Desk Copy

ECE 301
Fall Semester, 2007
HW Set # 4

Due: October 9, 2007
wlg

Name green
Print(last, first)

Use Engineering Paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 15 points.

4.7 (a) $v_c = 5o$ for $t < 0$, $= 50e^{-50t}$ V $t > 0$: $v_R = 0$ for $t < 0$, $= 50e^{-50t}$ V $t > 0$

(b) $p_R(t)$ $2500e^{-100t}$ W (c) 25 μJ

4.11 R = 4.33 MΩ

4.17 (a) $i(0^+) = 1$ mA,

(b) $$\frac{di(t)}{dt} + \frac{1}{R}\left[\frac{1}{C_1} + \frac{1}{C_2}\right] i(t) = 0$$

(c) 50 msec

(d) $i(t) = e^{-20t}\ mA$

(e) 50 V

4.23 i = 5 mA, $v_x = 10$ V, $v_c = -15$ V

4.28 $v_R = 0$, $t < 0$ $v_R = 10e^{-0.5t}\ V,\ t \geq 0$

4.32 i(t) = 0 for t < 0; $i(t) = (1 - e^{-20t})A,\ t \geq 0$

4.35 $i(t) = (0.5 - 0.5e^{-200t})A,\ t \geq 0$

$v_L = 100e^{-200t}\ V \quad t \geq 0$

4.48 $i(t) = (1/60)e^{-300t} - (1/60)\cos 300t + (1/60)\sin 300t\ A \quad t \geq 0$

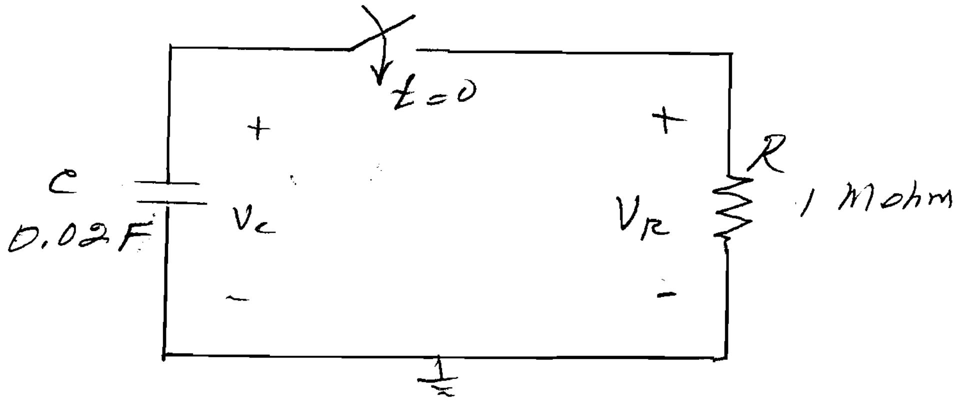
Use MATLAB to plot the above function. Plot for the time range of t = 0: 0.001: 0.25;
A copy of the expected plot is attached to this sheet. Turn in both your plot and a copy of your MATLAB program.

ECE 301

Homework #4

Fall Semester, 2007

4.7 In the circuit below, the capacitor is charged to 50 V prior to closing the switch at $t=0$.



(a) Find expressions for the voltage across the capacitor and voltage across the resistor

Using nodal analysis

$$C\frac{dV_c}{dt} + \frac{V_c}{R} = 0$$

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = 0 \qquad (1)$$

Assume $V_c = Ke^{st}$

substitute into (1), perform the math leads to

$$s + \frac{1}{RC} = 0$$

4.7 (continued)

We have $\tau = RC = 1\times10^{6} \times .02\times10^{-6}$

$$\tau = .02 \qquad \frac{1}{\tau} = 50$$

so

$$v_c(t) = k e^{-50t}$$

To evaluate k, use initial condition of capacitor voltage. We know that for the capacitor

$$v_c(0^-) = 50\text{ V} = v_c(0^+)$$

$$v_c(t)\Big|_{t=0} = 50 = k e^{-50t}\Big|_{t=0} = k$$

$$\therefore \boxed{v_c(t) = 50 e^{-50t}\text{ V} \quad t \geq 0}$$

From the circuit diagram we know that $v_c(t) = v_R(t)$, then

$$\boxed{v_R(t) = 50 e^{-50t}\text{ V} \quad t \geq 0}$$

QED

4.7 (cont)

(b) Find the expression for the power delivered to the resistor.

