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ECE 301
Fall Semester, 2007
HW Set # 5

Due: October 23, 2007 Name wlg
wlg Print(last, first)

Use Engineering Paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations.** **<u>Be sure to show how you got your answers.</u>** Problem 4.58 counts 50% and 4.61 counts 30%

4.58 Work the problem as stated in the text.

Answer: $v(t) = 50 - 53.87e^{-0.268x10^4 t} + 3.867e^{-3.73x10^4 t}\ V\ u(t)$

Supplemental work for this problem.

(a) What value of R will cause the response v(t) to have a ξ =0.3? R = 12 ohms (my solution)
(b) Write out the differential equation with the correct coefficients with this zeta.
(c) Use MATLAB solution method passed out in class to solve this differential equation with the stated initial conditions given in the text.
(d) Use MATLAB to plot the response for v(t) from (c) out to five time constants.

4.61 Work the problem as stated in the text.

Answers: damping coefficient = $20x10^6$; undamped resonant frequency = $10x10^6$ rad/sec

Damping ratio, $\xi = 2$ (overdamped)

$v(t) = 28.87e^{2.68x10^6 t} - 28.87e^{-37.32x10^6 t}\ V\ u(t)$ (general solution)

ECE 301
H.W. 5

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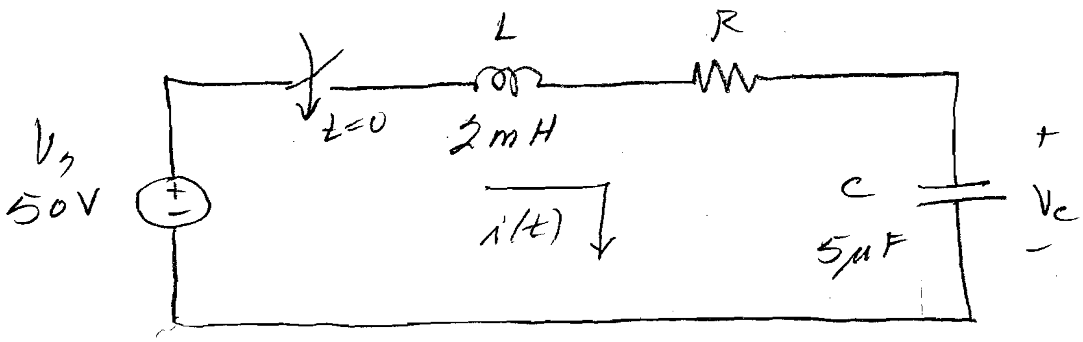


Assume: $i(0^+) = 0$, $V_c(0^+) = 0$; $R = 80\,\Omega$

write the general D.E. use V_s, R, L and C

$$Ri + L\frac{di}{dt} + V_c(t) = V_s \qquad (1)$$

but $$i = C\frac{dV_c}{dt} \qquad (2)$$

substitute (2) into (1)

$$RC\frac{dV_c}{dt} + LC\frac{d^2V_c}{dt^2} + V_c(t) = V_s \qquad (3)$$

$$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c(t)}{LC} = \frac{V_s}{LC} \qquad (4)$$

4.58 continued

use the given values of R, L, C, V_s

$$\frac{d^2 V_c}{dt^2} + \frac{80}{2\times10^{-3}}\frac{dV_c}{dt} + \frac{V_c}{2\times10^{-3}\times5\times10^{-6}} = \frac{50}{2\times10^{-3}\times5\times10^{-6}} \quad (5)$$

$$\frac{d^2 V_c}{dt^2} + 40\times10^{3}\frac{dV_c}{dt} + 0.1\times10^{9} = 5\times10^{7}$$

Characteristic equation

$$s^2 + 40\times10^3 s + .1\times10^9 = 0$$

$$(s + 2.68\times10^3)(s + 3.73\times10^4) = 0$$

$\therefore$

$$V_c(t) = V_{c_p} + V_{c_c}$$

$$V_{cp} = k_p$$

substituting into (5) gives

$V_{cp} = 50$ V (as expected) $\rightarrow V_c(\infty)$

$$V_c(t) = 50 + k_1 e^{-2.68\times10^3 t} + k_2 e^{-3.73\times10^4 t} \quad (6)$$

