Example problems using the step by step are presented below.

Example 1:

(From Irwin, p 278, 8th Edition)

In the circuit below, the switch has been open for a long time. It is closed at $t=0$. Find $i(t)$ for $t>0$. Use the step-by-step method

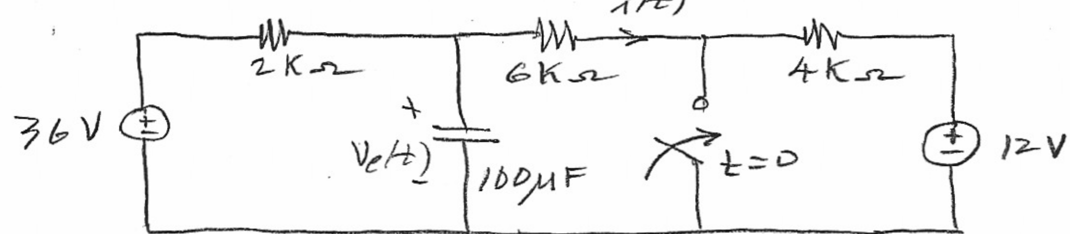


Fig 1

For $t<0$, we first determine $V_c(0^-)$ from the following circuit

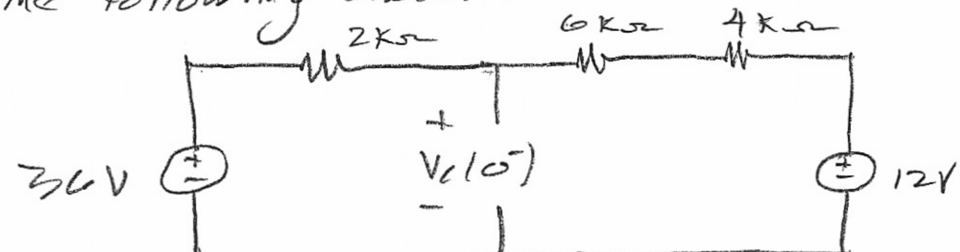


Fig 2

It is easy to find that $V_c(0^-) = 32V$

Now, at $t = 0^+$, we have the following circuit

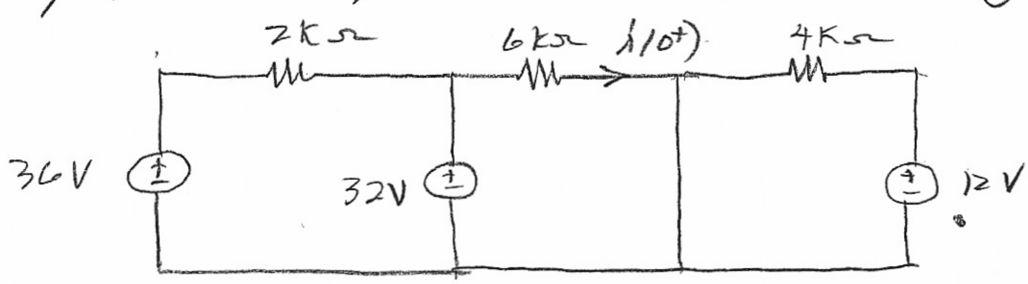


Fig 3.

Example 1 cont'd.

We find that

$$i(0^+) = \frac{32}{6K} = \frac{16}{3}\ mA$$

For $t \to \infty$, the circuit appears as follows.

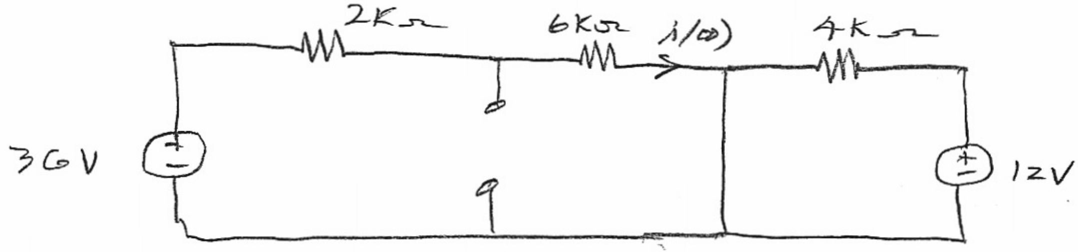


Fig 4

$$i(\infty) = \frac{36}{8} = \frac{9}{2}\ mA$$

To find τ

(Disable all independent sources and look into the terminals where the capacitor is located.

