wlg ECE 301 Lesson 16

Power Calculations In A.C. Circuits

Fall 2006
Nov 1

This lesson will consists entirely of example problems. Most of the problems will involve power calculations. Three reference sources, in addition to Rizzoni, were used as reference. They are

* Fundamentals of Electric Circuits; Alexander and Sadiku, 3rd Edition, 2006; McGraw-Hill
* Electric Circuits; Nilsson & Riedel; 7th Edition, 2005, Prentice Hall
* Basic Engineering Circuit Analysis; Irwin & Nelms Eight Edition; 2005, Publisher: John Wiley.

Maximum Power Transfer

Example 16.1

Given the circuit of Figure 16.1
- Find Z_L for max power transfer and the power delivered

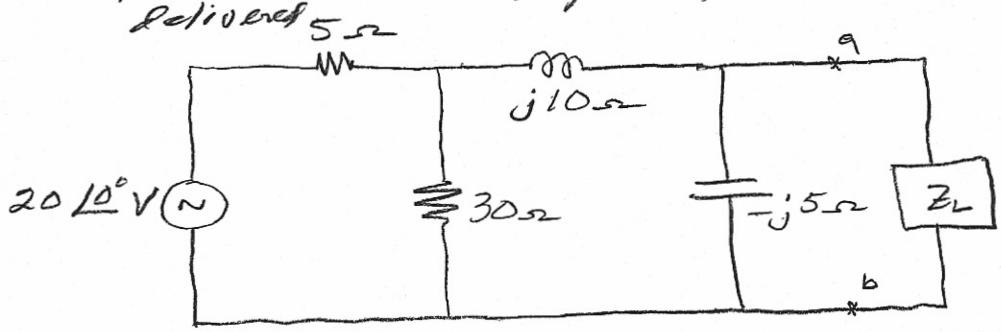


Figure 16.1: Circuit for Example 16.1 NRp476

We remove the load Z_L, deactivate all independent sources to the left of a-b and determine Z_{TH}. The circuit of Fig 16.2 is used to determine Z_{TH}.

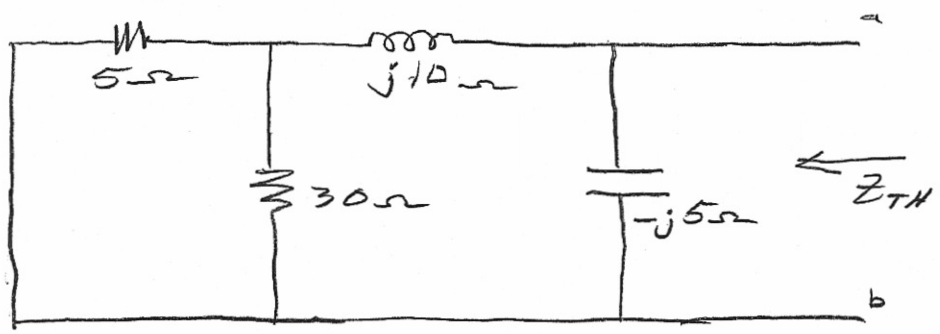


Figure 16.2: Circuit for determining Z_{TH}, Example 16.1.

$$5\Omega \parallel 30\Omega = \frac{5 \times 30}{5+30} \doteq 4.29\Omega$$

$$Z_{TH} = (-j5) \parallel (4.29+j10) \doteq \frac{(-j5)(4.29+j10)}{4.29+j10-j5}$$

$$Z_{TH} \doteq 2.471 - j7.880\ \Omega$$

$\therefore\ Z_L = Z_{TH}^{*} = 2.471 + j7.880\Omega$ for maximum power transfer.

Next find $\hat{V}_{TH}$. Refer back to Fig 16.1 with Z_L removed to find the open circuit voltage.