We need $V_c(0^+)$ (given as 0)

Go to Eq (2) $i = C\frac{dv}{dt}$

4.58 cont

We have

$$\frac{dV_c}{dt}(0^+) = \frac{i(0^+)}{C}$$

but $i(0^+) = 0 \quad \therefore \quad \frac{dV_c}{dt}(0^+) = 0$

We have

$$V_c(0^+) = 0 \quad ; \quad \dot{V}_c(0^+) = 0 \qquad (7)$$

From (6)

$$0 = 50 + k_1 + K_2$$

OR

$$\boxed{k_1 + K_2 = -50}$$

Now find $\frac{dV_c}{dt}$, from (6)

$$\frac{dV_c}{dt} = -2.68\times10^3 k_1 e^{-2.68\times10^3 t} - 3.73\times10^4 K_2 e^{-3.73\times10^4 t}$$

evaluate at $t=0$, using $\dot{V}_c(0^+) = 0$

$$0 = -2.67\times10^3 K_1 - 3.73\times10^4 K_2$$

OR

$$\boxed{2670\, K_1 + 37300\, K_2 = 0}$$

$$\begin{bmatrix} 1 & 1 \\ 2670 & 37300 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -50 \\ 0 \end{bmatrix}$$

4.58 cont.

$$K_1 = -53.9 \qquad K_2 = 3.86$$

$\therefore$

$$\boxed{V_c(t) = 50 - 53.9e^{-2670t} + 3.86e^{-37300t}} \qquad (7)$$

This is where the problem in the book ends

(a) What value of R will cause $V_c(t)$ to have $\zeta = 0.3$?

Go to equation (4), Ch. Eq.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad (8)$$

compare with

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad (9)$$

Put the numbers in (8)

$$s^2 + \frac{R}{2\times10^{-3}}s + 1\times10^8 = 0$$

or

$$s^2 + 500R\,s + 1\times10^8 = 0 \qquad (10)$$

4.58 cont.

Comparing (10) with (9)

$\omega_n^2 = 1 \times 10^8$

$\omega_n = 10{,}000$ rad/sec

then

$2\zeta\omega_n = 500R \qquad \zeta = .3, \ \omega_n = 10{,}000$

$2 \times 0.3 \times 10000 = 500R$

$$\boxed{R = \frac{6000}{500} = 12\,\Omega}$$

(b)

$$\frac{d^2 v_c}{dt^2} + 6000\frac{dv_c}{dt} + 1 \times 10^8 v_c = 50 \times 10^8$$