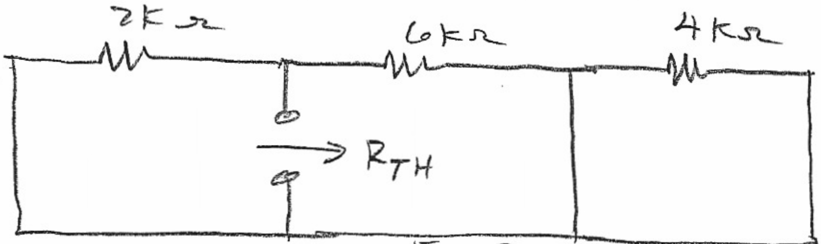


Fig 5

$$R_{TH} = \frac{2K \times 6K}{2K + 6K} = \frac{3}{2}\ kΩ \Rightarrow \tau = .15\ sec$$

Then $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/.15}$,

$$i(t) = \left[\frac{9}{2} + \frac{5}{6}e^{-\frac{t}{.15}}\right]\ mA$$

Example 2

You are given the following circuit, find $V_o(t)$ for $t>0$. Assume the switch has been closed for a long time and is opened at $t=0$.

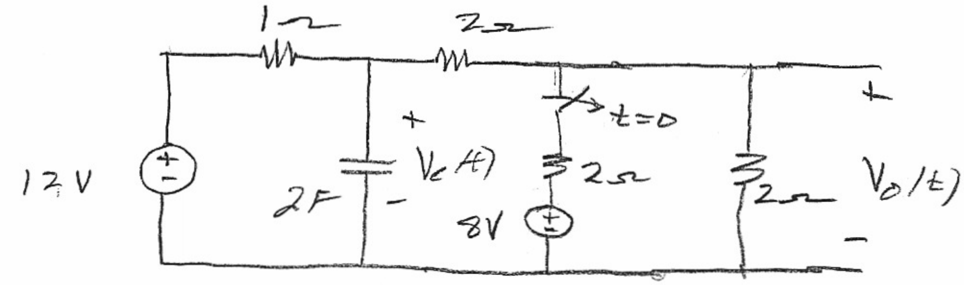


Fig 6

We use the following circuit to find $V_o(0^-)$ and $V_c(0^-)$.

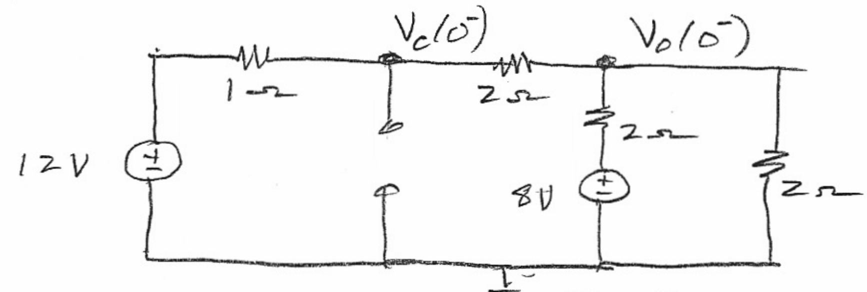


Fig 7

By nodal analysis;

At $V_c(0^-)$
$$\frac{V_c(0^-)-12}{1} + \frac{V_c(0^-)-V_o(0^-)}{2} = 0$$

OR

$$2V_c(0^-) - 24 + V_c(0^-) - V_o(0^-) = 0$$

$$\boxed{3V_c(0^-) - V_o(0^-) = 24}$$

At $V_o(0^-)$
$$\frac{V_o(0^-)-V_c(0^-)}{\cancel{2}} + \frac{V_o(0^-)-8}{\cancel{2}} + \frac{V_o(0^-)}{\cancel{2}} = 0$$

$$\boxed{-V_c(0^-) + 3 \cdot V_o(0^-) = 8}$$

Example 2: cont.

