wly

Transformers

Lesson 17
Lesson 18

To start with transformers — due to the nature of magnetic circuits, are not as linear as RLC (dc & Ac) circuits. We make simplifying assumptions that lead to linear analysis. The analytical calculations produced compared to laboratory (field) measurements are not as close in agreement as RLC passive circuit. However, the results are tolerable.

Because of time constraints, the analysis presented here is abbreviated.

Basic Considerations

Linear Transformers

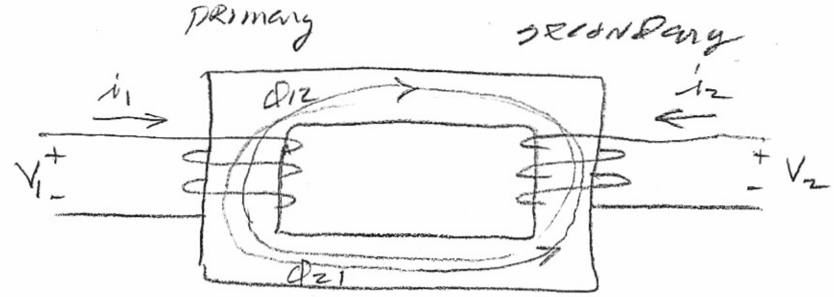


Figure 17.1: Basic coupled circuit

We can write (for the configuration shown)

$$V_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

If we reverse the winding of, say, the secondary the signs on the mutual inductance terms would change. The equations would become:

$$v_1(t) = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$
$$v_2(t) = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Rather than draw the simulated core and coils of a transformer we use a convention (called dot polarity) to explain what is going on with induced voltages.

Consider the transformer sketch of Figure 17.2

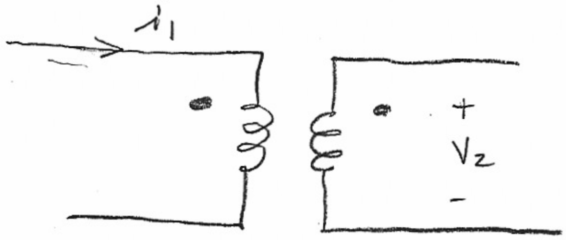


Figure 17.2: Dot markings

We say that if the assumed direction of current i_1 enters the dot then the voltage induced at the terminals of the coil in the right side will be positive at dot and given by

$$+M\frac{di_1}{dt}$$

If we assume V_2 is positive at this dot then

$$V_2 = +M\frac{di_1}{dt}$$

This is perhaps better illustrated by the cases presented in Figure 17.3.

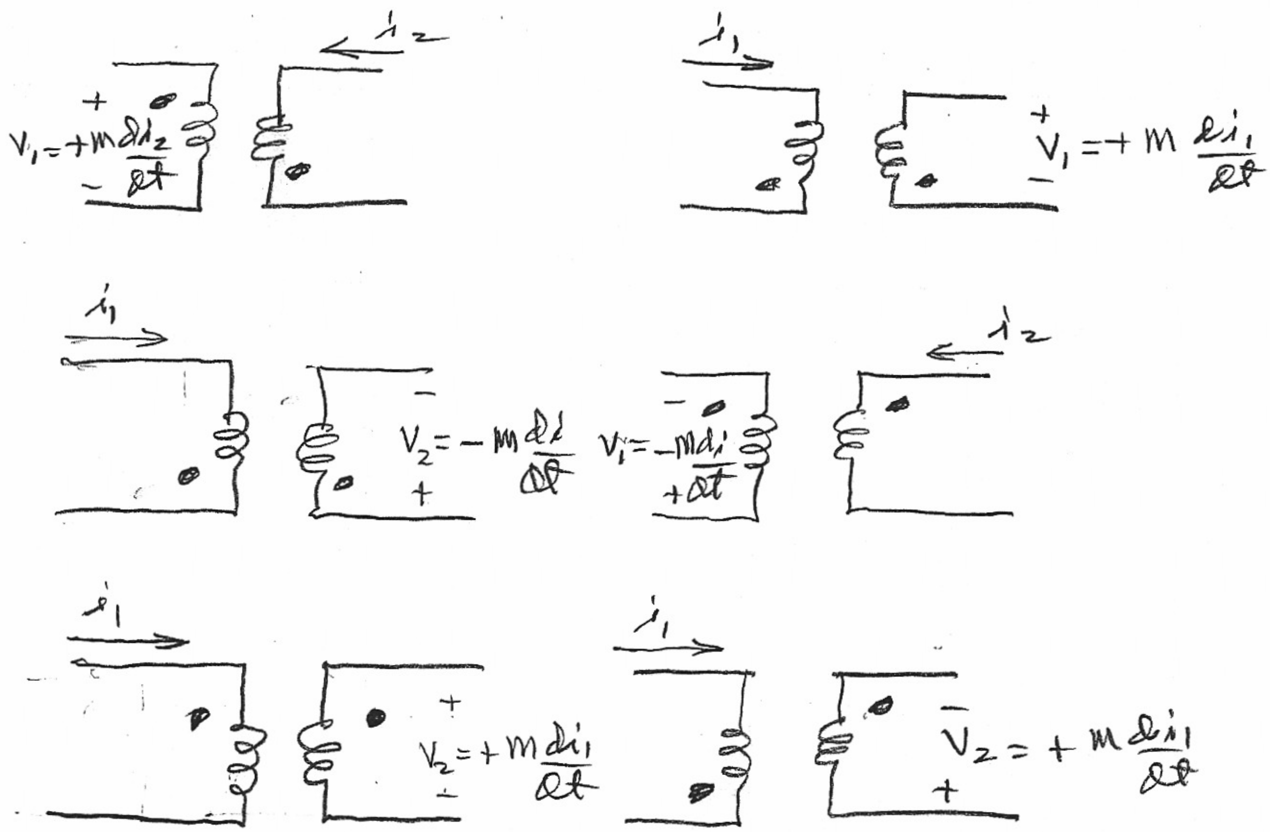


Figure 17.3: Illustrating dot polarity markings.

We next recall that in going from time Domain to steady state AC circuits we use

$$v(t) \longrightarrow \hat{V}$$

$$L\frac{di}{dt} \longrightarrow jwL\hat{I}$$

In light of this we consider the transformer circuit shown in Figure 17.4

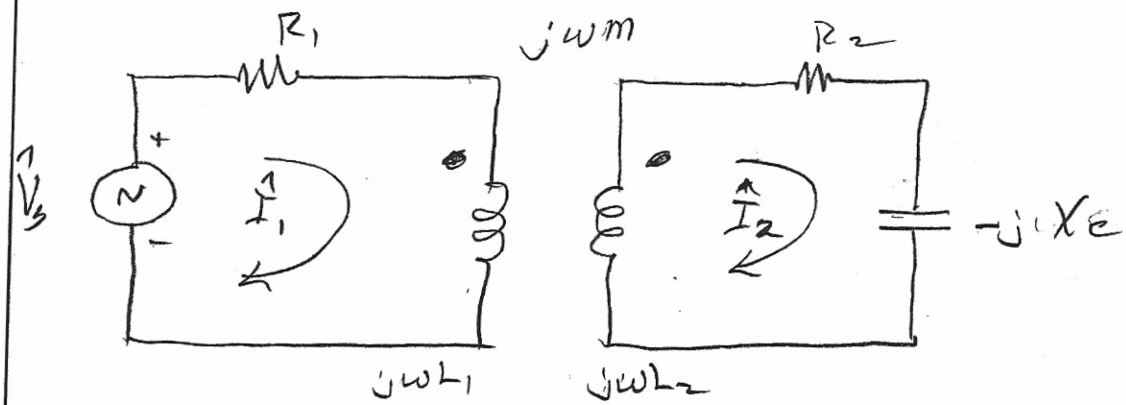


Figure 17.4: A linear transformer with AC circuit conditions.

We can write on the primary side,

$$R_1 I_1 + jwL_1\hat{I}_1 - jwM\hat{I}_2 = \hat{V}_s$$

or

$$\boxed{(R + jwL_1)\hat{I}_1 - jwm\hat{I}_2 = \hat{V}_s}$$

and for the secondary

$$jwL_2\hat{I}_2 + R_2\hat{I}_2 - jwcI_2 - jX_c\hat{I}_1 = 0$$

or

$$\boxed{-jwM\hat{I}_1 + (R_2 + j(wL_2 - X_c)) = 0}$$

In matrix form

$$\begin{bmatrix} [R+j\omega L_1], & -j\omega M \\ -j\omega M & [R_2+j(\omega L_2 - X_c)] \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} \hat{V}_s \\ 0 \end{bmatrix}$$

which of course can be easily solved when given parameter and source values.