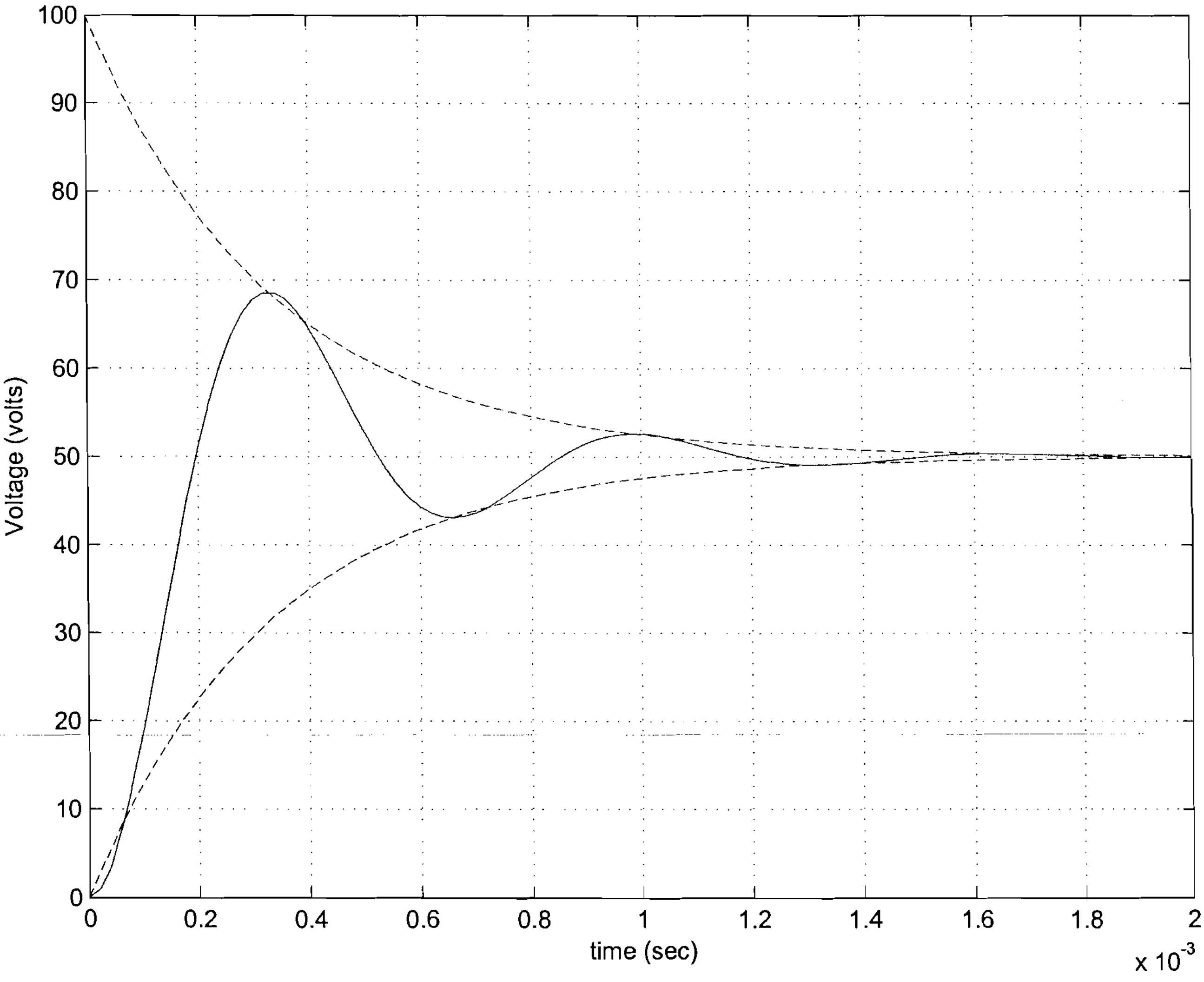
Use MATLAB D.E. solution method to find $V_c(t)$.

```
>>
>> dsolve('D2v + 6000*Dv + 100000000*v = 50*100000000', 'v(0) = 0', 'Dv(0) = 0')

ans =

50-50*exp(-3000*t)*cos(1000*91^(1/2)*t)-150/91*91^(1/2)*exp(-3000*t)*sin(1000*91^(1/2)*t)

>>
```



100
90
80
70
60
50
40
30
20
10
0
Voltage (volts)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (sec)
x 10^-3

```
% History:  Solution of Differential Equation part (b) supplement
% for HW #5, problem 4.58.  Office computer, October 21, 2007
% W. Green:  Program name solve4_58.m

% dsolve('D2v + 6000*Dv + 100000000*v = 50*100000000', 'v(0) = 0', 'Dv(0) = 0')

% The solution for the above is

t = 0:.00002: .002;

v = 50-50*exp(-3000*t).*cos(1000*91^(1/2)*t)-150/91*91^(1/2)*exp(-3000*t).*sin(1000*91^(1 ↙
/2)*t);

% define the following to illustrate the envelope

x1 = 50*(1 - exp(-3000*t));
x2 = 50*(1 + exp(-3000*t));


plot(t, v, t, x1,'--', t, x2,'--')
grid
ylabel('Voltage (volts)')
xlabel('time (sec)')
```

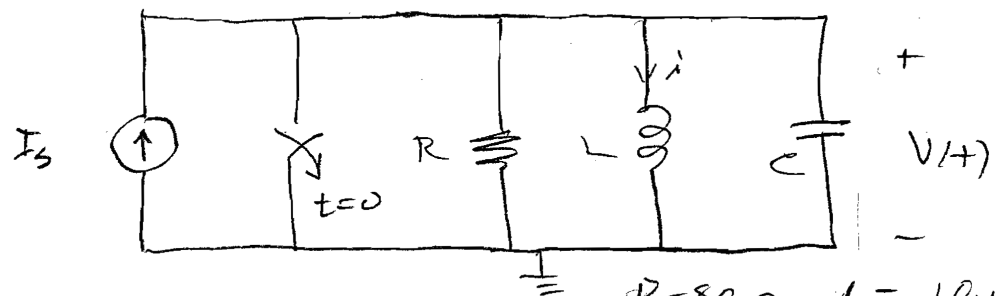
4.61 Work the problem as stated in the text.