We know on generally that $P_R = \frac{V_R^2}{R}$ and for this case

$$P_R = \frac{\left[50 e^{-50t}\right]^2}{1 \times 10^6}$$

$$P_R = \frac{2500 e^{-100t}}{1 \times 10^6} = .2500 e^{-100t} \mu W$$

(c)

$$W = \int_0^{t=\infty} P_R \, dt = \int_0^{t=\infty} 2500 \times 10^{-6} e^{-100t} \, dt$$

$$W = \frac{2500 \times 10^{-6}}{-100} e^{-100t} \Big|_0^{\infty}$$

$$W = 0 - \left[-\frac{2500}{100} \times 10^{-6} \right]$$

$$\boxed{W = 25 \times 10^{-6} J = 25 \mu J}$$

4.7 (cont)

(d) show that energy delivered to the resistor = energy stored in the capacitor.

The initial energy stored in the capacitor is

$$W_C = \frac{1}{2} C V_C(0^+)^2$$

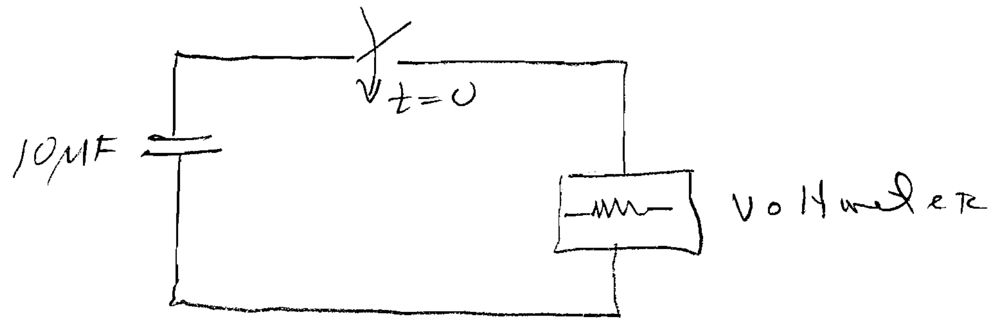
$$= \frac{1}{2} \times .02 \times 10^{-6} \times 50^2$$

$$= .01 \times 10^{-6} \times 2500$$

$$W_C = 25 \times 10^{-6}\,J = 25\,\mu J$$

QED

4.11 Consider the circuit below



The capacitor is initially charged. The switch closes at $t=0$. At $t=0$, the voltmeter reads 50 V. The voltmeter can be represented as a resistor. At $t=30$ sec, the meter reads 25 V. Find the value of R for the meter.

Solution

We know

$$V_c(t) = V_R(t) = V_c(0^+)e^{-\frac{t}{RC}}$$

The given values are

$C = 10 \times 10^{-6}$ F, $V_c(0^+) = 50$ V

$V_c(30) = 25$.

So

$$25 = 50\, e^{-\frac{30}{RC}}$$

4.11 (cont)

so taking the ln of both sides

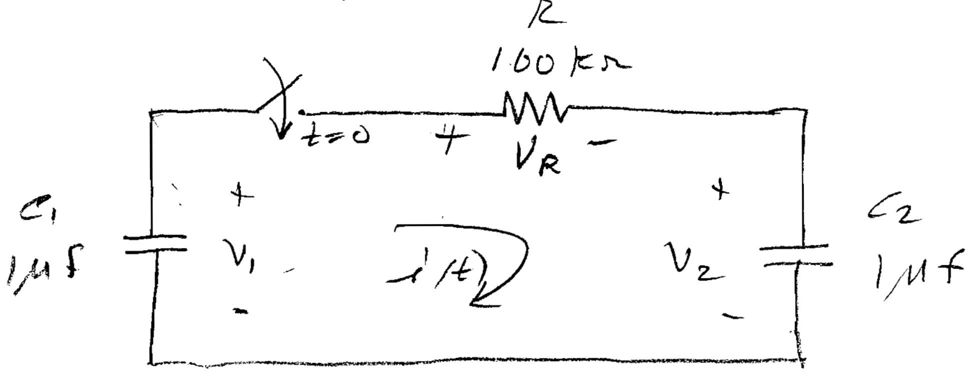
$$\ln(.5) = \ln\left(e^{-\frac{30}{RC}}\right) = \frac{-30}{RC}$$

$$-.69315 = -\frac{30}{RC}$$

$$RC = \frac{30}{.69315} \quad \text{but } C = 10 \times 10^{-6}$$

$$\boxed{R = \frac{30 \times 10^{6}}{6.9315} = 4.328\ M\Omega}$$

4.17 We are given the circuit below



Given initial conditions.

$V_1(0^-) = 100\ V$, $V_2(0^-) = 0\ V$

(a) Immediately after the switch is closed, what is $i(0^+)$?

