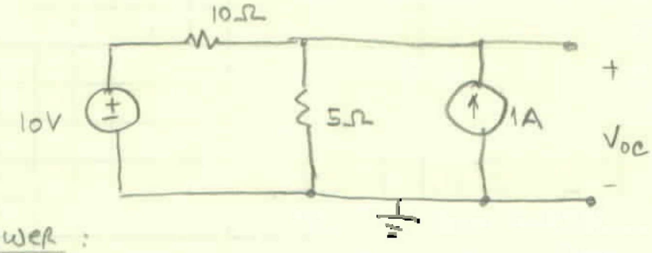
HW #3
SOLUTION

P. 2.75



FIND THEVENIN & NORTON EQUIVALENT CIRCUIT

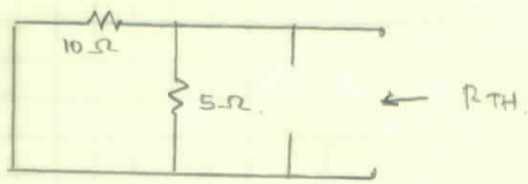
ANSWER :

- Open circuit solution :

(i) use nodal analysis to solve for the open-ckt voltage.

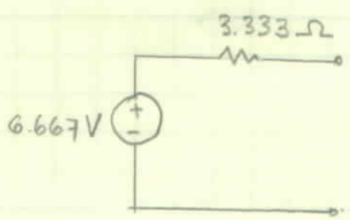
$$\frac{V_{oc}-10}{10} + \frac{V_{oc}}{5} = 1 \quad \Rightarrow \quad V_{oc} = \underline{\underline{6.667 \text{ Volts}}}$$

(ii) To find R_{TH}, we turn off all the sources,

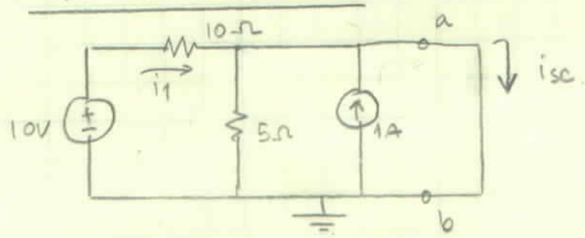


Therefore, $R_{TH} = 5 \parallel 10 = \frac{5 \times 10}{15} = 3.333\Omega$

∴ The thevenin Equivalent ckt :



- <u>short circuit solution</u>.



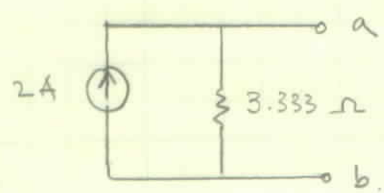
$$(4)\quad \frac{V_a - 10}{10} + \frac{V_a}{5} + i_{sc} - 1A = 0 \qquad \text{but } V_a = V_b = 0$$

$$\frac{-10}{10} + i_{sc} - 1A = 0$$

$$\underline{\underline{i_{sc} = 2A = I_N}}$$

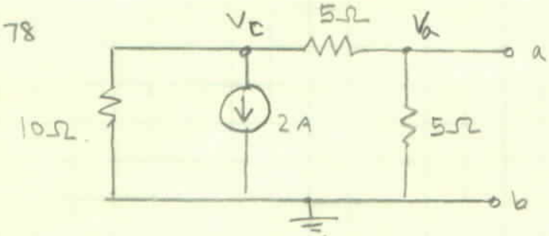
From the open circuit solution, $R_{Th} = 3.333\ \Omega$

∴ The Norton equiv. ckt.



Also, we verify that $i_{sc} = I_N = V_{TH}/R_{TH} = 2A$.

P2.78



Find the thevenin & Norton Equivalent ckt.