$$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} V_{c\infty} \\ V_{o\infty} \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \end{bmatrix}$$

$V_c(0^-) = V_c(0^+) = 10\ V$

$V_o(0^-) = 6\ V$

For $t = 0^+$, we have

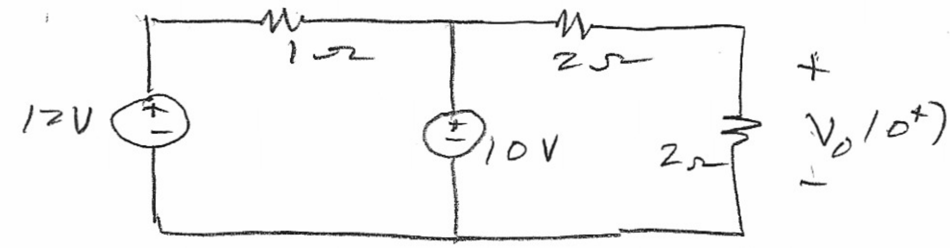
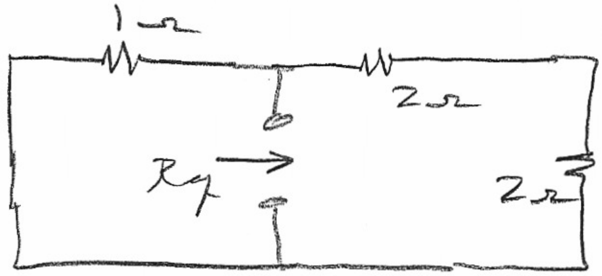


Fig 8

$$V_o(0^+) = \frac{10 \times 2}{2+2} = 5\ V$$

For determining R_{eq}



$R_{eq} = \frac{1 \times 4}{1+4} = \frac{4}{5}$

$R_{eq} = 0.8\ \Omega$

$R_{eq}C = 2 \times 0.8 = 1.6$ sec

Fig 9

We need $V_o(\infty)$. We find this by assuming the capacitor acts like an open circuit. We use Fig 10 to find $V_o(\infty)$.

Example 2; cont

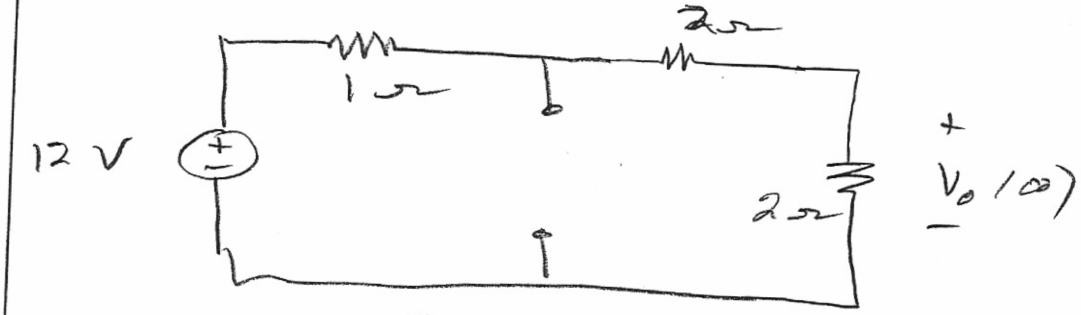


Fig 10

$$V_o(\infty) = \frac{2 \times 12}{1+2+2} = \frac{24}{5}\ V$$

so

$$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]e^{-\frac{t}{\tau}}$$

$$V_o(0^+) = \frac{24}{5} + \left[5 - \frac{24}{5}\right]e^{-\frac{5}{8}t}$$

$$V_o(0^+) = \left(\frac{24}{5} + \frac{1}{5}e^{-\frac{5}{8}t}\right)V, \quad u(t)$$

Example 3

In the following circuit, find $V_o(t)$. Sketch the waveform with the time axis calibrated.