We know

$$i(0^+) = \frac{V_1(0^+) - V_2(0^+)}{R}$$

$$\boxed{i(0^+) = \frac{100-0}{100\,k} = 1\ mA}$$

(b) Write KVL in terms of $V_1(t)$, $V_2(t)$ and $V_R(t)$

$$-V_1(t) + i(t)R + V_2(t) = 0$$

$$-\left[-\frac{1}{C_1}\int_0^t i(t)\,d\ell - V_1(0)\right] + i(t)R + \frac{1}{C_2}\int_0^t i\,d\ell + V_2(0) = 0$$

Take the derivative of the above wrt t.

4.17 (cont)

$$\frac{i(t)}{C_1} + R\frac{di}{dt} + \frac{i(t)}{C_2} = 0$$

$$\boxed{\frac{di}{dt} + \frac{1}{R}\left[\frac{1}{C_1} + \frac{1}{C_2}\right] i(t) = 0}$$

(c) What is the value of the time constant?

The time constant is

RC_{eq}

so for this case

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\tau = R\frac{C_1 C_2}{C_1 + C_2} = 100\times10^3 \times 0.5\times10^{-6}$$

$$\boxed{\tau = 50\,msec}$$

(d) We have

$$\frac{di}{dt} + 20\,i(t) = 0$$

$$i(t) = Ke^{-20t}$$

$$i(0^+) = 1\times10^{-3}$$

$$\boxed{i(t) = 1\times10^{-3}e^{-20t}\,A \qquad t \geq 0}$$

4.17 cont

(e) Find the value that V_2 approaches as t becomes very large.

$$V_2(t) = \frac{1}{C_2}\int_0^t i(t)\,dt + V_2(0^+)$$

$$V_2(t) = \frac{1}{C_2}\int_0^\infty 1\times10^{-3} e^{-20t}\,dt + 0$$

$$= \frac{1\times10^{-3}}{1\times10^{-6}}\left[-\frac{1}{20}e^{-20t}\right]_0^\infty + 0$$

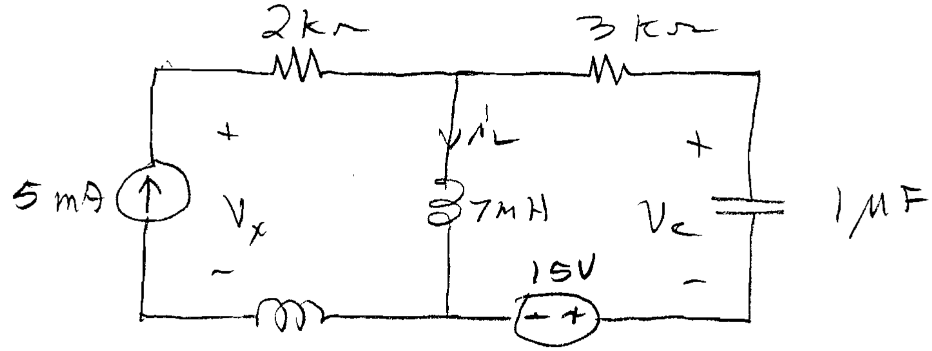
$$\boxed{V_2(\infty) = \frac{1\times10^{3}}{20} = 50\,V}$$

50

.05

4.23

We are given the following circuit

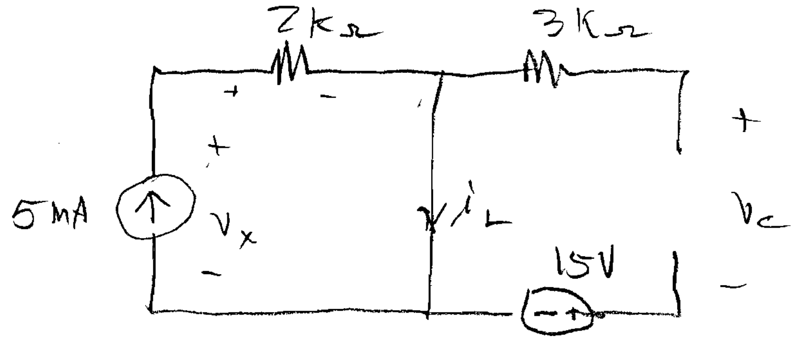


Find i_L, V_x and V_c in steady state.

