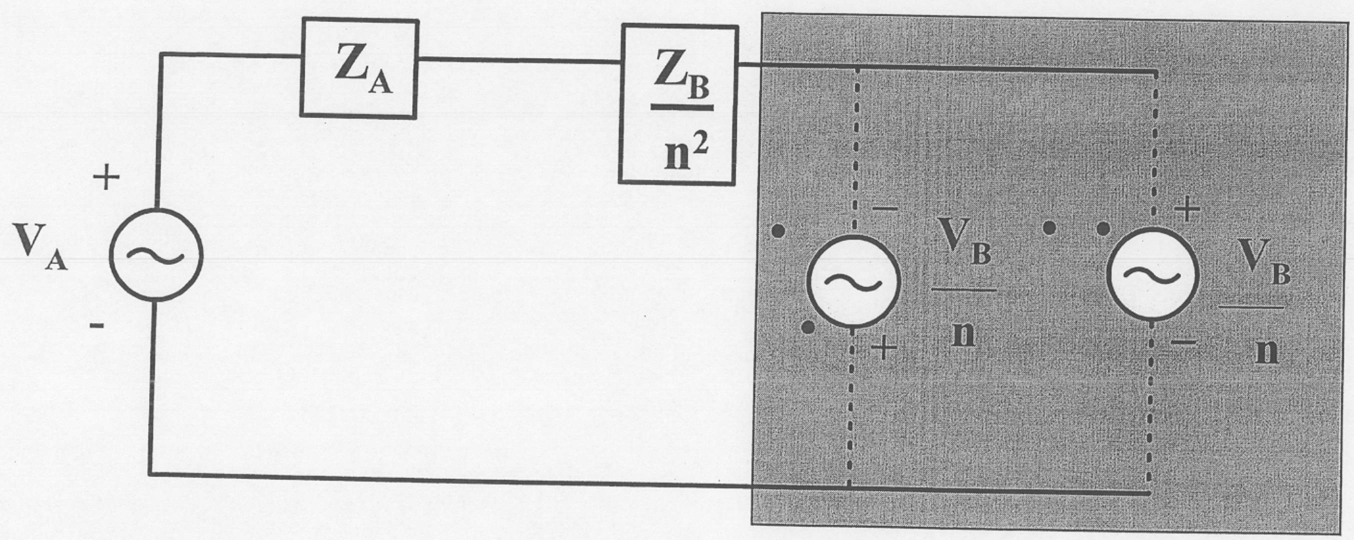


Thevenin Considerations:



THE IDEAL TRANSFORMER





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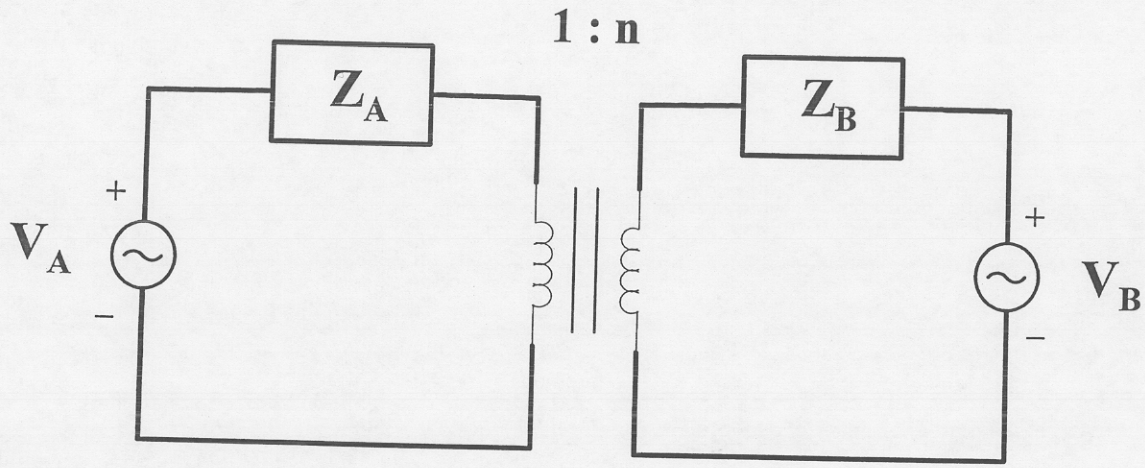