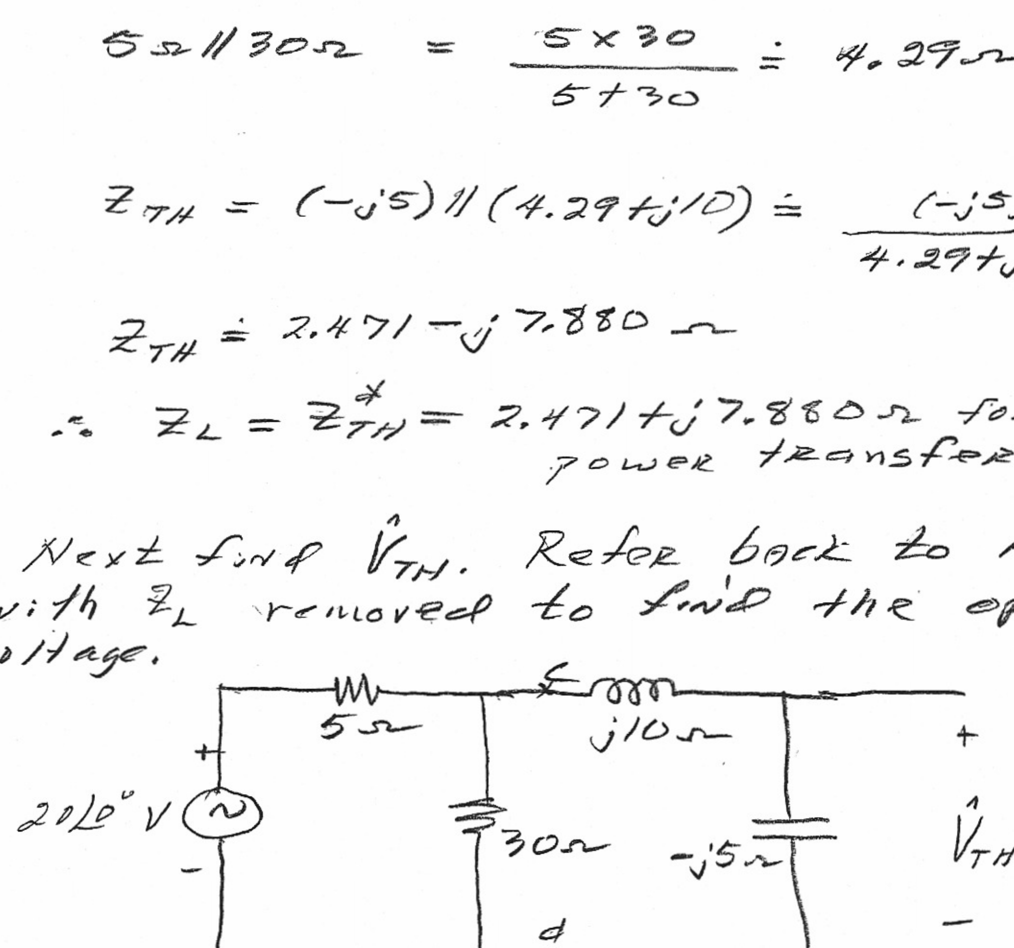


Figure 16.3: Circuit for finding $\vec{V}_{TH}$, Example 16.1

There are several ways to find $\vec{V}_{TH}$. Here I will find the Thevenin to the left of c-d and connect back to the original circuit to find V_{TH}. Reconnecting gives the circuit of Figure 16.4

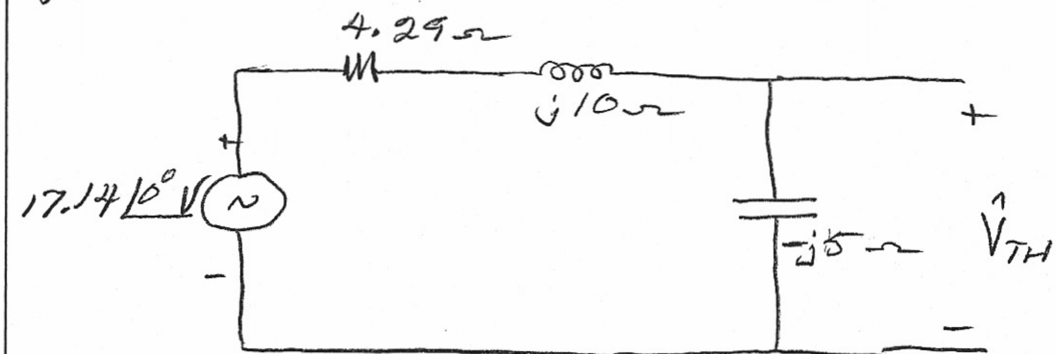


Figure 16.4: Modified circuit for finding V_{TH}.