<u>Solution</u>

Draw the steady state circuit.

coil $\longrightarrow$ short capacitor $\longrightarrow$ open



$i_L = 5\text{ mA}$ (no current through 3k)

<u>For V_x</u>

Write KVL

$-V_x + 2\times10^3 \times 5\times10^{-3} = 0$ (short is present)

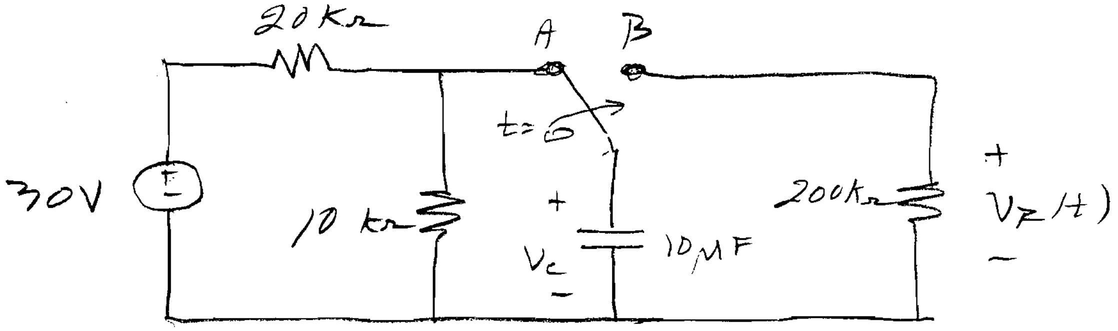
$$\boxed{V_x = 10\text{ V}}$$

<u>For V_c</u>

$V_c + 15 = 0$

$$\boxed{V_c = -15\text{ V}}$$

4.28 We are given the following circuit.



Switch is in the "A" position for a very long time. At t=0, it is moved to position B.

Find the expression for $V_R(t)$ and sketch the waveform for $-2 \le t \le 10$ s.

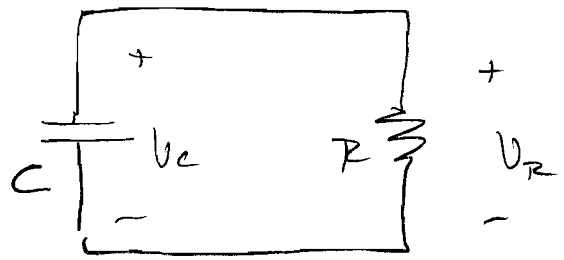
Solution

The switch being in position A for a very long time establishes $V_c(0^-)$ = Voltage across the 100kΩ resistor. This voltage is easily found by voltage Division.

$$V_c(0^-) = V_c(0^+) = 10\ V$$

Now look at the circuit after switching to B.

4.28 (cont)



We know from previous work,

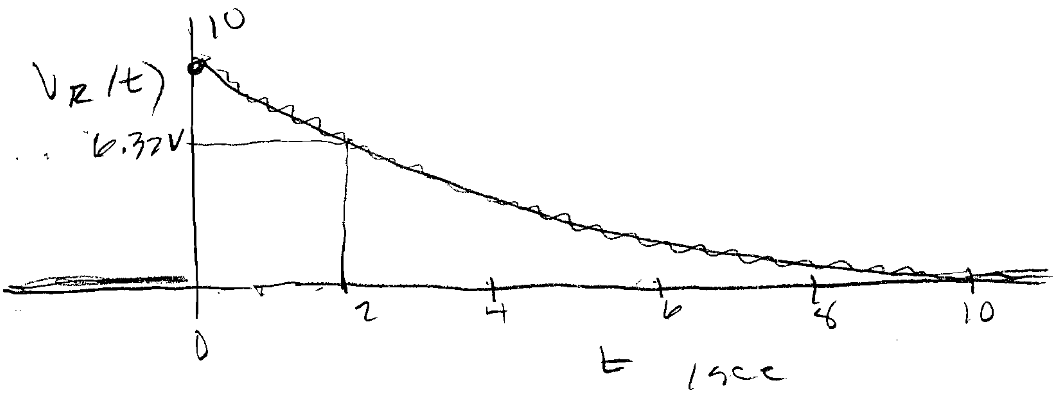
$$V_c(t) = V_R(t) = V_c(0^+)\, e^{-\frac{t}{RC}}$$

$$RC = 200 \times 10^3 \times 10 \times 10^{-6}$$

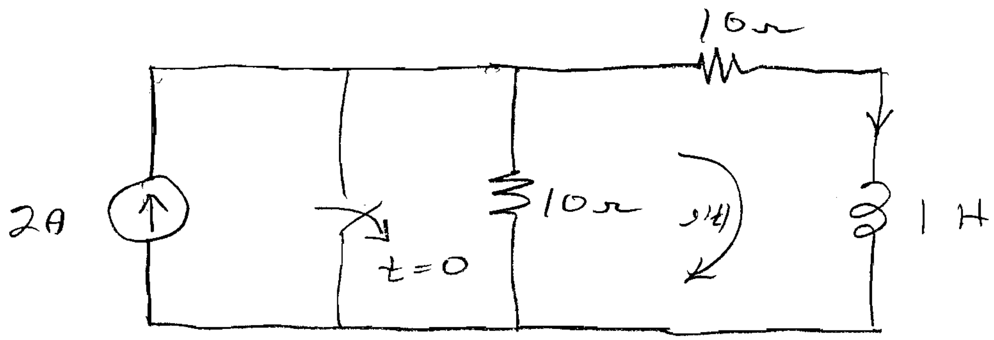
$$RC = 2000 \times 10^{-3} = 2 \text{ sec} = \tau$$

