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ECE 301
AC Power
Part II

Lesson 15
Fall '06

Complex Power

Thus far we have only considered Average real power.

$$P = \frac{V_m I_m}{2} \cos(\theta_V - \theta_I) \qquad (15.1)$$

Earlier, average real power was also expressed as

$$P = \frac{1}{2} Re\left[\hat{V}\hat{I}^*\right] = Re\left[\hat{V}_{rms}\hat{I}^*_{rms}\right] \qquad (15.2)$$

We define complex power as

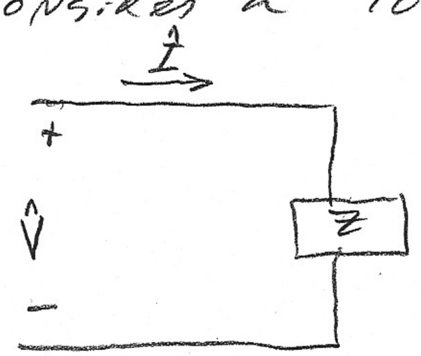
$$\hat{S} = \frac{1}{2}\left[\hat{V}\hat{I}^*\right] = \hat{V}_{rms}\hat{I}^*_{rms} \qquad (15.3)$$

We can further express this as

$$\hat{S} = \frac{V_m I_m}{2} \angle \theta_V - \theta_I = |V_{rms}||I_{rms}| \angle \theta_V - \theta_I \qquad (15.4)$$

$$\hat{S} = \frac{V_m I_m}{2}\left[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)\right] \qquad (15.5)$$

We consider a load, $\hat{Z}$



$$\hat{Z} = \frac{\hat{V}}{\hat{I}} = \frac{\hat{V}_{rms}}{\hat{I}_{rms}} = \frac{|V_{rms}|}{|I_{rms}|} \angle \theta_V - \theta_I \qquad (15.6)$$

$$\hat{Z} = \frac{V_m}{I_m} \angle \theta_V - \theta_I \qquad (15.7)$$

We recall

$$\hat{V}_{rms} = \hat{Z}\hat{I}_{rms} \qquad (15.8)$$

Then

$$\hat{S} = \hat{V}_{rms}\hat{I}^{*}_{rms} \qquad (15.9)$$

Using (15.8) in (15.9) gives

$$\hat{S} = \hat{I}_{rms}\hat{I}^{*}_{rms}\hat{Z} \qquad (15.10)$$

but $\hat{I}_{rms}\hat{I}^{*}_{rms} = I^2_{rms} = |\hat{I}_{rms}|^2$

so

$$\hat{S} = |I_{rms}|^2 \hat{Z} \qquad (15.11)$$

$$\hat{S} = \hat{V}_{rms} \times \frac{V^{*}_{rms}}{\hat{Z}^{*}} = \frac{|V_{rms}|^2}{\hat{Z}^{*}} \qquad (15.12)$$

With a little reflection we see that

$$\hat{S} = \frac{I_m^2}{2}\hat{Z} = \frac{V_m^2}{2\hat{Z}^{*}} \qquad (15.13)$$

Going back to (15.5)

$$\vec{S} = \frac{V_m I_m}{2}\left[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)\right] \quad (15.14)$$

$$= |V_{rms}||I_{rms}|\left[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)\right]$$

$$\vec{S} = P + jQ$$

where

$$P = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) = |V_{rms}||I_{rms}|\cos(\theta_V - \theta_I)$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_V - \theta_I) = |V_{rms}||I_{rms}|\cos(\theta_V - \theta_I)$$

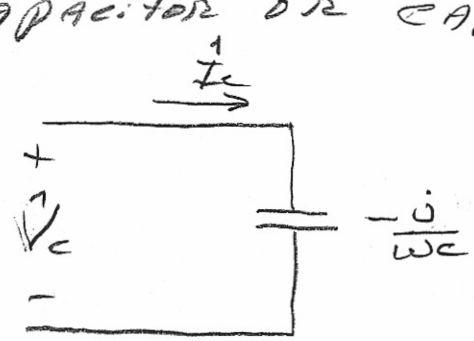
P is called the average real power
units of Watts

Q is called the reactive power
also the quadature power,
units of Volt-Amps-reactive
OR VARs

P is always positive.