We assume here a linear transformer. Such transformers are so characterized because of the core around which the wire is wound to form the transformer. Common materials for linear transformers core (a) air, (b) bakelite (c) plastics, (d) wood --- The type transformers are often used in communication circuits. Basically, a linear transformer is one with a core material that has a $\hat{B}-\hat{H}$ curve that is linear. You should see a more advanced treatment for an explanation of $\hat{B}-\hat{H}$ curves.

Without justification (but justification is presented in basic circuit text)

$$M = k\sqrt{L_1 L_2}$$

where k is called the coefficient of coupling.

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad (0 < k \leq 1)$$

With linear transformers we have a fair amount of flux leakage so $k \neq 1$

Consider the following example problem.

Example 17.2

Determine the phasor currents $\hat{I}_1$ and $\hat{I}_2$ in the following transformer circuit.

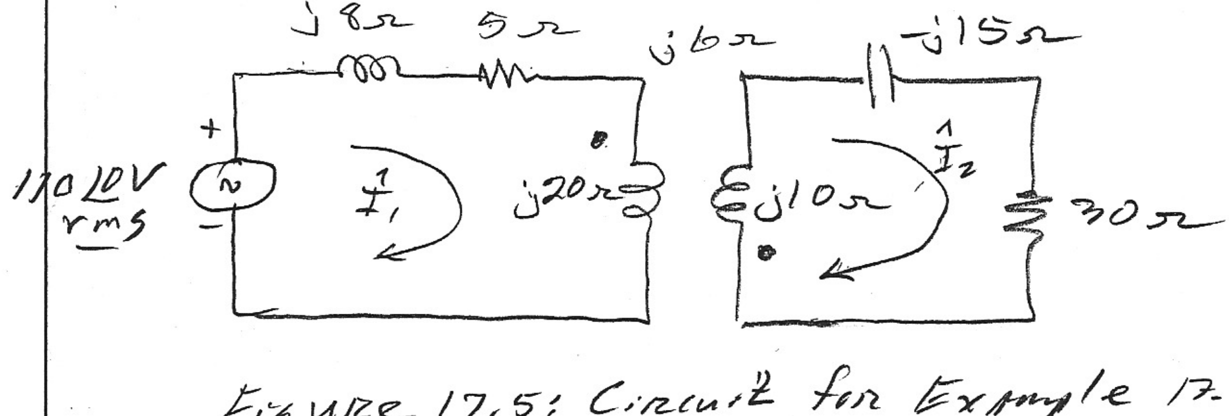


Figure 17.5: Circuit for Example 17-2.

Around the primary

$$(j8 + 5 + j20)\vec{I}_1 + j6\vec{I}_2 = 110\angle 0$$

Around the secondary

$$(j10 - j15 + 30)\vec{I}_2 + j6\vec{I}_1 = 0$$

In matrix form

$$\begin{bmatrix} 5 + j28, & j6 \\ j6 & 30 - j5 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 110\angle 0 \\ 0 \end{bmatrix}$$

which can easily be solve with your hand calculator.

When we have an ordinary AC circuit, typically as shown in Figure 17.6 we can find the impedance seen by the source $\hat{V}_s$ by forming

$$Z = \frac{\hat{V}_s}{\hat{I}_1}$$

This is sometimes called the driving point impedance. With transformers the problem is a little different.

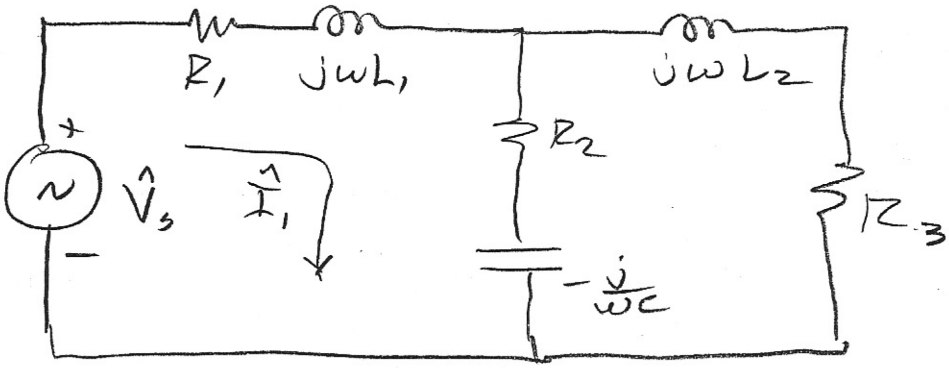


Figure 17.6: Illustrating driving point impedance.

Consider the transformer circuit of Figure 17.7.

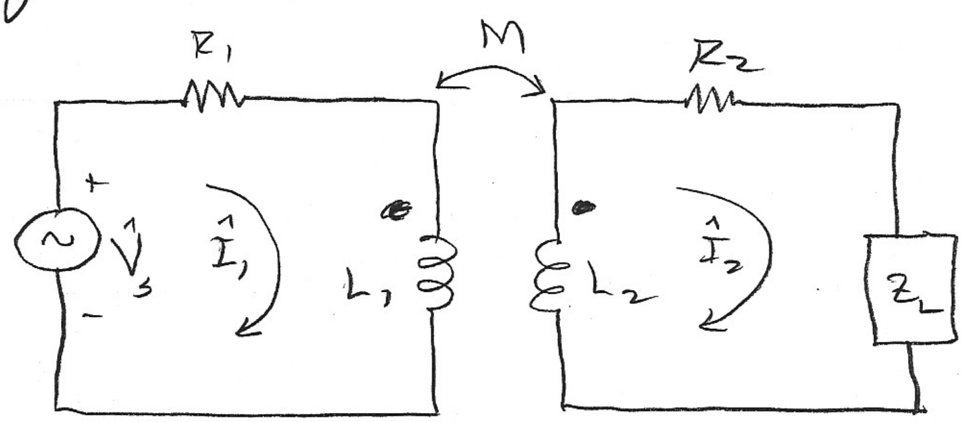


Figure 17.7: Circuit used to illustrate reflected or driving point impedance.

We write

$$\hat{V}_s = R_1\hat{I}_1 + j\omega L_1 I_1 - j\omega M I_2 \qquad (17.1)$$

$$0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)\hat{I}_2 \qquad (17.2)$$

Solve for $\vec{I}_2$ in terms of $\vec{I}_1$ from Eq (17.2) that is

$$\vec{I}_2 = \frac{j\omega M I_1}{R_2 + j\omega L_2 + Z_L} \qquad (17.3)$$

Substitute this in (17.1)

$$\vec{V}_s = (R_1 + j\omega L_1) I_1 - j\omega M \left[\frac{j\omega M I_1}{R_2 + j\omega L_2 + Z_L} \right]$$

OR

$$\vec{V}_s = (R_1 + j\omega L_1)\vec{I}_1 + \frac{\omega^2 M^2 \vec{I}_1}{R_2 + j\omega L_2 + Z_L}$$

OR

$$Z_{IN} = \frac{\vec{V}_s}{\vec{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

One might say, in light of the direct impedance of the primary of $R_1 + j\omega L_1$, that

$$Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} \qquad (17.4)$$

If you change the dots or I_2 direction you still get the same results.

Ideal Transformers

An ideal transformer is so-called ideal because of certain assumptions that are made. It is true that the way the transformer is made, (the core material, the winding) lead to help make the ideal assumption closer to reality.

In the following analysis these ideal assumption are brought to bear on the work. We start with a configuration that one might first assume is a linear transformer configuration, as shown in Figure 17.8.