$$V_R(t) = 10 e^{-0.5t} \text{ V} \quad t \geq 0$$

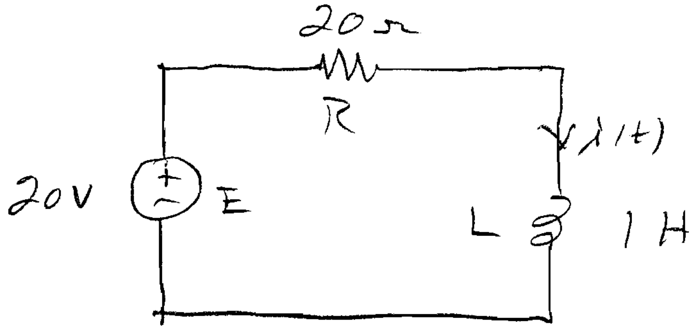
$$V_R(t) = 0, \quad t < 0$$



4.32 We are given the following circuit which is operating in steady state prior to $t=0$



Prior to $t=0$, $i(0^-) = 0\,A$ by inspection (All the 2A goes thru the short)
After $t=0$, we have, with a source transformation,



We can write

$$E = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}\,i(t) = \frac{E}{L}$$

4.32 (cont)

putting in numbers

$$\frac{di}{dt} + 20i(t) = 20$$

$$i(t) = i_p + i_c$$

$$i_p = K$$

$$20K = 20$$

$$K = 1$$

Find i_c from

$$\frac{di}{dt} + 20i = 0$$

$$i_c(t) = K_c e^{-20t}$$

$$i(t) = 1 + K_c e^{-20t}$$

Find K_c using $i_c(0^+)$

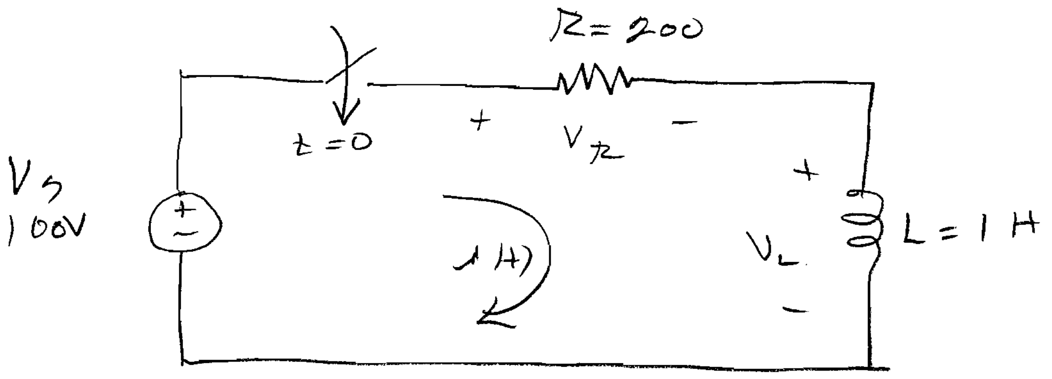
We know $i_c(0^-) = i(0^+) = 0A$

$$0 = 1 + K_c e^{-20t} = 1 + K_c$$

$$K_c = -1$$

$$\boxed{i(t) = 1 - e^{-20t} \text{ A} \qquad t \geq 0}$$

4.35 We are given the following circuit.



Find i(t), sketch the waveform to scale.

Find an expression for $V_L(t)$

Solution

We can write

$$V_s = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i(t) = \frac{V_s}{L}$$

Putting in numbers

$$\frac{di}{dt} + 200 i(t) = 100 \qquad (1)$$

$$i(t) = i_p + i_c = i_{ss} + i_t$$

$$i_{ss} = K_{ss}$$

4.35 (cont)

putting into (1) gives

$200 k_{ss} = 100$

$k_{ss} = 0.5$

solve

$$\frac{di}{dt} + 200 i(t) = 0$$

for i_t

$$i_t = k_t e^{-200t}$$

$$i(t) = 0.5 + k_t e^{-200t} \text{ A} \qquad (2)$$

$i(0^-) = i(0^+) = 0$

current can't change inst.