Q can be positive or negative.
This depends on the value of
$\sin(\theta_V - \theta_I)$ (15.15)

We take note that θ_I is greater than θ_V when are dealing with a capacitor or capacitive load.



$$\vec{V}_c = \frac{-j}{\omega C} I_c$$

$$V_{cm} \angle \theta_V = \frac{-j}{\omega C} I_{cm} \angle \theta_I = \frac{1}{\omega C} I_{cm} \angle \theta_I - 90^\circ$$

or

$$\theta_I = \theta_V + 90^\circ$$

This establishes that for a pure capacitor that $\theta_V - \theta_c = -90^\circ$

$$\sin(-90) = -1$$

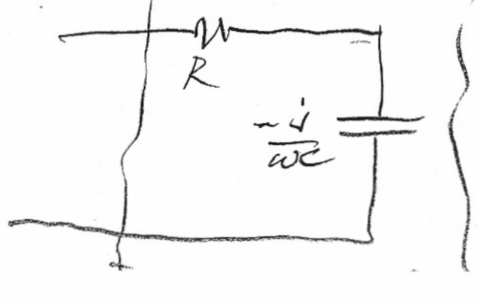
since

$$Q = \frac{V_m I_m}{2} \sin(\theta_V - \theta_I)$$

Then Q is negative.

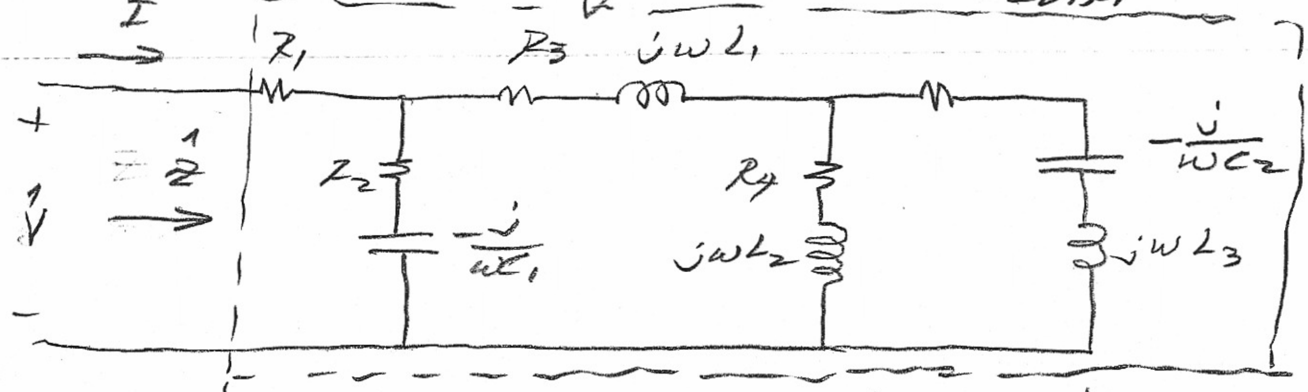
A capacitive load is one in which the angle of Z is negative.

For example



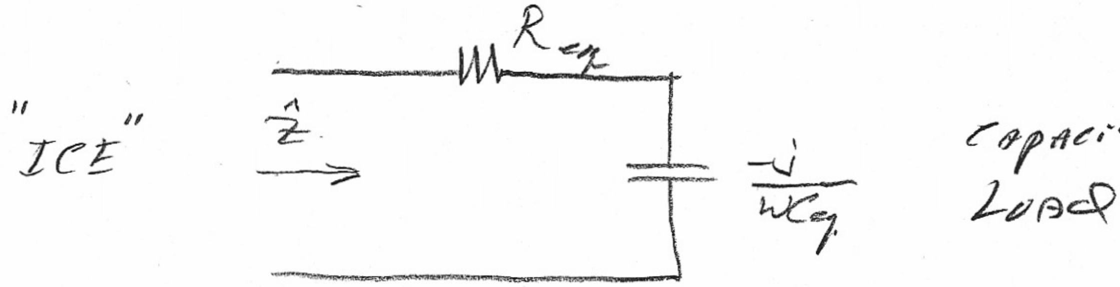
Z

Suppose you look into the terminals of the following network.



Depending on ω and the values of the passive circuit element parameters, the angle of $\hat{Z}$ can be positive or negative. If the angle of $\hat{Z}$ is negative we say we have a capacitive load.