ANSWER

- the open circuit solution

(i) using Nodal analysis w V_b as the reference Node.

$$\frac{V_a}{5} + \frac{V_a - V_c}{5} = 0$$

$$2V_a - V_c = 0 \quad \text{—— ①}$$

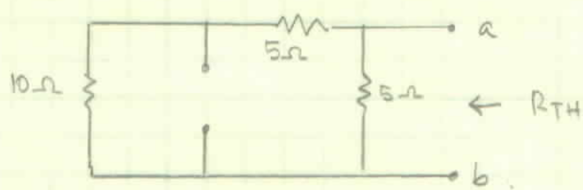
$$\frac{V_c}{10} + 2 + \frac{V_c - V_a}{5} = 0.$$

$$-2V_a + 3V_c = -20 \quad \text{—— ②}$$

Solving equations ① and ②, we get $V_a = -5\,V$, $V_c = -10\,V$

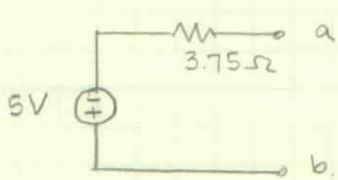
thus, $V_{TH} = \underline{\underline{V_{ab} = -5V}}$

(ii) to find the R_{TH}, we turn off the sources:

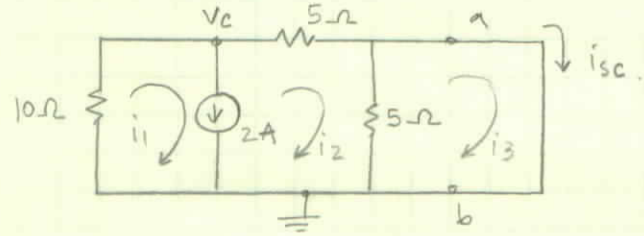


$$R_{TH} = (10+5) /\!/ 5 = 15 /\!/ 5 = \frac{15 \times 5}{15+5} = \frac{75}{20} = \underline{\underline{3.75\,\Omega}}$$

Therefore, the Thevenin equiv. ckt



- the short circuit soln.



(i) Supermesh 1 and 2: w Constraint

$i_1 - i_2 = 2A$ —— ②

$$10i_1 + 5i_2 + 5(i_2 - i_3) = 0$$
$$10i_1 + 10i_2 - 5i_3 = 0 \quad \text{—— ①}$$

Mesh 3: $5(i_3 - i_2) = 0$

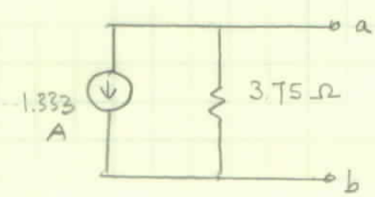
$$5i_2 - 5i_3 = 0 \quad \text{—— ③}$$

Solve for eqn. ①, ② and ③, we get $i_1 = 0.667$ A

$i_2 = -1.333$ A

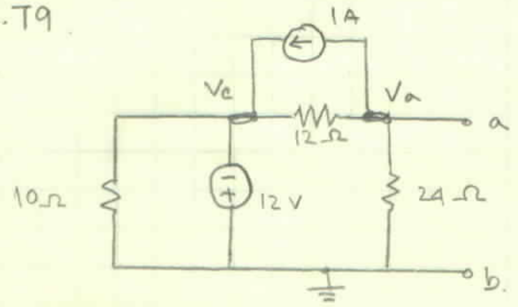
$i_3 = -1.333\text{ A} = i_{sc} = I_N$.

Therefore, the Norton equiv. ckt:



We verify that $i_{sc} = I_N = -1.333\text{ A} = \frac{-5\text{ V}}{3.75\,\Omega} = V_{TH}/R_{TH}$

P. 2.T9



a. Find the Thévenin & Norton equivalent ckt.
b. What does the 10Ω - Resistor have on the equiv. ckt ? Explain your answer.

(a) • the open - circuit solution.

