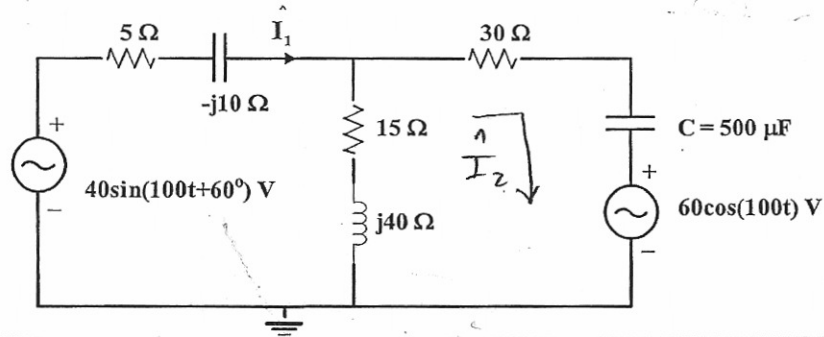


ECE 301
Test 3A

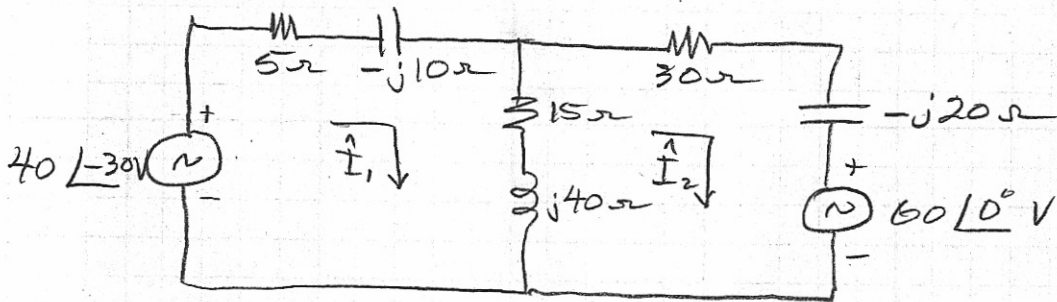
(1) You are given the circuit of Figure 1. Find the current \hat{I}_1 as an rms phasor current.



$$40 \sin(100t + 60^\circ) \rightarrow 40 \cos(100t - 30^\circ) \rightarrow 40 \angle -30^\circ$$

$$\frac{1}{\omega C} = \frac{1}{1 \times 10^2 \times 5 \times 10^{-4}} = 0.2 \times 10^2 = 20$$

Circuit becomes



This results in the following matrix

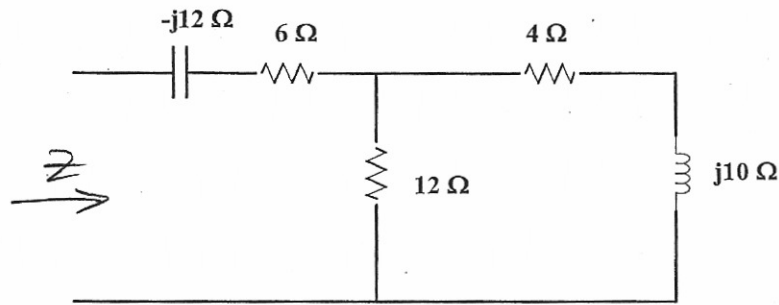
$$\begin{bmatrix} 20 + j20, & -15 - j40 \\ -15 - j40, & 45 + j20 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 40 \angle -30^\circ \\ -60 \end{bmatrix}$$

$$\hat{I}_1 = (0.109 - j1.59) \text{ V}$$

$$\hat{I}_1 = 1.596 \angle -86^\circ \text{ V}$$

$$\hat{I}_1 = 1.1285 \angle -86^\circ \text{ V rms}$$

(2) Find the power factor for the following circuit. Indicate which case of leading/lagging.