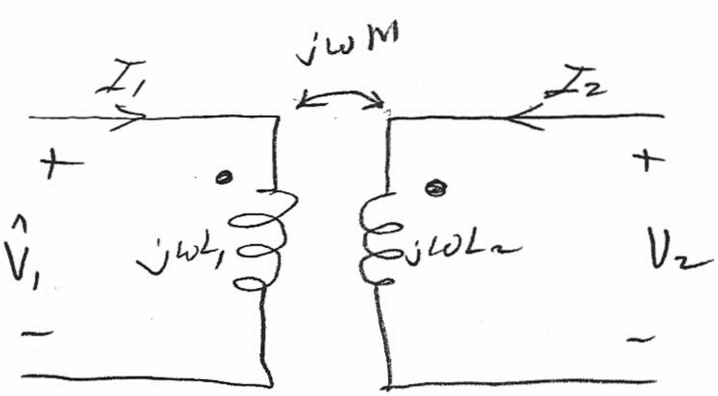


Figure 17.8: Basic transformer

We can write

$$j\omega L_1 \hat{I}_1 + j\omega M \hat{I}_2 = \hat{V}_1 \qquad 17.5$$

$$j\omega M \hat{I}_1 + j\omega L_2 \hat{I}_2 = \hat{V}_2 \qquad 17.6$$

Solve for $\hat{I}_1$ from Eq 17.5;

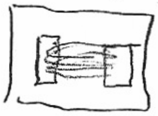
$$\hat{I}_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1} \qquad (17.7)$$

Substitute this for $\hat{I}_1$ in Equation 17.6

$$j\omega M \frac{[V_1 - j\omega M \hat{I}_2]}{j\omega L_1} + j\omega L_2 \hat{I}_2 = \hat{V}_2$$

OR

$$\hat{V}_2 = j\omega L_2 \hat{I}_2 + \frac{M V_1}{L_1} - \frac{j\omega M^2}{L_1} \hat{I}_2 \qquad (17.8)$$

At this point we introduce the assumption that we have perfect coupling. We come close to realizing this by using a common core  and by using a transformer material with a very high permeability. Doing this means most all the flux stays in the center core without leakage. Also, with high permeability and a large number of turns we recall

$$L = \mu \frac{N^2 A}{l}$$

so L goes very, very high with high μ and high N.

In Eq 17.8, we substitute

$$(M)^2 = \left(\sqrt{L_1 L_2}\right)^2 \qquad (k=1)$$

or $$M^2 = L_1 L_2$$

giving

$$\hat{V}_2 = \cancel{j\omega L_2 \hat{I}_2} + \frac{\sqrt{L_1 L_2}}{L_1} V_1 - \cancel{j\omega \frac{L_1 L_2}{L_1} \hat{I}_2}$$

$$\hat{V}_2 = \sqrt{\frac{L_2}{L_1}}\, V_1$$

with a common core, same permeability,

$$L_1 = \mu \frac{N_1^2 A}{l} \; ; \; L_2 = \mu \frac{N_2^2 A}{l}$$

and $$\frac{L_2}{L_1} = \frac{\mu N_2^2 A / l}{\mu N_1^2 A / l} = \left(\frac{N_2}{N_1}\right)^2$$

Thus $$\boxed{\hat{V}_2 = \frac{N_2}{N_1} \hat{V}_1} \qquad 17.9$$

This is a very fundamental equation.

The basic assumptions made for an ideal transformer are

(1) Coupling is perfect; $k=1$

(2) The impedances are infinite L_1, L_2 and $M \longrightarrow \infty$

(3) There is no power loss (we know this is not true because we have resistance in the windings and we have what is called eddy currents in the magnetic material, all of which causes power loss)

Nonetheless, in the ideal transformer we assume $R_1 = R_2 = 0$

So, we have

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

and with no power loss; power into the transformer = power out of the transformer, or

$$V_1 I_1 = V_2 I_2 \qquad (17.10)$$

From (17.10)

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

In words:

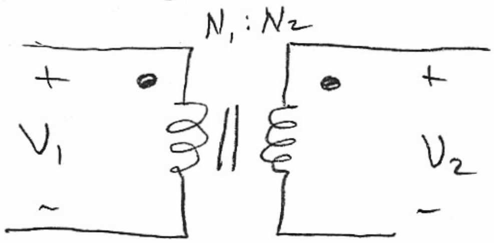
When we step up the voltage from primary to secondary we step down the current. Vise-versa.

Key:

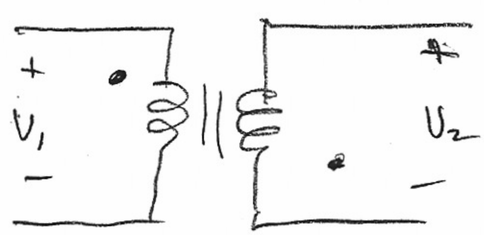
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \qquad (17.11)$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = n \qquad (17.12)$$

As with the linear transformer, we use dots to keep us straight on polarity.



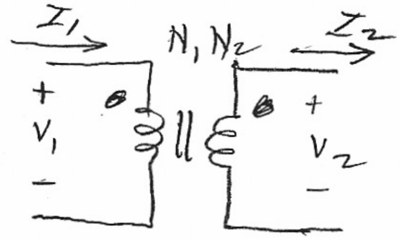
$$\frac{V_2}{V_1} = +\frac{N_2}{N_1}$$

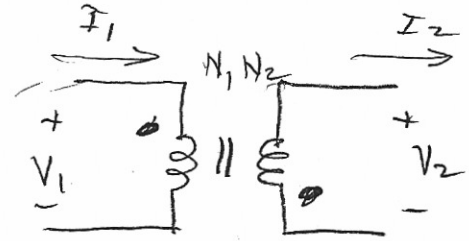


$$\frac{V_2}{V_1} = -\frac{N_2}{N_1}$$

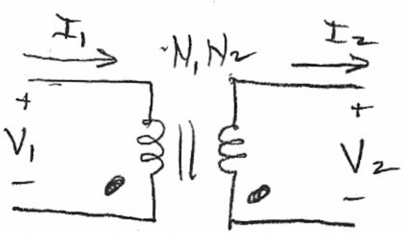
speaks for itself

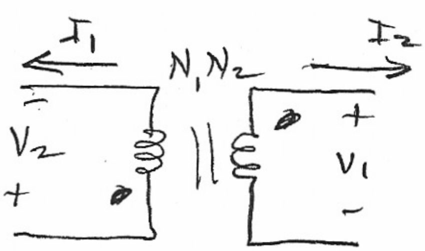
A collection of cases is given in Figure 17.8 to illustrate dot markings



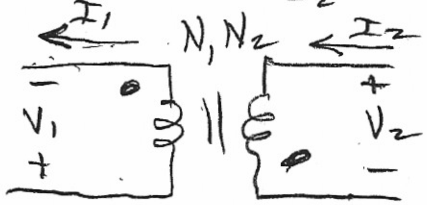


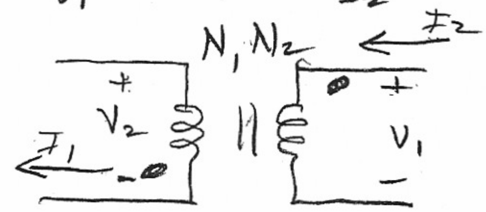
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1} \qquad \frac{V_2}{V_1} = -\frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = -\frac{N_2}{N_1}$$



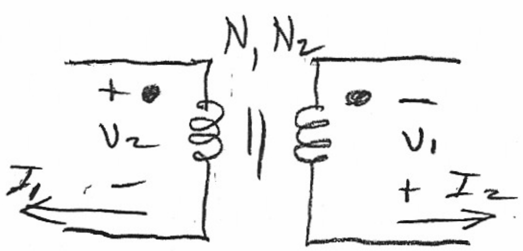


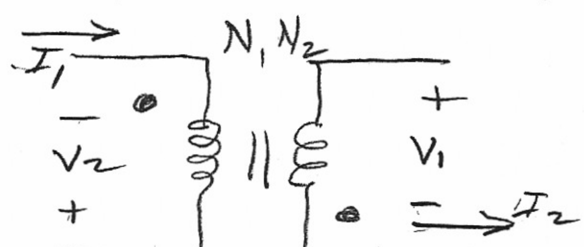
$$\frac{V_2}{V_1} = +\frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = +\frac{N_2}{N_1} \qquad \frac{V_2}{V_1} = +\frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = +\frac{N_2}{N_1}$$





$$\frac{V_2}{V_1} = +\frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = -\frac{N_2}{N_1} \qquad \frac{V_2}{V_1} = -\frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = +\frac{N_2}{N_1}$$





$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = -\frac{N_2}{N_1} \qquad \frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Figure 17.8: Illustrating dot markings for ideal transformer

Example 17.3

You are given the ideal transformer circuit of Figure 17.9. Solve for $\hat{V}_1, \hat{V}_2, \hat{I}_1, \hat{I}_2$

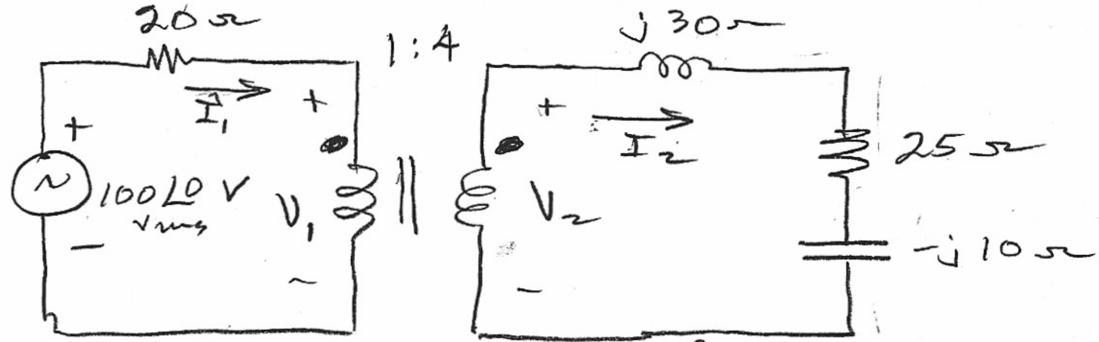


Figure 17.9: Circuit for Example 17.3.