(i) use nodal analysis w V_b as the Ref. Node:

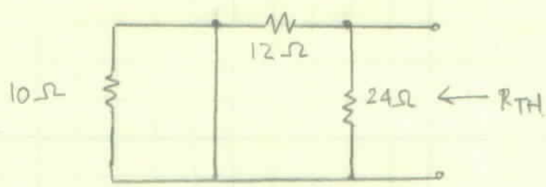
$$\frac{V_a}{24} + \frac{V_a - V_c}{12} + 1 = 0.$$

$$3V_a - 2V_c = -24. \quad \text{—— (1)}$$

but $V_c = -12V$, therefore

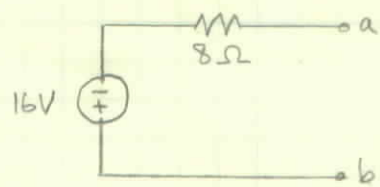
$$3V_a + 24 = -24$$
$$3V_a = -48$$
$$\underline{\underline{V_a = -16\,V = V_{TH}.}}$$

(ii) Turn all sources off to find R_{TH}.

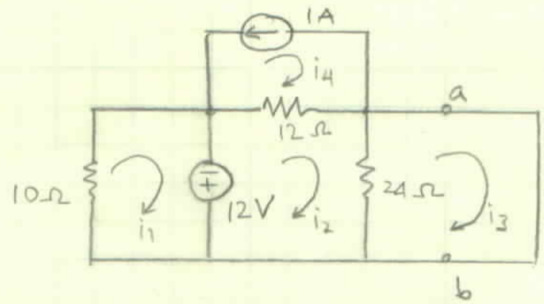


$$R_{TH} = 12 \,//\, 24$$
$$= \frac{12 \times 24}{12 + 24} = \underline{\underline{8\,\Omega = R_{TH}}}$$

Therefore, the Thévenin equivalent ckt:



- The short-circuit solution



(i) Using mesh analysis:

<u>mesh 1</u>:

$10 i_1 - 12\,V = 0$

$10 i_1 = 12$

$i_1 = 1.2\,A$.

<u>mesh 2</u>:

$12V + 12(i_2 - i_4) + 24(i_2 - i_3) = 0$, $i_4 = -1A$.

$12 i_2 + 12 + 24 i_2 - 24 i_3 = -12$

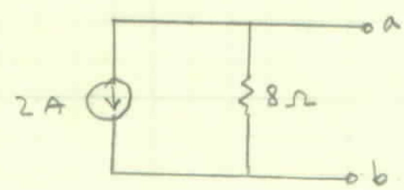
$36 i_2 - 24 i_3 = -24$ — ①

<u>mesh 3</u>: $24(i_3 - i_2) = 0$

$i_2 = i_3$ — ②

Solving for ① and ②, we get $i_3 = i_{sc} = \underline{\underline{I_N = -2A}}$.

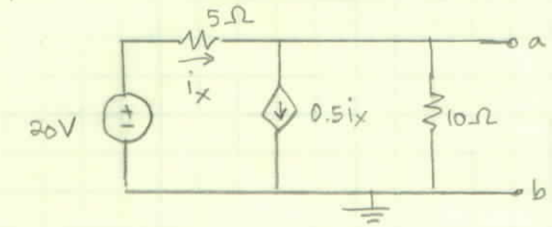
Therefore, the Norton equiv circuit.



This verify that $i_{sc} = I_N = -2A = \dfrac{-16V}{8\Omega} = V_{TH}/R_{TH}$.

(b) The 10Ω resistor has no effect on the equiv ckt because the voltage across the 12V source is independent of the resistor value.

P 2.83.



Find Thévenin & Norton Equiv. ckt.

ANSWER

(i) use nodal analysis w/ V_b as Ref. Node.

$$\frac{V_a}{10} + \frac{V_a - 20}{5} + 0.5\,i_x = 0 \quad , \text{ but } i_x = \frac{20 - V_a}{5}$$