Find Z

$$Z = (6 - j12) + 12 \parallel (4 + j10)$$

$$= (6 - j12) + \frac{12(4 + j10)}{16 + j10}$$

$$Z = 14 \angle -34.6^\circ \Omega$$

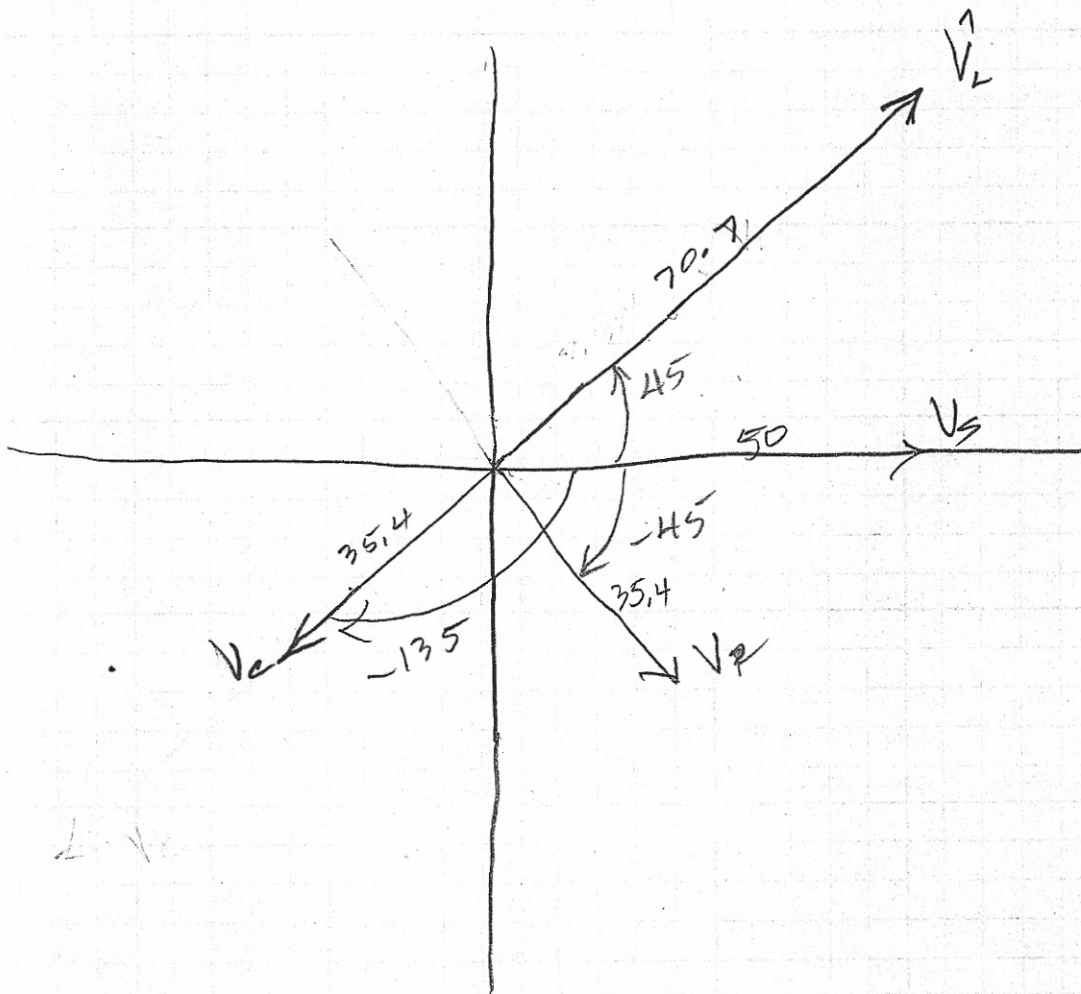
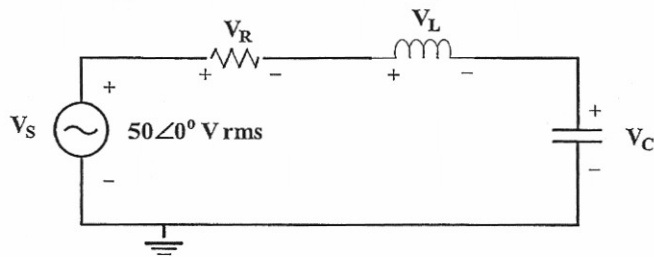
[pf = angle of Z