We visualize this as

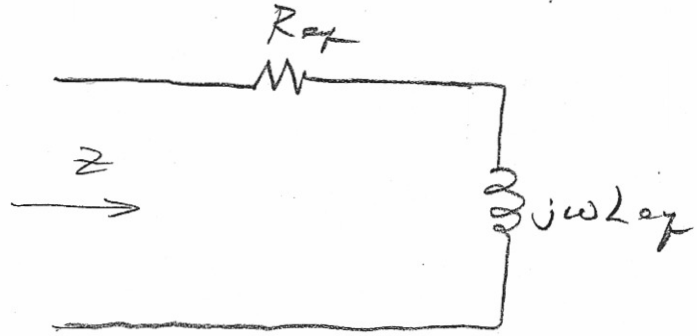


$\hat{Z} = |Z| \angle \theta_Z \ ; \quad \theta_Z < 0$

$Q < 0$; leading power factor.

If we look into the terminals of a circuit and the angle of $\hat{Z}$ is positive, we say we have an inductive load.

"ELI"



Inductive Load

$$\hat{Z} = |Z| \angle \theta_Z \; ; \; \theta_Z > 0$$

For the inductive load, Q is positive.
lagging power factor

We have already seen

$$\frac{\hat{V}}{\hat{I}} = \frac{V_m \angle \theta_V}{I_m \angle \theta_I} = \hat{Z} = \frac{V_m}{I_m} \angle \theta_V - \theta_I$$

The pf angle is defined as $\theta_V - \theta_I$

$$pf = \cos(\theta_V - \theta_I)$$

We note from Equation (15.11)

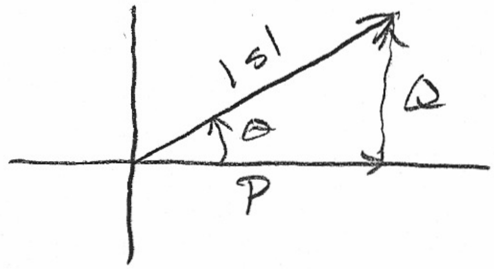
$$\hat{S} = |I_{rms}|^2 \hat{Z}$$

$$\hat{S} = |I_{rms}|^2 |Z| \angle \theta_V - \theta_I \qquad (15.16)$$

This tells us that the angle of $\hat{S}$ is the same as the angle of the impedance.

Now we can think of $\bar{S}$ in the following graphical since.

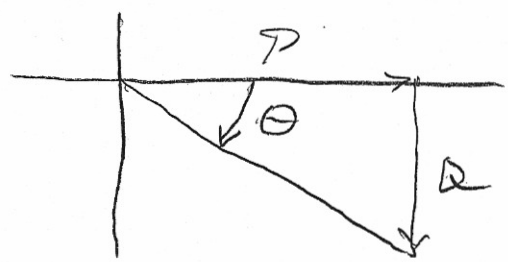
"ELI"



$Q > 0$
p.f. lagging
$\theta = \theta_V - \theta_I > 0$

An inductive load. Current lags the voltage. pf is lagging.
$\angle Z$ is positive

"ICE"



$Q < 0$
pf leading
$\theta = \theta_V - \theta_I < 0$

A capacitive load. Current leads the voltage. p.f. is leading
$\angle Z$ is negative.

If we keep the triangle idea of $\bar{S}$ before our mind we are not likely to get confused when we deal with complex power.

$$\vec{S} = \frac{1}{2}\vec{V}\vec{I}^* \qquad (15.17)$$

$$\vec{S} = \vec{V}_{rms}\vec{I}^*_{rms} \qquad (15.18)$$

where

$$\vec{V} = V_m \angle\theta_V \ ; \ \vec{I} = I_m \angle\theta_I$$

$$\vec{V}_{rms} = \frac{\vec{V}}{\sqrt{2}} = \frac{V_m}{\sqrt{2}}\angle\theta_V \ ; \ \vec{I}_{rms} = \frac{\vec{I}}{\sqrt{2}} = \frac{I_m \angle\theta_I}{\sqrt{2}}$$

using $\vec{Z} = \dfrac{\vec{V}_{rms}}{\vec{I}_{rms}} = \dfrac{\vec{V}}{\vec{I}}$