Applying voltage division gives,

$$\vec{V}_{TH} = \frac{(17.14)(-j5)}{4.29 + j10 - j5} = 13.01\angle{-139.37^\circ}\ V$$

The circuit for finding maximum power transfer is shown in Figure 16.5.

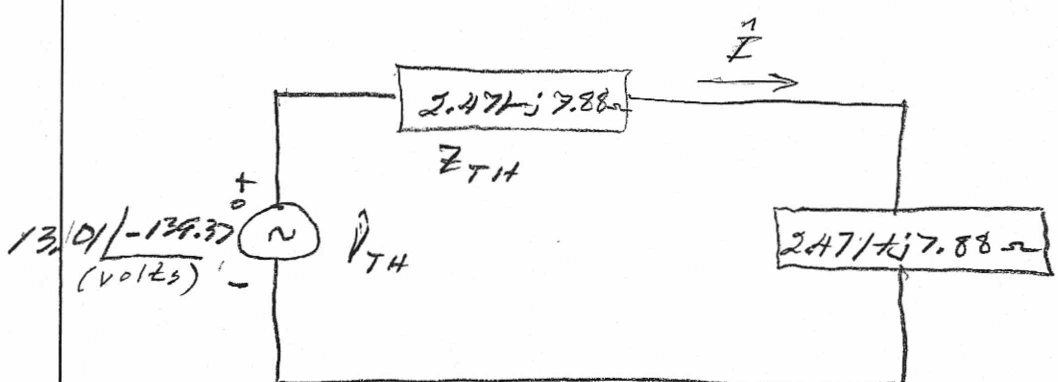


Figure 16.5: Circuit for Finding maximum power Transfer, Example 16.1.

By direct calculations,

$$\hat{I} = \frac{13.01\angle{-139.37}}{2 \times 2.471} \doteq 2.63\angle{-139.37^\circ}\ A$$

$$P_L = \frac{|\hat{I}|^2}{2} R_L = \frac{2.63^2 \times 2.471}{2} \doteq 8.55\ W$$

Using the expression

$$P_L = \frac{|\hat{V}_s|^2}{8 R_L} \doteq \frac{(13.01)^2}{8 \times 2.47} = 8.56\ W$$
(close enough)

QED

Example 16.2

Given the circuit of Figure 16.6.

(a) Find R_L for maximum power to be delivered to the load.

(b) Find the power delivered to R_L as found in (a)

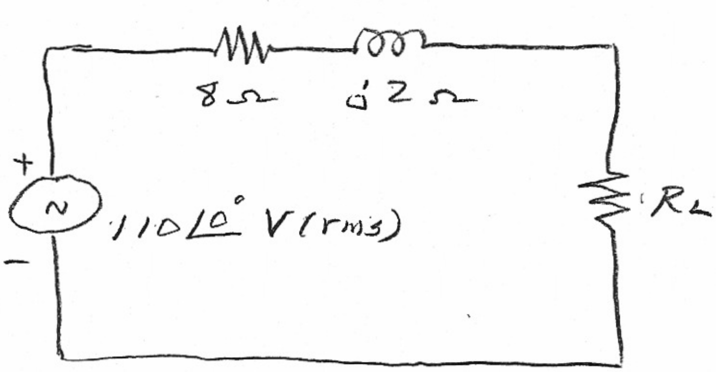


Figure 16.6: Circuit for maximum power transfer, Example 16.2.