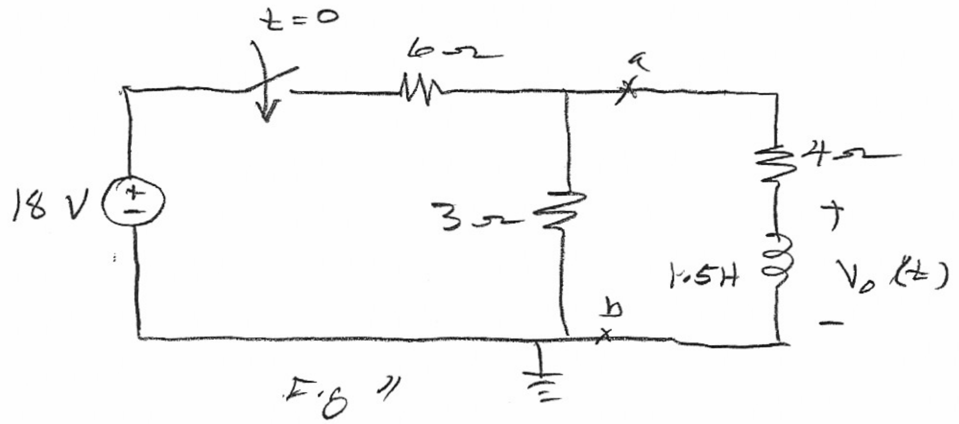


Fig 11

For $t<0$, $V_o(0^-)=0$, $i(0^-)=0=i(0^+)$

Just after the switch is closed ($t=0^+$) the inductor looks like an open circuit. No current flows through the 4Ω resistor. So the voltage $V_o(0^+) = V_{ab}(0^+)$. We find

$$V_{ab}(0^+) = V_o(0^+) = \frac{18 \times 3}{3+6} = 6V$$

We find R_{eq} in the following ckt.

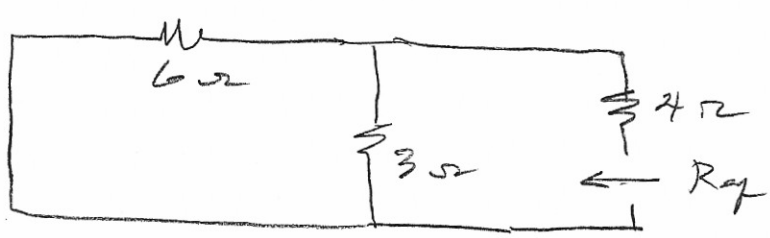


Fig 12

Example 3: cont

Therefore

$$R_{eq} = 4 + 3 \| 6 = 4 + 2$$

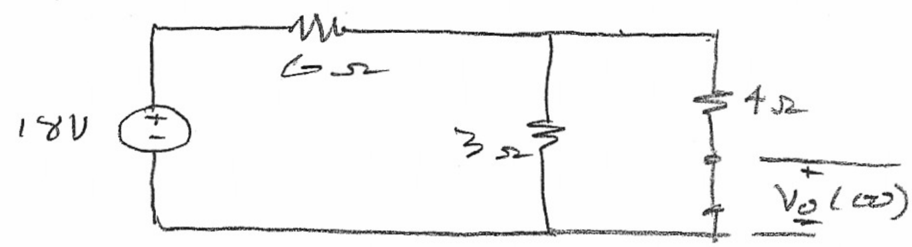
$$R_{eq} = 6\,\Omega$$

$$\tau = \frac{L}{R} = \frac{1.5}{6} = \frac{1}{4}$$

We then have

$$v_o(t) = v_o(\infty) + [v_o(0^+) - v_o(\infty)]\, e^{-\frac{t}{\tau}}$$

We find $v_o(\infty)$ from Fig 13



$v_o(\infty) = 0$

$$v_o(t) = v_o(0^+)\, e^{-\frac{t}{\tau}}$$

$$v_o(t) = 6\, e^{-4t}\ V, \quad u(t)$$