$$pf = \cos(-34.6) = 0.823 \text{ leading}$$

$$pf = 0.823 \text{ leading}$$

A.

- (3) For the circuit of Figure 3, the RMS phasor voltages (as indicated) are $V_R = 35.4 \angle -45^\circ$ V rms, $V_L = 70.7 \angle 45^\circ$ V rms, $V_C = 35.4 \angle -135^\circ$ V rms. Draw the phasor diagram (approx. to scale) showing these voltages and the source voltage V_S all on the same diagram.



(4) You are given the circuit of Figure 4.

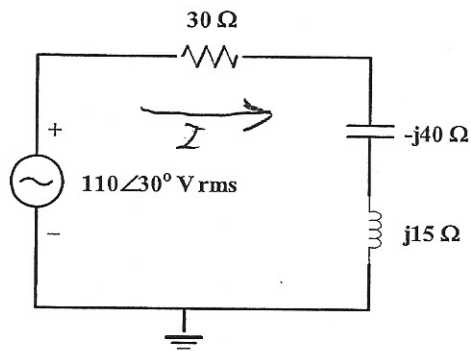


Figure 4: Circuit for problem 4.

- Determine the average complex power supplied by the source.
- Determine the average real power delivered (absorbed) by the resistor.
- Determine the average reactive power supplied to the inductor.
- Determine the average reactive power supplied to the capacitor.
- Draw (sketch) the complex power triangle for the circuit.
- Determine the power factor including leading/lagging.

$$(a) \vec{I} = \frac{110 \angle 30^\circ}{(30 - j25)} = 2.82 \angle 70^\circ \text{ A rms}$$

$$\vec{S} = \vec{V}_s \vec{I}^* = (110 \angle 30^\circ)(2.82 \angle -70^\circ) = (237.6 - j199.4) \text{ VA}$$

$$\vec{S} = 310.2 \angle -40^\circ \text{ VA} = (237.6 - j199.4) \text{ VA}$$

$$(b) P = \text{Re}[\vec{S}] = 237.6 \text{ W}$$

$$(c) \vec{S}_L = jQ_L = |\vec{V}_L \vec{I}^* = j15 \vec{I} \times \vec{I}^* = j15 |\vec{I}|^2$$

$$\vec{S}_L = jQ_L = j119.3 \text{ VA (VARs)}$$

$$Q_L = 119.3 \text{ VARs}$$

$$(d) \text{ similarly: } \vec{S}_C = jQ_C = (-j40) |\vec{I}|^2 = -j318.1 \text{ VA}$$

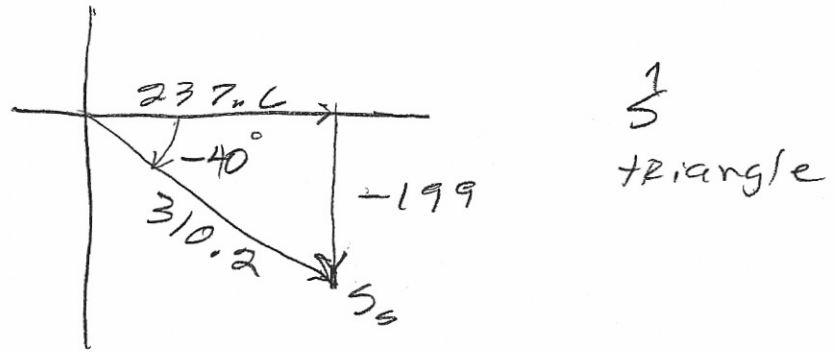
$$Q_C = -318.1 \text{ VARs}$$

$$\text{Note: } \vec{S}_{\text{overall}} = \vec{S}_{\text{supplied}} = 237.6 + j(119.3 - 318.1)$$

$$\vec{S}_{\text{overall}} = (237.6 - j199) \text{ VA}$$

(4) - continued

(e)