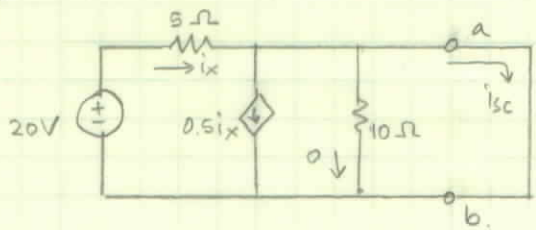
$$\frac{V_a}{10} + \frac{V_a - 20}{5} + \frac{20 - V_a}{10} = 0 \quad \times 10$$

$$2V_a - 20 = 0$$

$$2V_a = 20$$

$$V_a = \underline{\underline{10\text{ V} = V_{OC} = V_{TH}}}$$

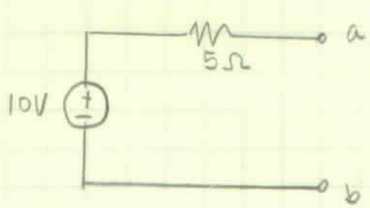
(ii) For the short circuit condition:



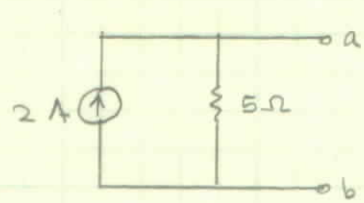
$$i_x = \frac{20\text{ V}}{5\Omega} = 4\text{A} \Rightarrow i_{sc} = i_x - 0.5i_x = 0.5i_x = \underline{\underline{2\text{A}}}$$

Therefore, $R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{10\text{V}}{2\text{A}} = \underline{\underline{5\Omega}}$

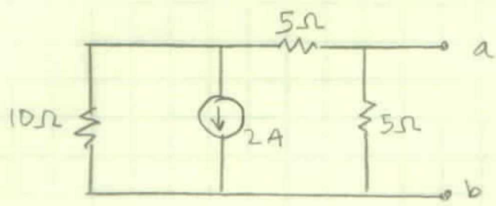
The thévenin equiv. ckt



The Norton equiv. ckt



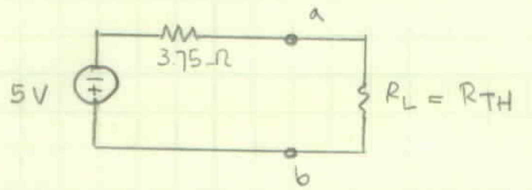
P 2.85



Find the max. power that can be delivered to a resistive load by the circuit.

ANSWER.

From P 2.78 we got the Thevenin equiv circuit:

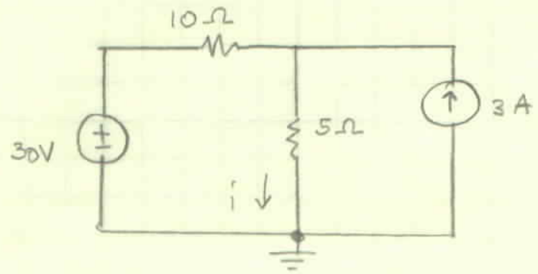


$$V_L' = V_{TH} \frac{R_{TH}}{2R_{TH}} = \frac{V_{TH}}{2}$$

Then, the maximum power is obtained for a load resistace equal to R_{TH}.

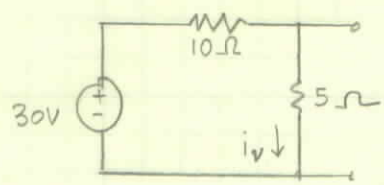
$$P_{max} = \frac{V^2}{R} = \frac{(V_{TH}/2)^2}{R_{TH}} = \frac{(5/2)^2}{3.75} = \underline{\underline{1.667\ W}}$$

P 2.89



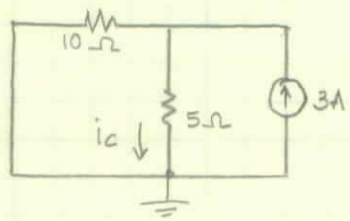
Find i using superposition principle

(i) turn off the current source, leave the 30V on.



$$i_v = \frac{30\,V}{(10+5)\,\Omega} = 2\,A$$

(ii) turn off the voltage source, leave the 3A source on.



$$i_c = \frac{10}{10+5}\,(3A)$$
$$= 2A.$$

Therefore, $i = i_v + i_c$

$= (2+2)\,A$

$= \underline{\underline{4\,A}}$.