Mesh Equations

$$20\hat{I}_1 + \hat{V}_1 = 100\angle 0$$

$$(25 + j20)I_2 - \hat{V}_2 = 0$$

OR

$$20I_1 + 0I_2 + V_1 + 0V_2 = 100\angle 0 \quad 17.13$$

$$0I_1 + (25+j20)I_2 + 0V_1 - V_2 = 0 \quad 17.14$$

Transformer Equations

$$\frac{V_2}{V_1} = 4$$

OR

$$0I_1 + 0I_2 + 4V_1 - V_2 = 0 \quad 17.15$$

$$\frac{I_1}{I_2} = 4$$

OR

$$I_1 - 4I_2 + 0V_1 + 0V_2 = 0 \quad 17.16$$

We have 4 equations, 4 unknowns:

Placing 17.13, 17.14, 17.15 and 17.16 in matrix form

$$\begin{bmatrix} 20 & 0 & 1 & 0 \\ 0 & 25+j20 & 0 & -1 \\ 0 & 0 & 4 & -1 \\ 1 & -4 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Although the work on this example looks straightforward (and it is), it is so very, very easy to make a careless mistake.

$\hat{I}_1 = 4.62 - j0.268$ A rms $\quad \hat{I}_2 = 1.156 - j0.067$ A rms

$\hat{V}_1 = 7.56 + j5.36$ V rms $\quad \hat{V}_2 = 30.23 + j21.44$ V rms

Power Consideration

We have assume an ideal transformer which amoung other things means that power in = power out. This should hold for complex power.

We know

$$\hat{S}_1 = \hat{V}_1 \hat{I}_1^* \qquad 17.17$$

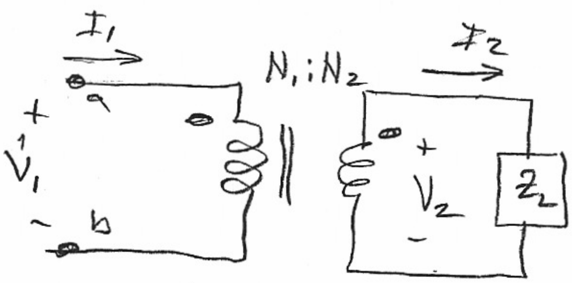
where $\hat{V}_1$ is the voltage across the transformer terminals and $\hat{I}_1$ is the current into the transformer primary winding.

Carrying on with Equation 17.17

$$\hat{S}_1 = \hat{V}_1 \hat{I}_1^* = \frac{\hat{V}_2}{N} \hat{I}_2^* N = \hat{V}_2 \hat{I}_2 \qquad 17.18$$

which shows that the complex power in equals the complex power out, as should be.

Consider the following sketch



We want to find $\frac{V_1}{I_1}$ which will be the impedance seen at a-b

$$\hat{V}_1 = \frac{\hat{V}_2}{N} = \frac{Z_L I_2}{N} = \frac{Z_L I_1}{N N} = \frac{Z_L I_1}{N^2}$$

$$\frac{\hat{V}_1}{I_1} = Z_{IN} = \frac{Z_L}{N^2} \qquad (17.19)$$

The impedance of the secondary is reflected to the primary as the load impedance divided by N^2. This is important.

We might as "is there a more general result than this?" The answer is yes and we turn to Thevenin's theorem to explore this.

Consider the circuit of Figure 17.10

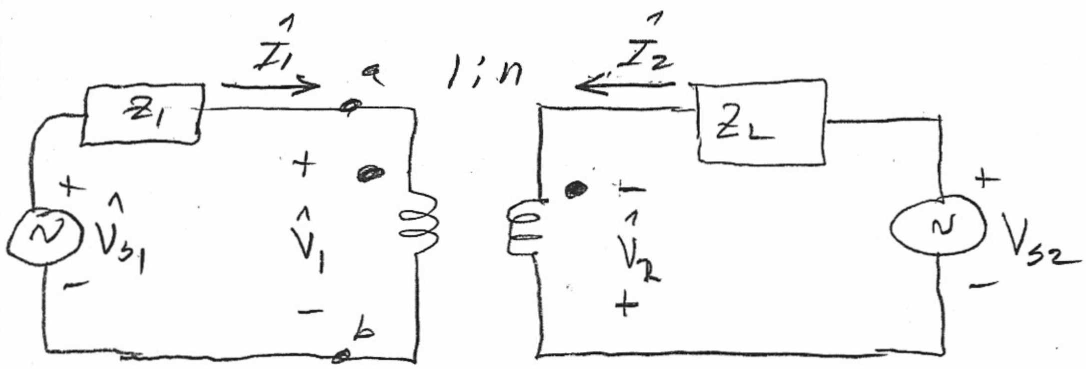


Figure 17.10: Circuit used to Develop The Thevenin Circuit.

We want to remove V_{S1} and Z_1 and look into Terminals a-b and find $\hat{V}_{TH}$ and Z_{TH}. The polarity of the voltage $\hat{V}_2$ has been reversed as well as the direction of $\hat{I}_2$, on purpose.

If we want to find V_{TH} we note that $\hat{I}_1 = 0$ Therefore since $\frac{I_1}{I_2} = -n$ $\hat{I}_2$ is also zero. If $\hat{I}_2 = 0$,

$$\hat{V}_2 = -V_{S2} \qquad (17.20)$$

Now $\frac{\vec{V}_2}{\vec{V}_1} = -n$ OR $\vec{V}_1 = -\frac{\vec{V}_2}{n}$

using (17.20) gives

$$\vec{V}_1 = \frac{-(-V_{S2})}{n} = \frac{V_{S2}}{n} = V_{TH}$$

So the Thevenin voltage is

$$\boxed{\vec{V}_{TH} = \frac{\vec{V}_{S2}}{n}} \qquad (17.21)$$

Now we find Z_{TH}. Recall that a general way of find Z_{TH} is to disable all independent voltage sources and apply a known voltage, say $\vec{V}_x$, and find

$$Z_{TH} = \frac{\vec{V}_x}{\vec{I}_1} \qquad (17.22)$$

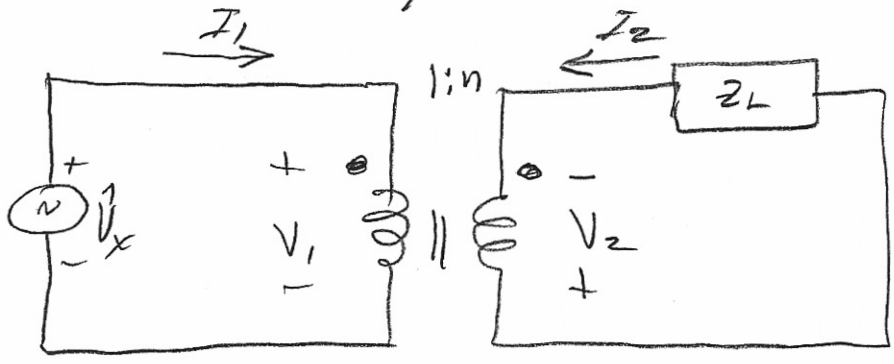


Figure 17.11: Circuit for find Z_{TH}.