(f)

$$pf = \cos(\text{angle of } \vec{S})$$

$$pf = \cos(-40) = 0.766 \text{ leading}$$

#

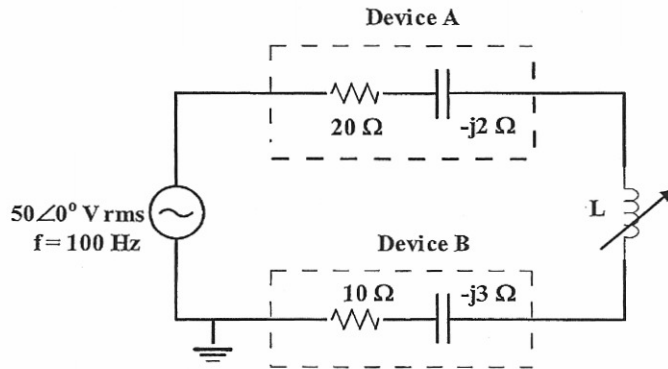
In part (a) you can find \vec{S} by other methods. For example

$$\vec{S} = \frac{|V_{\text{rms}}|^2}{Z_{\text{circuit}}} = \frac{(110)^2}{(30 - j25)^*}$$

$$\begin{aligned} \vec{S} &= (238.03 - j198.4) \text{ VA} \\ &= 310 \angle -40^\circ \text{ VA} \end{aligned}$$

A

- (5) The circuit shown in Figure 5 is used for experimental work in an aerospace project. It is required that the current in the circuit be in phase with the source voltage. The frequency of the source voltage is 100 Hz. The inductor at the output of the circuit is variable. What must be the value of L so that the requirements concerning the phase of the source voltage and current are met?



To get 0 phase shift between the circuit current and the supply voltage requires Z to have a phase angle of zero.

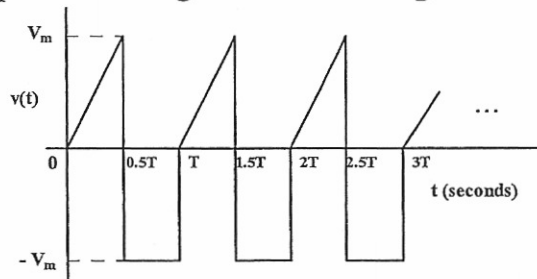
$$\text{Now, } Z = 30 - j5 + j\omega L$$

$$\text{so } \omega L = 5 \quad (\omega = 377)$$

$$L = \frac{5}{377} = 0.013 \text{ H}$$

A

(6) You are given the periodic voltage waveform of Fig 6. Find the RMS value of the voltage.



$$V_{RMS} = \sqrt{\frac{1}{T} \left[\int_0^{0.5T} \left(\frac{2V_m t}{T} \right)^2 dt + \int_{0.5T}^T V_m^2 dt \right]}$$

$$= \sqrt{\frac{1}{T} \left[\frac{4V_m^2 t^3}{3} \Big|_0^{0.5T} + \frac{V_m^2 T}{2} \right]}$$

$$= \sqrt{\frac{1}{T} \left[\frac{4V_m^2 T^3}{3 \times 8 \times T^2} + \frac{V_m^2 T}{2} \right]}$$

$$= V_m \sqrt{\frac{1}{6} + \frac{1}{2}}$$

$$= V_m \sqrt{\frac{2}{3}} = 0.816 V_m$$