Answers: damping coefficient = $20x10^6$; undamped resonant frequency = $10x10^6$ rad/sec

Damping ratio, $\xi = 2$ (overdamped)

$v(t) = 28.87e^{2.68x10^6 t} - 28.87e^{-37.32x10^6 t}\ V\ u(t)$ (general solution)

For the following circuit:



$R = 80\,\Omega$, $L = 10\,\mu H$
$C = 1{,}000\,pF$
$= 1 \times 10^{-9}$

By nodal analysis:

$$\frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int_0^t V(t)\,dt + i = I_s \quad (1)$$

Take the derivative wrt time

$$\frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{V(t)}{L} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{V(t)}{LC} = 0$$

With numbers

$$\frac{d^2v}{dt^2} + \frac{1}{25 \times 1 \times 10^{-9}}\frac{dv}{dt} + \frac{V(t)}{10 \times 10^{-6} \times 1 \times 10^{-9}} = 0$$

(2)

4.61

$$\frac{d^2 v}{dt^2} + 40 \times 10^6 \frac{dv}{dt} + 1 \times 10^{14} v(t) = 0$$

By text book

$$\left[\begin{array}{l} s^2 + 2\alpha s + \omega_0^2 = 0 \\ s^2 + 2\xi \omega_n s + \omega_n^2 = 0 \end{array}\right] \text{Char. Eq.}$$

(a) damping coefficient

$$2\alpha = 40 \times 10^6$$

$$\boxed{\alpha = 20 \times 10^6}$$

$$\omega_0 = \omega_n = \sqrt{1 \times 10^{14}} = 1 \times 10^7 \text{ rad/sec}$$

$$\boxed{\omega_0 = 10 \times 10^6 \text{ rad/sec}}$$

$$\xi = \frac{\alpha}{\omega_0} = \frac{20 \times 10^6}{10 \times 10^6} = 2$$

$$\xi = 2$$

(b) Given $v(0^+) = 0$ $i(0^+) = 0$

show that $\dot{v}(0^+) = 10^9$ V/sec

From (1)

$$\cancel{\frac{v(0^+)}{R}}^{\,0} + C\frac{dv(0^+)}{dt} + \cancel{i(0^+)}^{\,0} = 1.$$

$$\boxed{\frac{dv(0^+)}{dt} = \frac{1}{C} = \frac{1}{1 \times 10^{-9}} = 1 \times 10^9 \text{ V/sec}}$$

4.61

(c) Find the particular solution

From Equation (2), the particular solution is

$V_{cp} = 0$

(d) Equation (2) becomes

$$\frac{d^2v}{dt^2} + 40\times10^6 \frac{dv(t)}{dt} + 1\times10^{14} v(t) = 0$$

$$s_1 = -37.3\times10^6$$

$$s_2 = -2.68\times10^6$$

$$v(t) = K_1 e^{-37.3\times10^6 t} + K_2 e^{-2.68\times10^6 t}$$

$$v(0) = 0 = K_1 + K_2$$

$$\frac{dv}{dt} = -37.3\times10^6 K_1 e^{-37.3\times10^6 t} - 2.68\times10^6 e^{-2.68\times10^6 t}$$

$$\frac{dv(0)}{dt} = 1\times10^9 = -37.3\times10^6 K_1 - 2.68\times10^6 K_2$$

$$K_1 + K_2 = 0$$

$$37.3\times10^6 K_1 + 2.68\times10^6 K_2 = -1\times10^9$$

$$K_1 = -28.9, \quad K_2 = 28.9$$

4.61 cont.

$$\therefore \quad V(t) = -28.9e^{-37.3\times10^6 t} + 28.9e^{-2.68\times10^4 t} \; V, \; t \geq 0$$