It is clear from the circuit of Figure 17.11 that

$$\vec{V}_x = V_1$$

so

$$Z_{TH} = \frac{\vec{V}_1}{\vec{I}_1} \qquad (17.22)$$

but $\vec{V}_1 = -\frac{\vec{V}_2}{n}$ and $I_1 = -nI_2$

substituting into (17.22) gives

$$Z_{TH} = \frac{-V_2}{n(-nI_2} = \frac{V_2}{n^2 I_2} \qquad (17.23)$$

Examining the circuit of Figure 17.11 we see that $Z_L = \frac{V_2}{I_2}$. Therefore

$$Z_{TH} = \frac{Z_L}{n^2}$$

We can now draw the transformer circuit using the Thevenin equivalent. This eliminates the windings of the transformer. We still retain $\vec{V}_1$, $\vec{V}_2$, $\vec{I}_1$ and $\vec{I}_2$ as the case may be. This is shown in Figure 17.12.

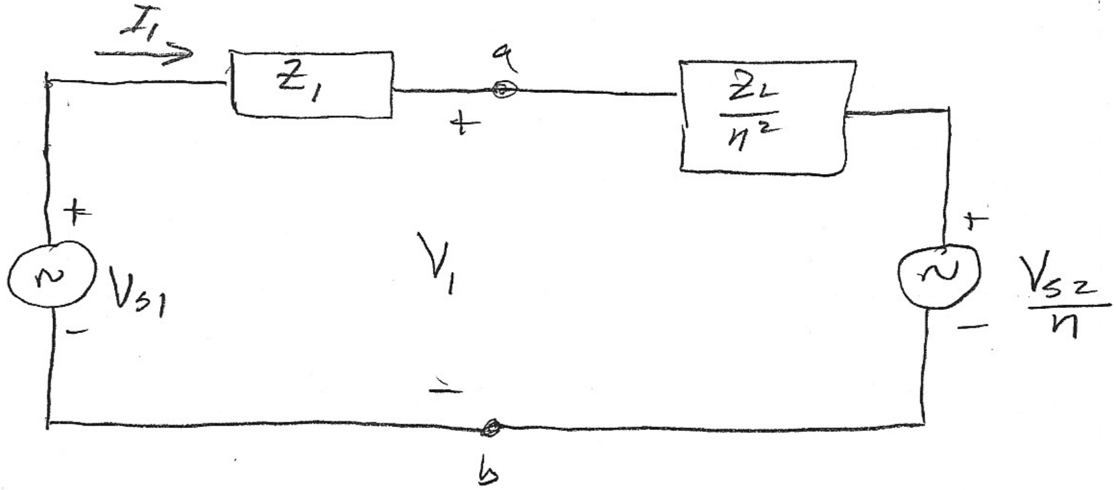


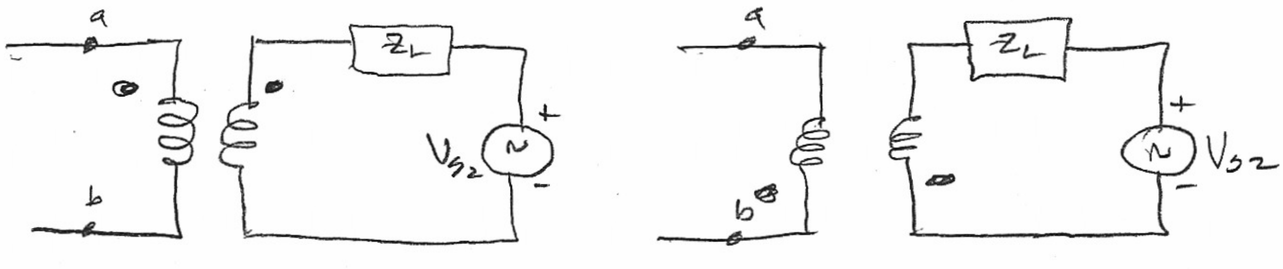
Figure 17.12: Thevenin equivalent transformer circuit of Figure 17.10.

Note that we retain the relationships between $\vec{V}_1$ and $\vec{V}_2$ and $\vec{I}_1$ and $\vec{I}_2$ of Figure 17.10

It can be shown (I did it by trial and observation, that regardless of how we assume $\vec{V}_1$, $\vec{V}_2$, $\vec{I}_1$ and $\vec{I}_2$ if we have dots located as in Figures 17.13a,b we will have an equivalent circuit as shown in Figure 17.14.

Now if we orient the dots as shown in Figure 17.15a,b the Thevenin equivalent will be as shown in Figure 17.16.

Again, this is regardless of assumed polarities of $\vec{V}_1$, $\vec{V}_2$, $\vec{I}_1$, $\vec{I}_2$.



(a) (b)

Figure 17.13: Two dot possibilities for ideal transformer.

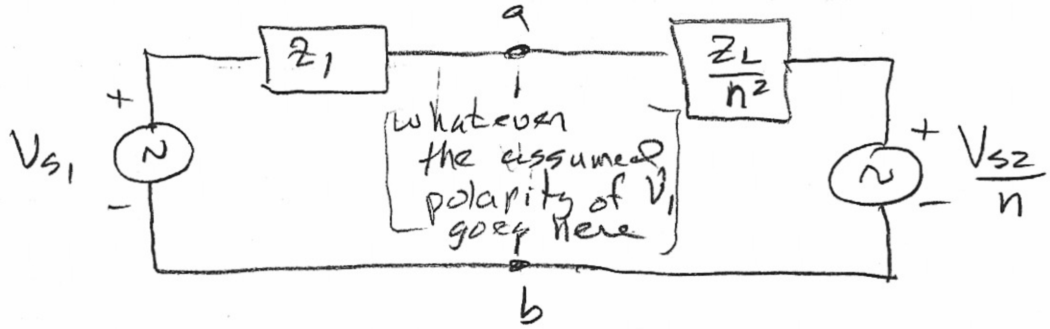
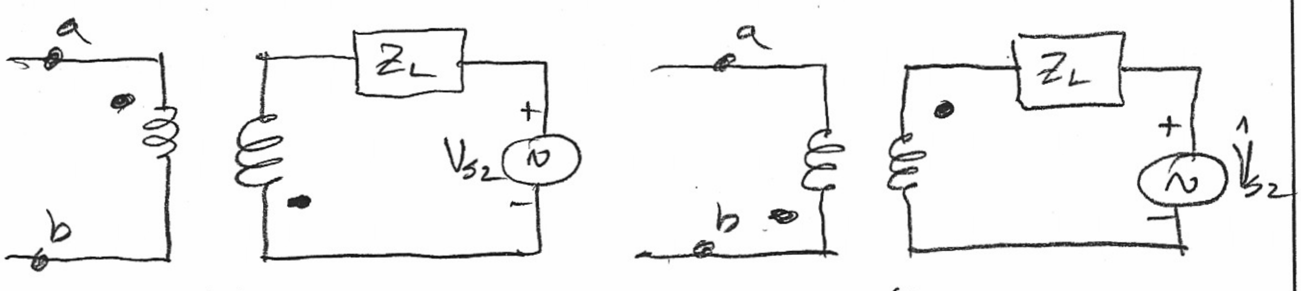


Figure 17.14: Thevenin equivalent for case shown in Figure 17.13



(a) (b)

Figure 17.15: Two dot configurations.

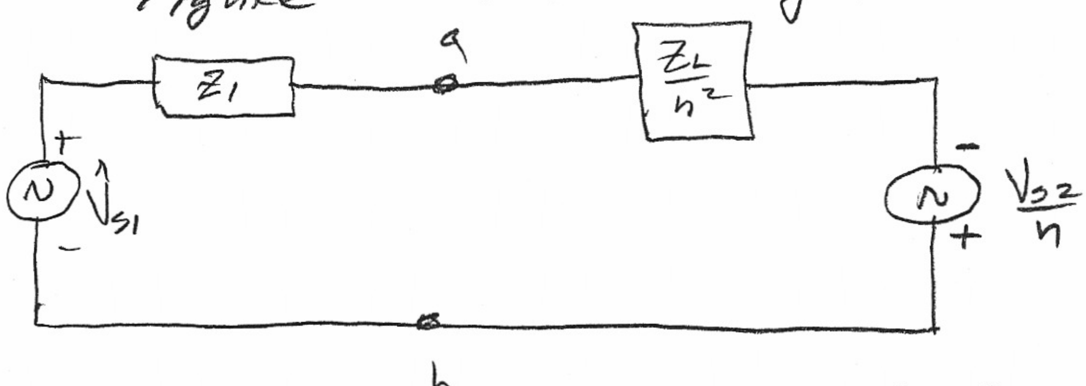


Figure 17.15: Thevenin equivalent for the cases shown in Figure 17.15.