It has been stated in class that for maximum power transfer, when the load is constrained to be a resistor, is

$$R_L = |Z_{TH}|$$

or

$$R_L = |8 + j2| = \sqrt{8^2 + 2^2} = 8.246\,\Omega$$

The circuit becomes as shown in Figure 16.7. Note Rms source voltage

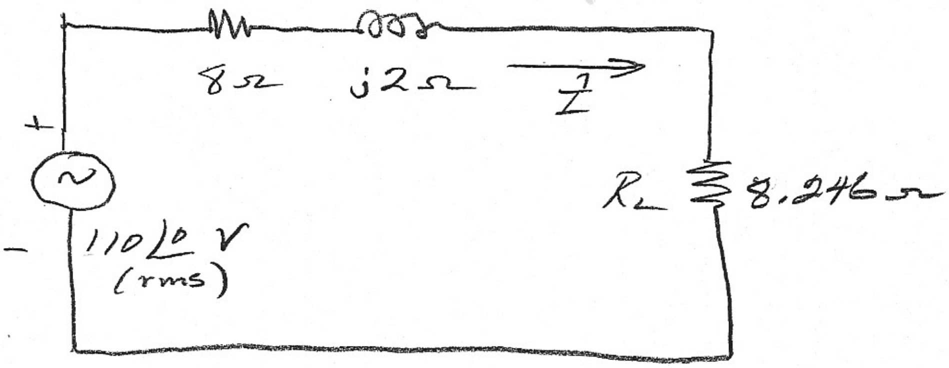


Figure 16.7: Final Circuit for Determining $P_{L_{max}}$ for Example 16.2.

From Figure 16.2;

$$\vec{I} = \frac{110\angle 0}{8 + j2 + 8.246} = 6.72\angle{-7.02^\circ}\ A \text{ (rms)}$$

$$P_{R_L} = |\vec{I}|^2 R_L = 6.72^2 \times 8.246 = 372.38\ W$$

Note: You cannot use

$$P_{R_L} = \frac{|V_s|}{8R_L}$$

Example 16.3

The purpose of this example is to illustrate how to calculate complex power, real power, Reactive power in an AC circuit.

Given the circuit of Figure 16.8

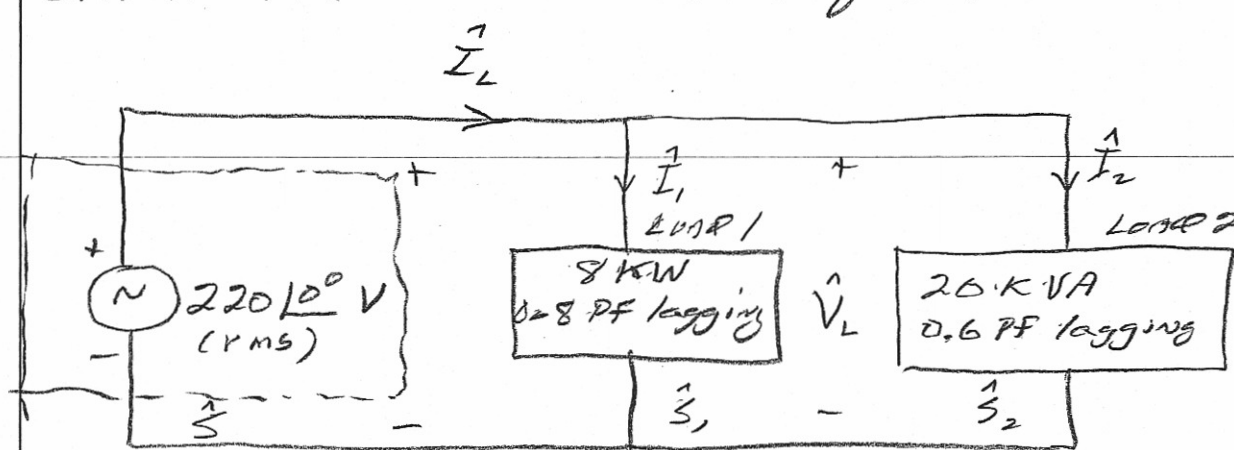


Figure 16.8: Circuit for Example 16.3.