$$\vec{S} = \vec{V}_{rms} I^*_{rms} = \vec{I}_{rms} I^*_{rms} \vec{Z} = |\vec{I}_{rms}|^2 \vec{Z}$$

$$\vec{S} = \frac{\vec{V}\vec{I}^*}{2} = \frac{\vec{I}\vec{I}^*\vec{Z}}{2} = \frac{|\vec{I}|^2\vec{Z}}{2} = \frac{I_m^2 \vec{Z}}{2}$$

so;

$$\vec{S} = |I_{rms}|^2 \vec{Z}$$

$$\vec{S} = \frac{I_m^2 \vec{Z}}{2}$$

$$\vec{S} = \frac{|V_{rms}|^2}{\vec{Z}^*}$$

$$\vec{S} = \frac{V_m^2}{2\vec{Z}^*}$$

$$\tilde{S} = P + jQ$$

$$|\tilde{S}| = \text{Apparent Power} = \sqrt{P^2 + Q^2}$$

$$|\tilde{S}| = |V_{rms}||I_{rms}| = \frac{V_m I_m}{2}$$

$$P = \text{Real Average power} = \frac{V_m I_m}{2} \cos(\theta_V - \theta_I)$$

$$P = |V_{rms}||I_{rms}|\cos(\theta_V - \theta_I)$$

$$P = |\tilde{S}|\cos(\theta_V - \theta_I)$$

$$Q = \text{Reactive Power} = \frac{V_m I_m}{2} \sin(\theta_V - \theta_I)$$

$$Q = |V_{rms}||I_{rms}|\sin(\theta_V - \theta_I)$$

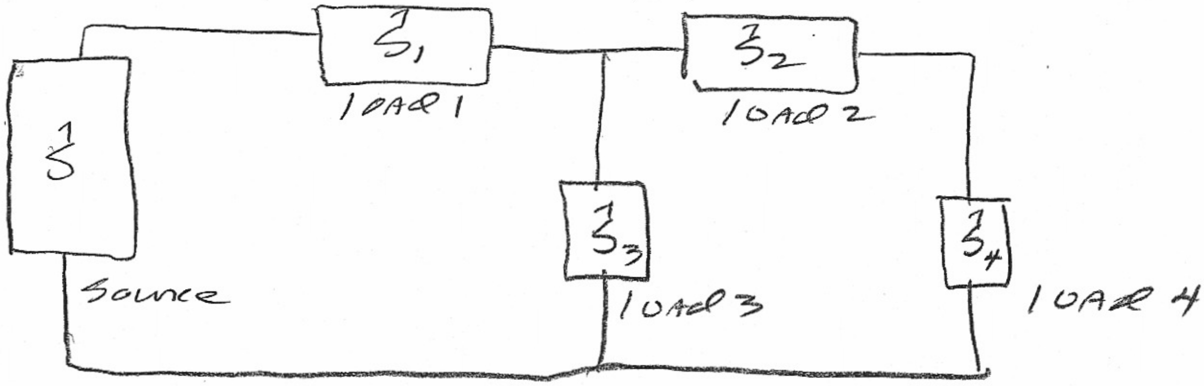
$$Q = |\tilde{S}|\sin(\theta_V - \theta_I)$$

Seems like a lot of equations but they stem from the fundamental equations.

Conservation of Complex Power

In a summarizing statement we can say, "The complex, real and reactive powers of the sources equal the respective sums of complex, real and reactive powers of individual loads." (Alexander, p 448)

Illustration



$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \vec{S}_4.$$

If

$$\vec{S} = P + jQ \qquad \vec{S}_3 = P_3 + jQ_3$$

$$\vec{S}_1 = P_1 + jQ_1 \qquad \vec{S}_4 = P_4 + jQ_4$$

$$\vec{S}_2 = P_2 + jQ_2$$

then

$$P = P_1 + P_2 + P_3 + P_4$$

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \vec{S}_4$$

Example 15-1

You are given the following circuit. The real average power delivered to the $60\,\Omega$ resistor is 240W.

- Find $\hat{V}$
- Find the complex power of each branch
- Find the complex power supplied

Assume the current through the $60\,\Omega$ resistor has zero phase shift.