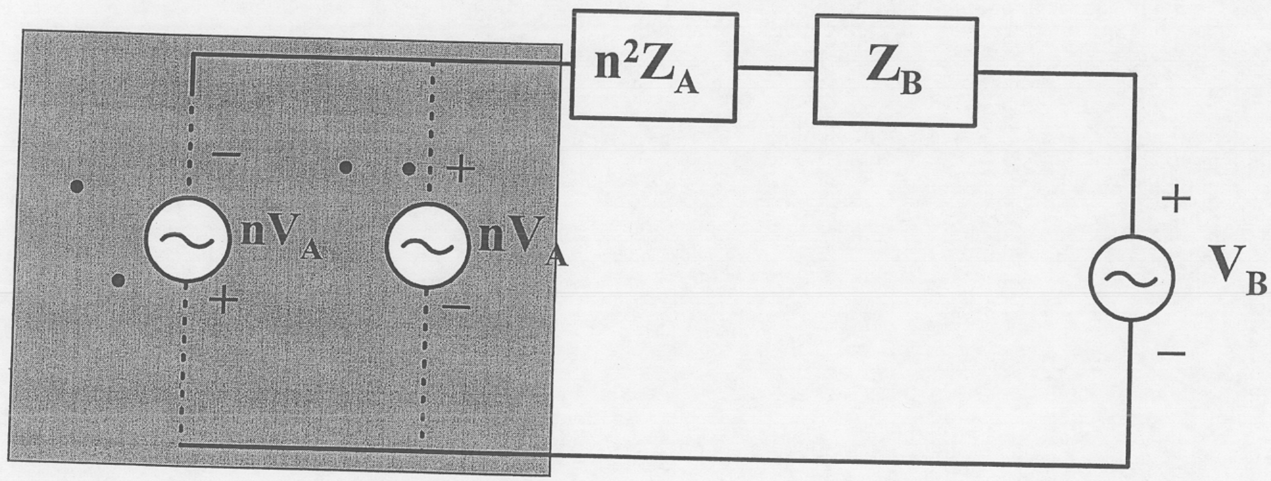

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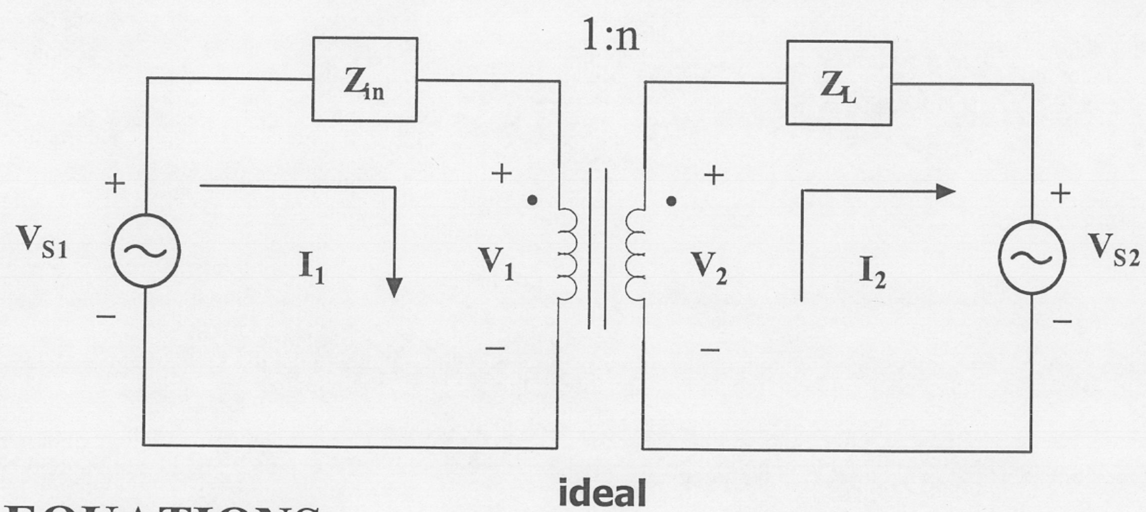
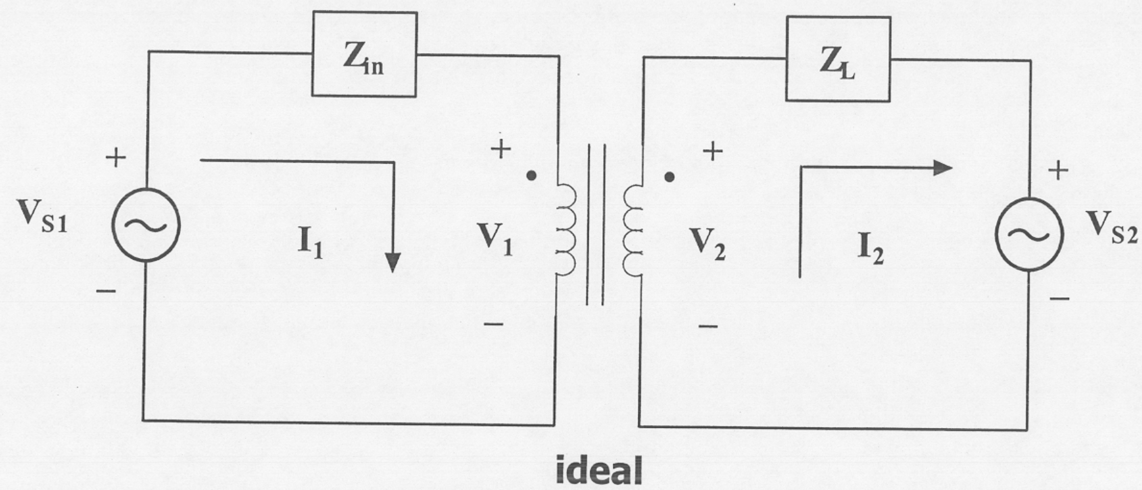


BASIC EQUATIONS:

$$\frac{V_2}{V_1} = n \qquad \frac{I_1}{I_2} = n$$

$$I_1 Z_{in} + V_1 = V_{S1}, \qquad I_2 Z_L - V_2 = -V_{S2}$$

THE IDEAL TRANSFORMER



Rearrange previous equations

Matrix

$$\begin{bmatrix} Z_{in} & 0 & 1 & 0 \\ 0 & Z_L & 0 & -1 \\ 0 & 0 & -n & 1 \\ 1 & -n & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{s1} \\ -V_{s2} \\ 0 \\ 0 \end{bmatrix}$$