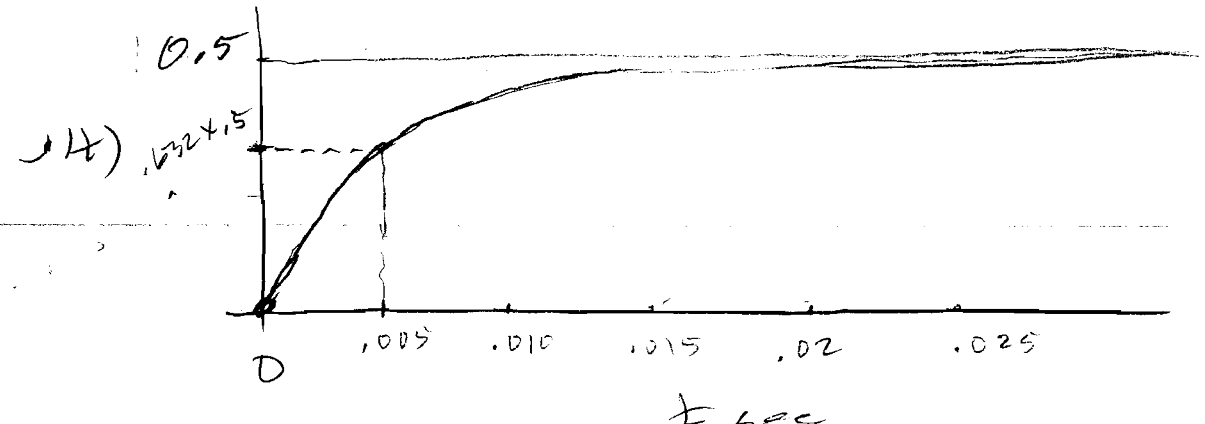
so, evaluate (2) at $t = 0^+$

$0 = 0.5 + k_t$

$k_t = -0.5$

then

$$i(t) = 0.5 - 0.5 e^{-200t} \text{ A}, \quad t \geq 0$$



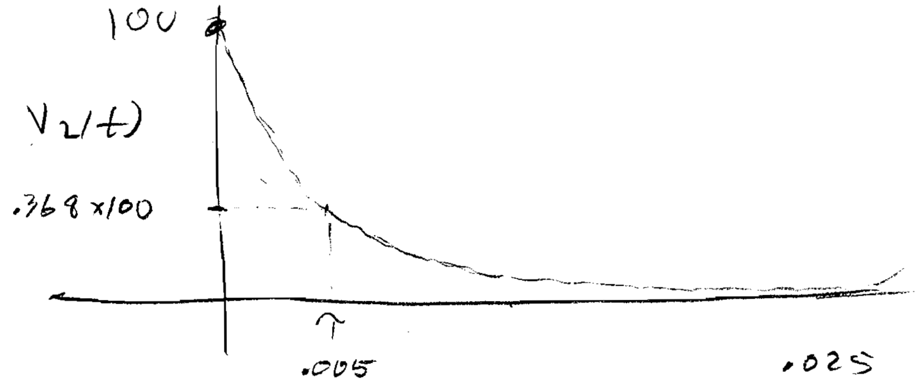
4.35 (cont)

$$V_L = \frac{L\,di}{dt}$$

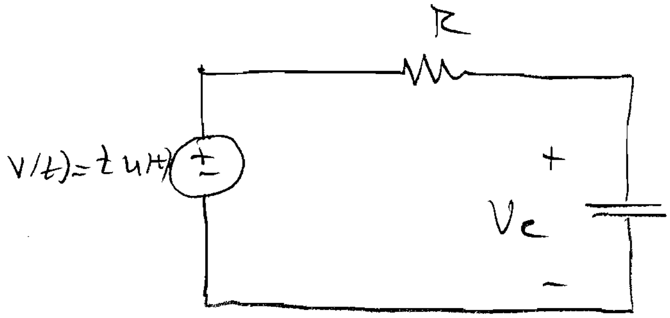
$$V_L = 1 \times \frac{d}{dt}\left[.5 - .5e^{-200t}\right]$$

$$= .5 \times 200\, e^{-200t}$$

$$= 100\, e^{-200t}$$



4.45 We are given the circuit below.



Assume $V_c(0^-) = 0$

Derive an expression for $V_c(t)$, $t \geq 0$.

$$i = C \frac{dV_c}{dt}$$

We write

$$V(t) = R\,i(t) + V_c(t)$$

$$V(t) = RC \frac{dV_c}{dt} + V_c(t)$$

$$\frac{dV_c}{dt} + \frac{V_c(t)}{RC} = \frac{t}{RC} \qquad (1)$$

$$V_c(t) = V_{c_{ss}} + V_{c_t}$$

$$V_{c_{ss}} = A + Bt$$

put this into (2)