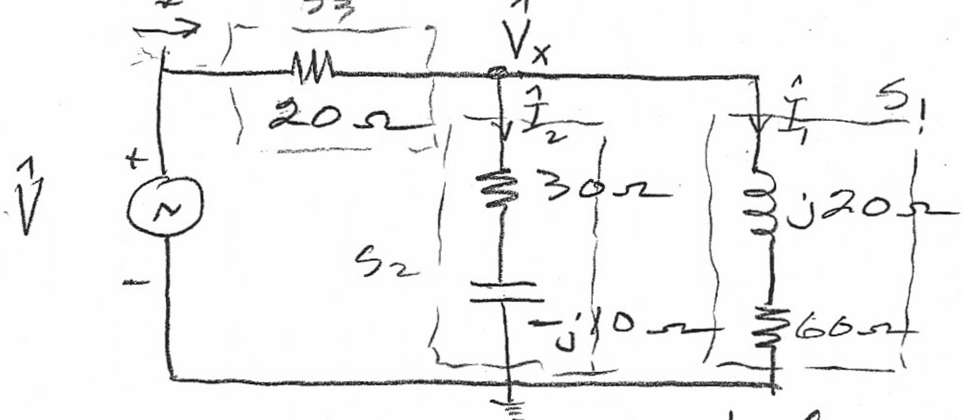


Figure 15.1: Circuit for Example 15.1

$$P_{60} = 240 = |I_{1rms}|^2 60 \;;\; |I_1| = \sqrt{4} = 2A$$

Give $\angle I_1 = 0$

$$\hat{I}_1 = 2\angle 0$$

$$\hat{V}_x = \hat{I}_1 \times (60 + j20) = 126.49\angle 18.43^\circ \ V_{rms}$$

$$\hat{S}_1 = |I_1|^2 \hat{Z}_1 = (2)^2(60+j20) = (240+j80)\ VA$$

$$\hat{S}_2 = \frac{|V_x|^2}{Z_2^*} = \frac{(126.49)^2}{30+j10} = (480 - j160)\ VA$$

$$\hat{I}_2 = \frac{\hat{V}_x}{30-j10} = \frac{126.49\angle 18.43}{(30-j10)} = 4\angle 36.87^\circ\ A_{rms}$$

$$\hat{I} = \hat{I}_1 + \hat{I}_2 = 2\angle 0 + 4\angle 36.87 = 5.73\angle 24.78^\circ\ A_{rms}$$

$$\hat{S}_3 = |\hat{I}|^2 \times 20 = 5.73^2 \times 20 = 656.7\ VA$$

$$\hat{V} = 20I + \hat{V}_x = 240.72\angle 21.45^\circ\ V_{rms}$$

$$\hat{S} = \hat{V}\hat{I}^* = (240.72\angle 21.45)(5.73\angle -24.78)$$

$$\hat{S} = 1377 - j80.1 \quad VA$$

$$\hat{S} \stackrel{?}{=} \hat{S}_1 + \hat{S}_2 + \hat{S}_3 \quad \text{(check this)}$$

$$\hat{S} = 240 + j80 + 480 - j160 + 656.7$$

$$\hat{S} = 1376.7 - j80 \quad VA$$

checks pretty close

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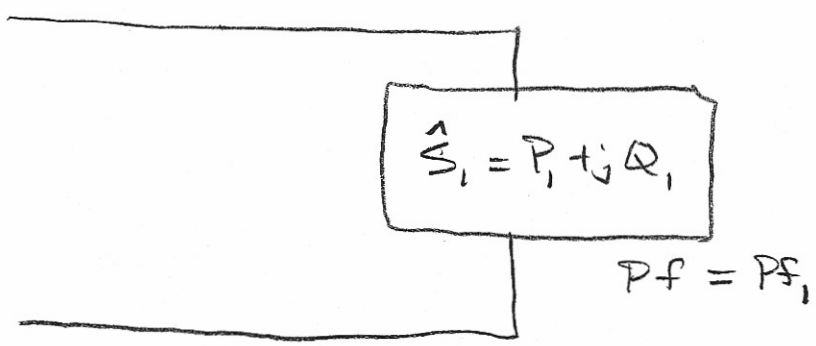
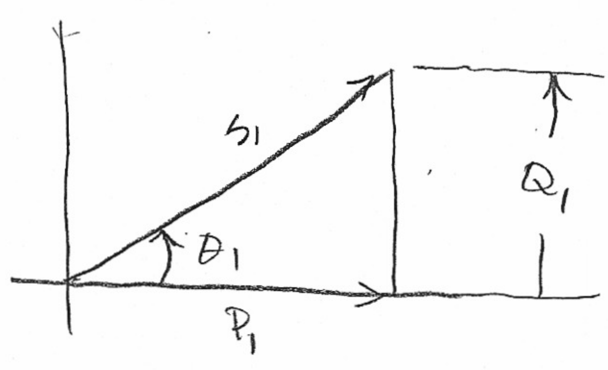
Power Factor Correction:

Suppose we are given the following situation





Suppose we would like to change the pf. to a value closer to 1. This can be done by placing a capacitor in parallel with the load as shown in Figure 15.2

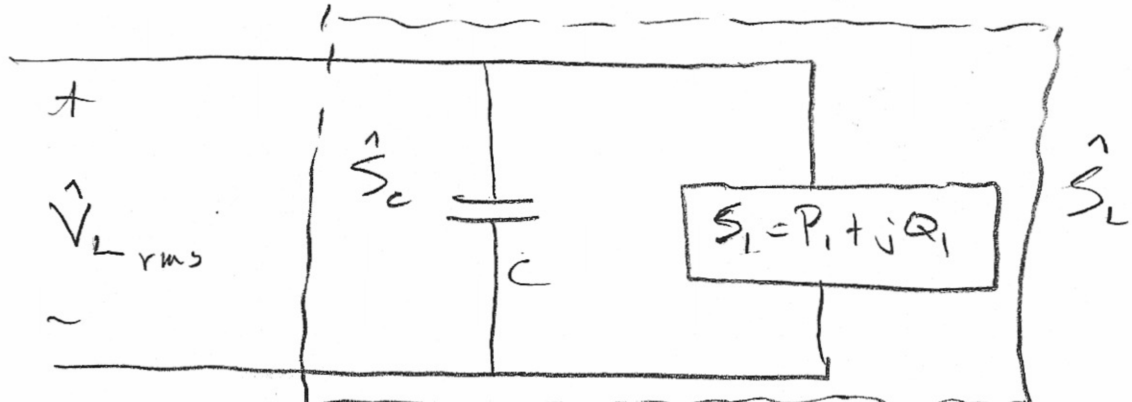


Figure 15.2: Correcting (changing) the pf of a load.

The diagram in Figure 15.3 shows what is happening to the complex power when we add the capacitor.

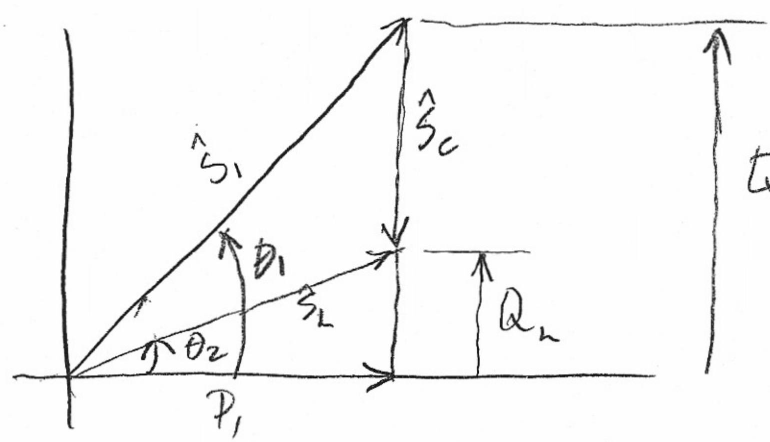


Figure 15.3: Power Factor correction Diagram.

We see that

$$\hat{S}_1 + \hat{S}_c = \hat{S}_L$$

$$\hat{S}_c = \hat{S}_L - \hat{S}_1$$

$$\hat{S}_2 = P_1 + jQ_L \; ; \; S_1 = P_1 + jQ_1$$

$$\hat{S}_c = P_1 + jQ_2 - P_1 - jQ_1$$

$$\hat{S}_c = j(Q_2 - Q_1) = jP_1[\tan\theta_L - \tan\theta_1]$$

but $\hat{S}_c = \dfrac{|V_{L_{rms}}|^2}{\hat{Z}_c^*} = \dfrac{V_{L_{rms}}^2 \, \omega C}{+j} = -j\omega C V_{rms}^2$

so

$$-j\omega C V_{rms}^2 = -jP[\tan\theta_1 - \tan\theta_2]$$

$$\boxed{C = \frac{P[\tan\theta_1 - \tan\theta_2]}{\omega V_{L_{rms}}^2}} \quad (15.17)$$