We note that regardless of the dot marking, the reflected Z is $\frac{Z_L}{n^2}$.

In the same manner, we can go to the transformer and look into c-d with Z_L and $\vec{V}_{S2}$ removed and find $\vec{V}_{TH}$ and Z_{TH}.

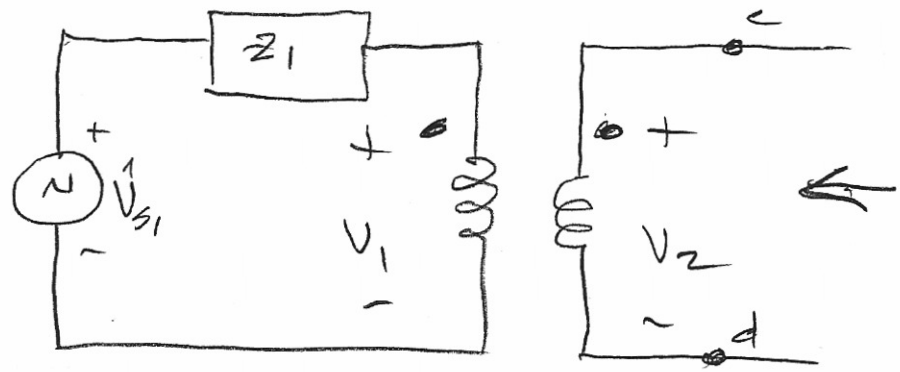


Figure 17.16: Finding Thevenin equivalent looking into the secondary.

The results are shown in Figure 17.17.

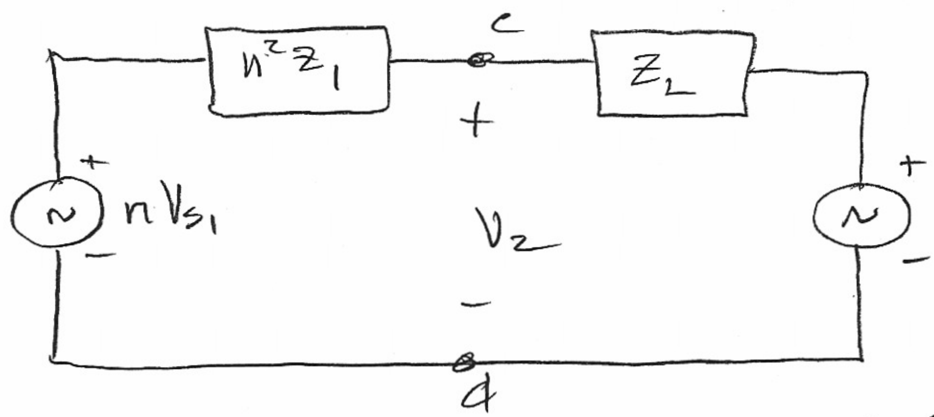


Figure 17.17: Thevenin equivalent seen looking into the secondary for ˙˙ or .. markings

If the dots are diagonal, the sign on $n\hat{V}_s$ changes to $-n\hat{V}_s$.

We now illustrate how to use this.

Consider again Example 17.3, page 16. We redraw for convenience.

Example 17.4

You are given the circuit of Figure 17.18.

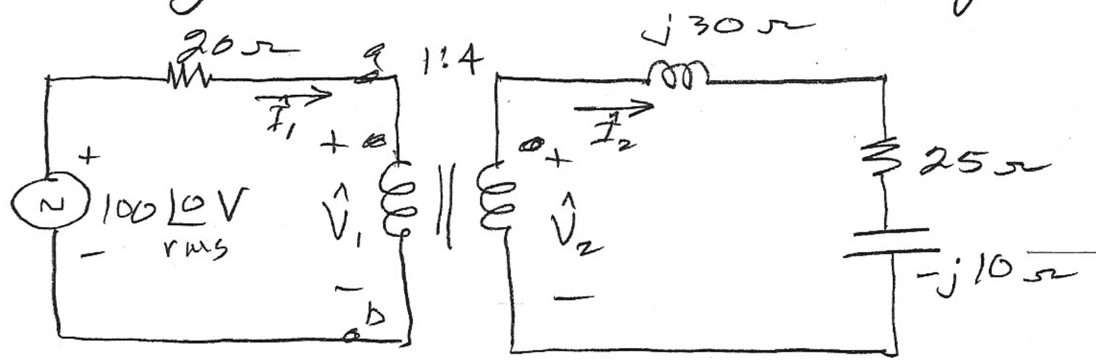


Figure 17.18: Transformer circuit for Example 17.4.

Reflect the circuit to the right of a-b to the primary and solve for $\hat{V}_1$, $\hat{V}_2$, $\hat{I}_1$, $\hat{I}_2$.

$$\frac{25+j20}{4^2}$$

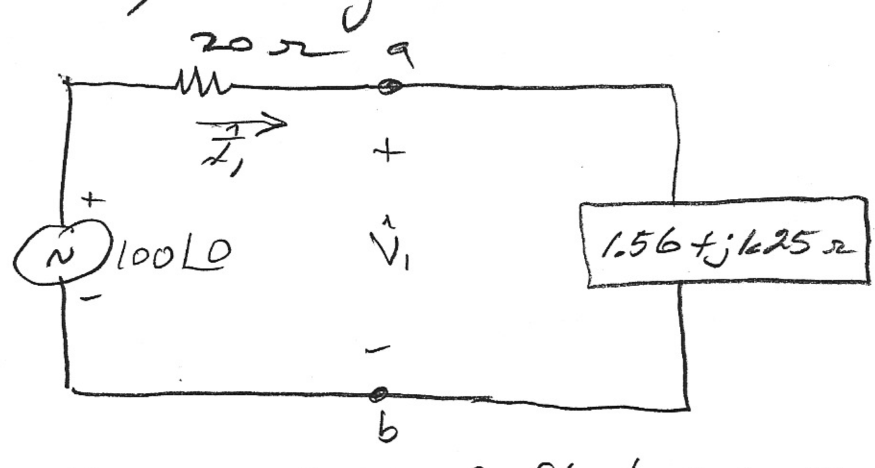


Figure 17.19: Reflected secondary of transformer from Figure 17.18.

The current $\vec{I}_1$ is given by

$$\vec{I}_1 = \frac{100}{20 + 1.56 + j1.25} = 4.62 - j0.268 \text{ Arms}$$

This agrees with the solution on page 17.

The voltage $\vec{V}_1$ is given by

$$\vec{V}_1 = \frac{100 \times (1.56 + j1.25)}{20 + 1.56 + j1.25}$$

$$\vec{V}_1 = (7.55 + j5.36) \text{ Vrms}$$

Referring to Figure 17.18,

$$\vec{V}_2 = 4 \times \vec{V}_1$$

$$\vec{V}_2 = 30.19 + j21.44 \quad \text{Vrms}$$

and

$$\vec{I}_2 = \frac{\vec{I}_1}{4} = 1.155 - j0.067 \text{ Arms}$$

All of these agree with the answers on page 17 which were obtained by direct circuit analysis.

Example 17.5

Ideal transforms, with turn rations properly selected, can be used to couple loads (speakers) to amplifiers so that maximum power is transfered to the load. Consider the circuit of Fig. 17.20.

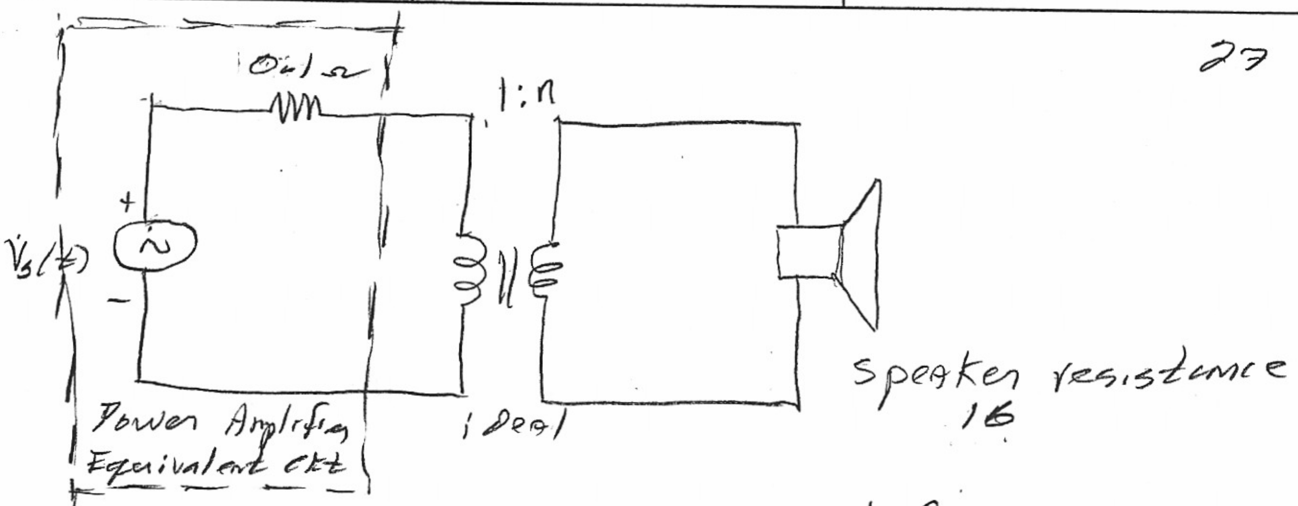


Figure 17.20: Circuit for Example 17.5.