4.45 (cont)

$$\frac{d}{dt}(A+Bt) + \frac{A+Bt}{RC} = \frac{t}{RC}$$

$$B + \frac{A+Bt}{RC} = \frac{t}{RC}$$

$$\left(\frac{A}{RC} + B\right) + \frac{B}{RC}t = \frac{t}{RC}$$

Equate coefficients of like powers.

$$\frac{A}{RC} + B = 0$$

$$\frac{B}{RC} = \frac{1}{RC}$$

$$\therefore B = 1$$

and

$$\frac{A}{RC} = -1$$

$$A = -RC$$

so

$$V_{c_{ss}} = -RC + t$$

4.45 (cont)

v_{c_t} satisfies

$$\frac{dv_c}{dt} + \frac{v_c}{RC} = 0$$

OR

$$v_{c_t} = K_t e^{-\frac{t}{RC}}$$

then

$$v_c(t) = v_{ss} + v_t$$

$$v_c(t) = -RC + t + K_t e^{-\frac{t}{RC}}$$

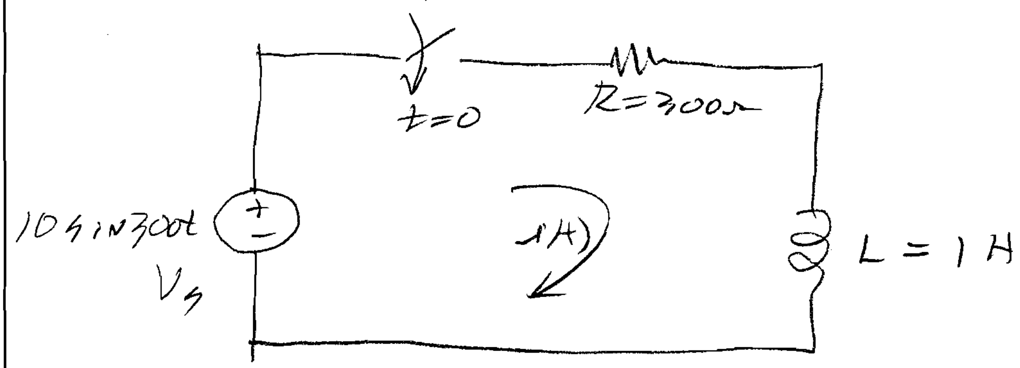
$$v_c(0^-) = v_c(0^+) = 0$$

$$0 = -RC + K_t$$

$$K_t = RC$$

$$\therefore \boxed{v_c(t) = -RC + t + RC e^{-\frac{t}{RC}}}$$

4.48 We are given the circuit below. $i(0^+) = 0$



Find the solution for $i(t)$.

We have

$$V_s = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i(t) = \frac{V_s}{L}$$

with numbers

$$\frac{di}{dt} + 300 i(t) = 10 \sin 300t \quad (1)$$

$$i(t) = i_{ss} + i_t = i_p + i_c$$

$$i_p = A\cos 300t + B\sin 300t$$

substitute into, group coefficients

4.48 (cont)

$$\frac{d}{dt}\left[A\cos 300t + B\sin 300t\right] + 300\left[A\cos 300t + B\sin 300t\right]$$
$$= 10\sin 300t$$

$$-300A\sin 300t + 300B\cos 300t$$
$$+ 300A\cos 300t + 300B\sin 300t$$
$$= 10\sin 300t$$

$$300(A+B)\cos 300t + 300(B-A)\sin 300t = 10\sin 300t$$

$$300A + 300B = 0$$

OR

$$A + B = 0 \qquad (3)$$

$$-300A + 300B = 10$$

OR

$$-30A + 30B = 1 \qquad (4)$$

multiply (3) by 30 and add

$$60B = 1$$

$$B = \frac{1}{60}$$

$$A = -\frac{1}{60}$$

4.48 (cont)