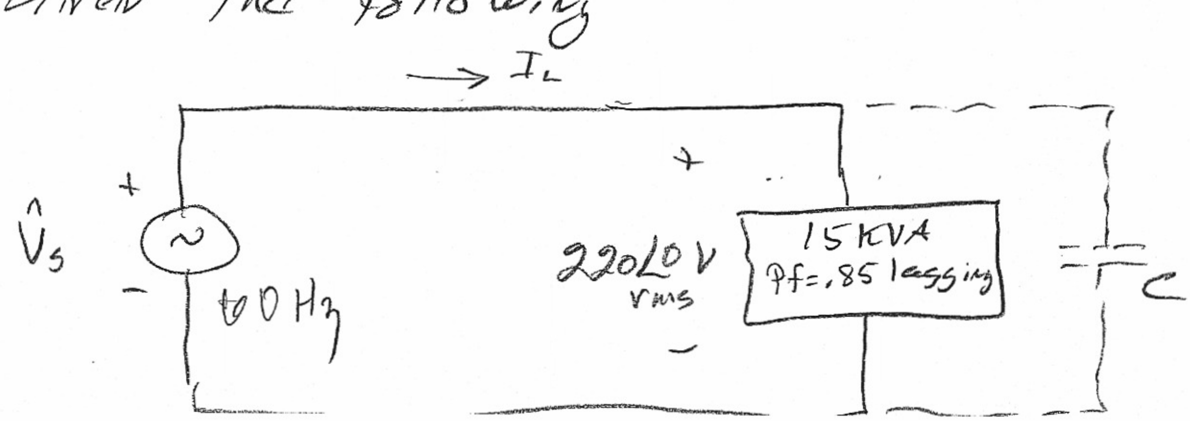
Handy dandy equation.

#

Power companies would like to see all loads have a unity p.f. short of this they like to keep the p.f. > 0.97. This reduces the reactive power. In the end, this reduces the line current (of the distribution system) which in turn reduces the real average power lost on the transmission line. The power company has to pay for this loss plus they must have heavier copper lines to carry the larger current. This also increases the debt load. A simple example is given below to see how this comes about as a savings if a load power factor is increased. The following problem is a little superficial but it shows the point.

Example 15.2

Given the following



(a) Find $|I_L|$

(b) Find the value of C that will change the pf to 0.98

(c) Find the new $|I_L|$

Solution

S_1 (present complex power without C)

$|S_1| = 15 \text{ KVA}$

$\cos\theta = .85$

$\theta = 31.79^\circ$

$\hat{S}_1 = 15\angle 31.79^\circ \text{ KVA} = (12.75 + j7.9) \text{ K.VA}$

$\hat{S}_1 = \hat{V}\hat{I}_L^*$

$\hat{I}_L^* = \dfrac{(15\angle 31.79)K}{220} = 68.18\angle 31.79 \text{ A rms}$

$\boxed{|\hat{I}_L| = 68.18 \text{ A}}$ (o.k., before C)

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{377 \times (220)^2}$$

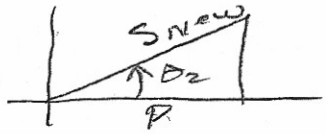
New pf = .98 $\rightarrow$ $\theta_2 = \cos^{-1}.98 = 11.48^\circ$

$$C = \frac{12.75 \times 10^3 [\tan 31.79 - \tan 11.48]}{377 \times 220^2}$$

(b)

$C = 291\,\mu F$

(c) Find New I_L



$$\cos\theta_2 = \frac{P}{S_{New}}$$

$$S_{New} = \frac{12.75\text{ kW}}{.98} = 13\text{ kVA}$$

$$\hat{S}_{New} = 13\angle 11.48\text{ kVA} = \hat{V}_L\,\hat{I}_L^*$$

$$|I_L| = \frac{13\,000}{220} = 59.1\text{ Amp}$$

Originally $|I_L| = 68.2$ Amps