Good power amplifier are characterized by having very low out resistance. We assume here, 0.1 Ω, which is typical. Find n so that maximum power is transferred to the speaker.

Solution: The resistance reflected is

$$Z_{ref} = \frac{Z_L}{n^2}$$

We want $Z_{ref} = 0.1$ for maximum power transfer. So

$$n^2 = \frac{Z_L}{Z_{ref}} = \frac{16}{.1} = 160$$

$$n = \sqrt{160} = 12.7 = \frac{N_2}{N_1}$$

This does not preclude that N_1 might be 300 turns and $N_2 = 12.7 \times 300 = 3810$ turns.

Returning to the question of Thevenin's circuits, reflecting secondary to primary and primary to secondary, the procedure is summarized in the next 3 pages. Look on my web site, ECE 300 S, 2005, under magnetic circuits for more explanation and example problems of linear and ideal transformers.

Example 17.6

You are given the circuit of Figure 17.21. Use the concept of reflected impedance to find

(a) the average complex power $\vec{S}$ supplied by the $110\angle 0$ V rms source

(b) the current $\vec{I}_2$,

(c) the average real power delivered to the 9 Ω resistor.

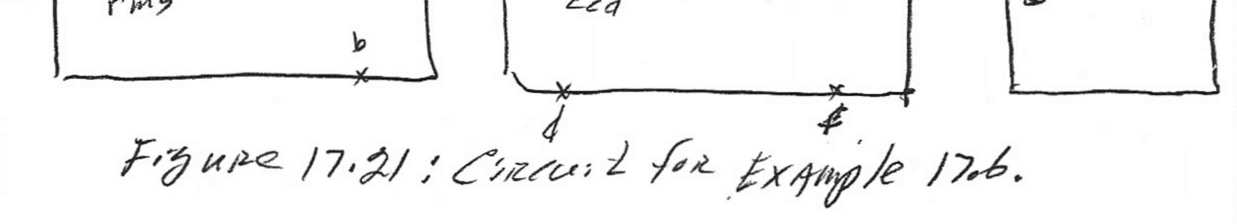
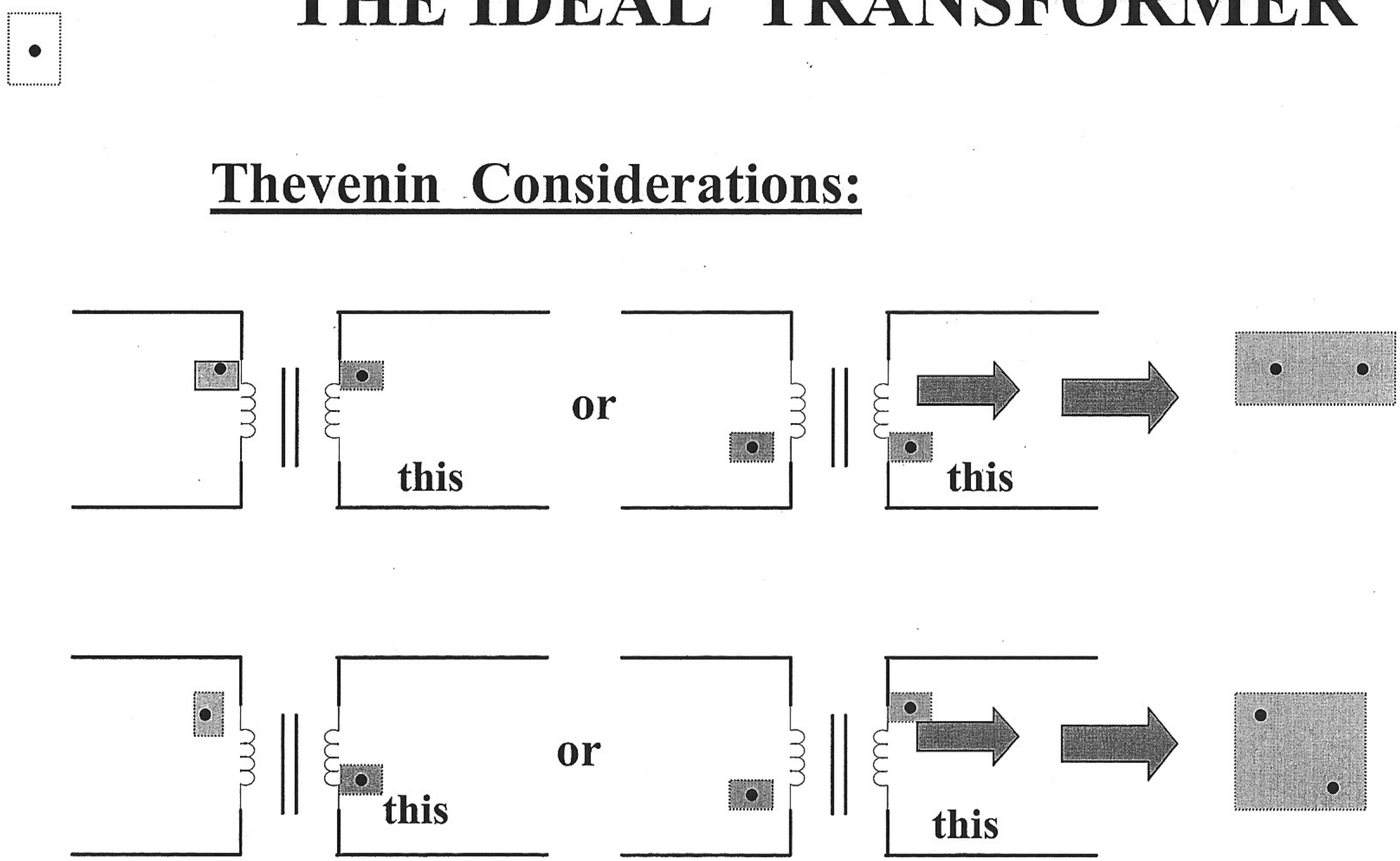


Figure 17.21: Circuit for Example 17.6.

THE IDEAL TRANSFORMER

Thevenin Considerations:



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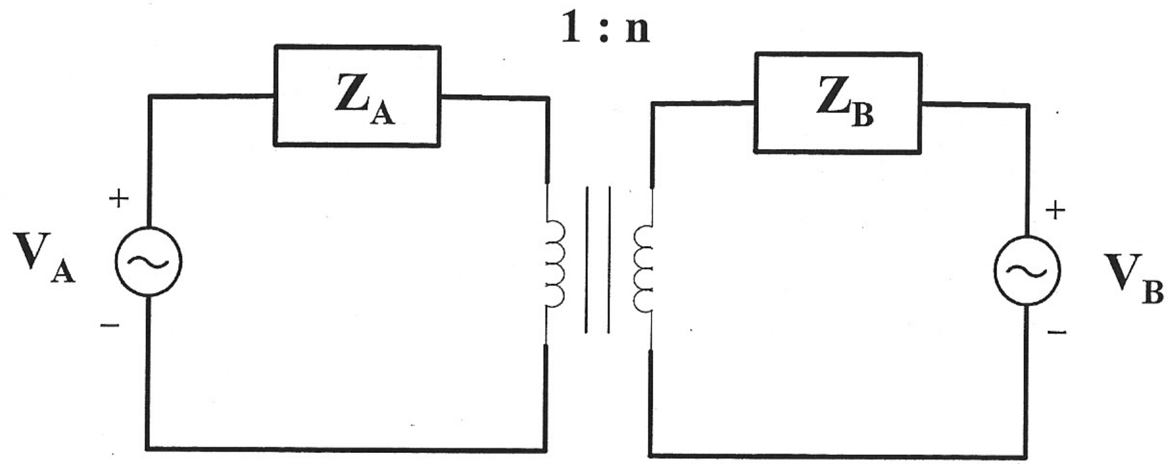
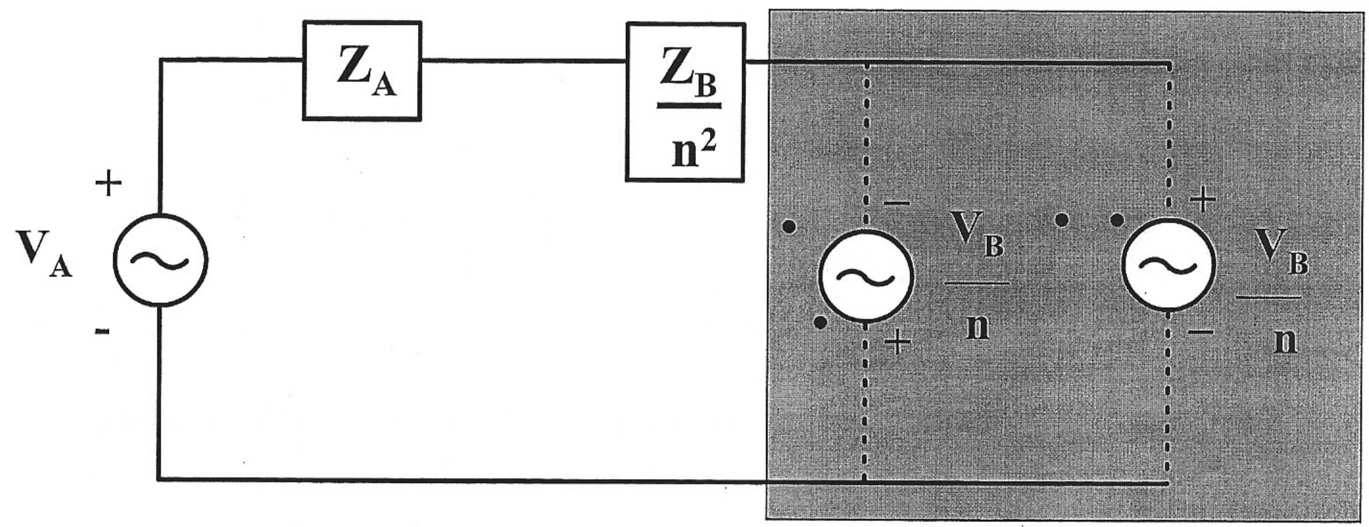
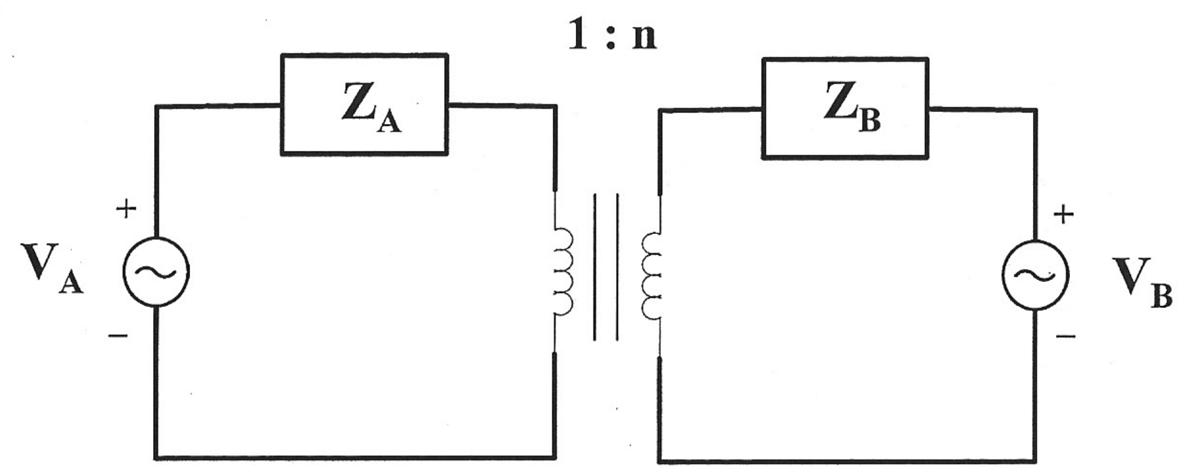
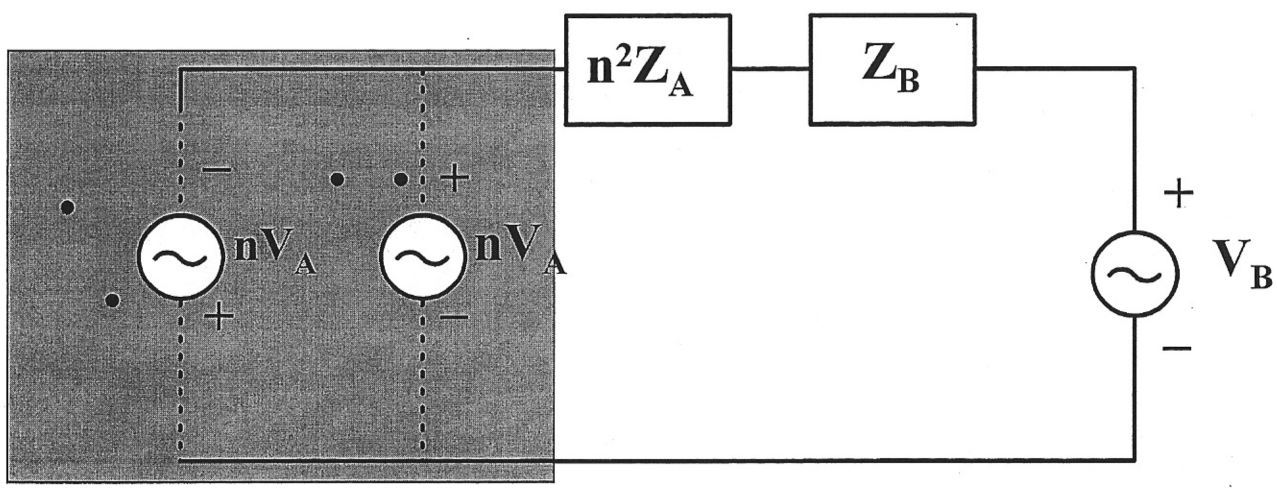
THE IDEAL TRANSFORMER





THE IDEAL TRANSFORMER





Reflecting the 3rd stage impedance to e-f give

$$Z_{ef} = \frac{9+j18}{3^2} = 1+j2\ \Omega$$

The impedance Z_{cd} is then

$$Z_{cd} = 6-j6+1+j2 = \underline{7-j4}\ \Omega$$

The impedance reflected to ab is

$$Z_{ab} = \frac{7-j4}{2^2} = (1.75-j1)\ \Omega$$

The impedance seen by the $110\angle 0$ volt source is

$$\hat{Z}_{in} = 4+Z_{ab}$$
$$\hat{Z}_{in} = 4+1.75-j1$$
$$\hat{Z}_{in} = (5.75-j1)\ \Omega$$

The current $\hat{I}_1$ is

$$\hat{I}_1 = \frac{110\angle 0}{(5.75-j1)} = 18.85\ \underline{/9.87^\circ}\ A\ rms$$

(a) The complex power at the source

$$\hat{S}_s = \hat{V}_{rms}\hat{I}^*_{1\,rms} = 110\times 18.85\angle -9.87$$

$$\hat{S}_s = 2.074\angle -9.87^\circ\ kVA$$

(b) The current $\vec{I}_2$

We know

$$\frac{\vec{I}_1}{\vec{I}_2} = 2$$

$$\vec{I}_2 = \frac{\vec{I}_1}{2} = \frac{18.85 \angle 9.87^\circ}{2} \text{ A rms}$$

$$\boxed{\vec{I}_2 = 9.425 \angle 9.87^\circ \text{ A rms}}$$

(c) We know

$$\frac{I_2}{I_3} = -3$$

$$I_3 = \frac{I_2}{-3} = \frac{9.425 \angle 9.87^\circ}{-3}$$

$$\vec{I}_3 = -3.14 \angle 9.87$$

$$\vec{I}_3 = 3.14 \angle -170.13^\circ \text{ A rms}$$

$$P_9 = |I_3|^2 \times 9 = (3.14)^2 9$$

$$P_9 = 88.74 \text{ W}$$