$$i_p = -\frac{1}{60}\cos 300t + \frac{1}{60}\sin 300t$$

$$i_c = K_c e^{-300t}$$

$$i(t) = i_p + i_c$$

$$i(t) = -\frac{1}{60}\cos 300t + \frac{1}{60}\sin 300t + K_c e^{-300t}$$

@ $t=0$, $i(0^+) = 0$

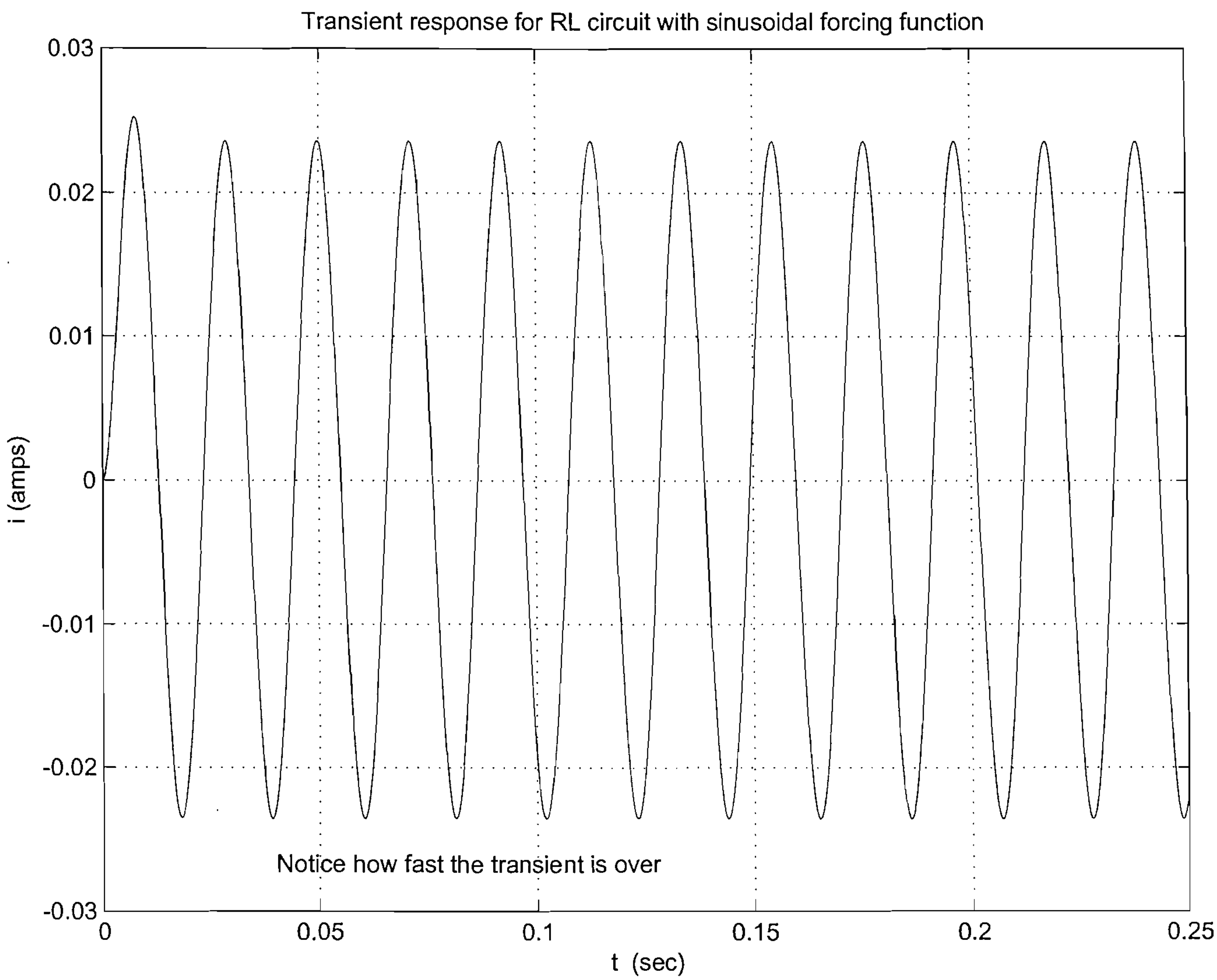
$$0 = -\frac{1}{60} + K_c$$

$$K_c = \frac{1}{60}$$

$$i(t) = \left(-\frac{1}{60}\cos 300t + \frac{1}{60}\sin 300t + \frac{1}{60}e^{-300t}\right)A$$

$$t \geq 0$$

```
%    The purpose of this problem is to use MATLAB to plot a
%    mixture of cosine, sine and exponential functions.  This
%    function arises in ECE 301 from HW problem 4.48 in Hambley
%    History:   wlg; October 2, 2007; Office PC
%               program name:  trig_functions.m

%    set up the span of time and the time stepping

     t = 0:  0.001:    0.25;

%    define the function to be plotted

    i = (1/60)*exp(-300*t)  -  (1/60)*cos(300*t)  +  (1/60)*sin(300*t);

%   plot the output

plot(t,i)
grid
gtext('Notice how fast the transient is over')
ylabel('i (amps)')
xlabel('t  (sec)')
title('Transient response for RL circuit with sinusoidal forcing function')
```


Transient response for RL circuit with sinusoidal forcing function
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
i (amps)
Notice how fast the transient is over
0
0.05
0.1
0.15
0.2
0.25
t (sec)