(a) Determine $\hat{S}_1$ and $\hat{S}_2$

(b) Determine $\hat{I}_L$.

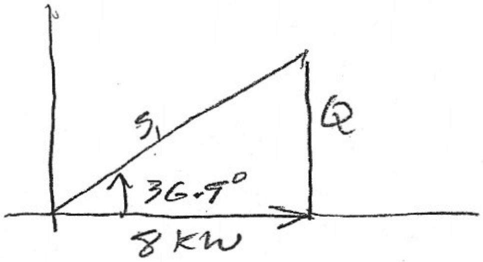
(c) Determine $\hat{S}$.

(d) Show that $\hat{S} = \hat{S}_1 + \hat{S}_2$; $P = P_1 + P_2$, $Q = Q_1 + Q_2$

Like in most circuit problems, there is more that one way to find the solution.

(a) The complex power triangle for load 1 is as shown in the following sketch.

Load 1



$Pf = .8$ lagging

$\cos\theta = .8$

$\theta = \cos^{-1}.8 = 36.9°$

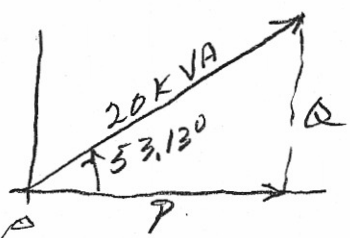
$$\cos 36.9 = .8 = \frac{8\,KW}{|S_1|}$$

$$|S_1| = \frac{8KW}{.8} = 10\ KVA$$

Knowing the angle from the triangle gives

$$\boxed{\hat{S}_1 = 10\angle 36.9\ KVA = (8 + j6)\ KVA}$$

Load 2



$Pf = .6$ lagging

$\theta = \cos^{-1}.6 = 53.13°$

$$\boxed{\hat{S}_2 = 20\angle 53.13°\ KVA = (12 + j16)\ KVA}$$

(b) Referring to Figure 16.8

$\hat{S}_1 = \hat{V}_L \hat{I}_1^*$ $\quad (\hat{V}_L = 220\angle 0\ V\ (rms))$ given

$$I_1^* = \frac{\hat{S}_1}{220\angle 0} = \frac{10\angle 36.9\ K}{220\angle 0} = 45.45\angle 36.9\ A$$

$$\hat{I}_1 = 45.45\angle -36.9\ A \ (rms)$$

(b) continued

$$\hat{I}_2^* = \frac{\hat{S}_2}{\hat{V}_L} = \frac{20\angle 53.12\ K}{220\angle 0}$$

$$I_2^* = 90.91\angle 53.12\ A_{rms}$$

$$\hat{I}_L = \hat{I}_1 + \hat{I}_2 = (45.45\angle -36.9) + (90.91\angle 53.12)$$

$$\hat{I}_L = 101.62\angle 26.6^\circ\ A_{(rms)}$$

(c) Determine $\hat{S}$

$$\hat{S} = \hat{V}_s \hat{I}_L^* = (220\angle 0)(101.62\angle -26.6)$$

$$\hat{S} = (20 - j10)\ KVA = 22.356\angle -26.6\ KVA$$

check

$$\hat{S} = \hat{S}_1 + \hat{S}_2$$

$$\hat{S} = (8 + j6)\ KVA + (12 + j16)\ KVA$$

$$\hat{S} = (20 + j22)\ KVA\ ??$$

What is wrong? Where has the error(s) been made? You find it. I made an error (on purpose) that is often made.

The correct answer is

$$\hat{S} = (20 + j22)\ KVA$$

Example 16.4

In the circuit shown in Figure 16.9, a load having an impedance of $40 + j20\ \Omega$ is fed from a voltage source through a line having and impedance of $0.5 + j2\ \Omega$. The rms value of the source voltage is $220\angle 0°$.

(a) Calculate the load current $\hat{I}_L$ and voltage $\hat{V}_L$.

(b) Calculate the average and reactive power delivered to the load.

(c) Calculate the average and reactive power delivered to the line.

(d) Calculate the average and reactive power supplied by the source.

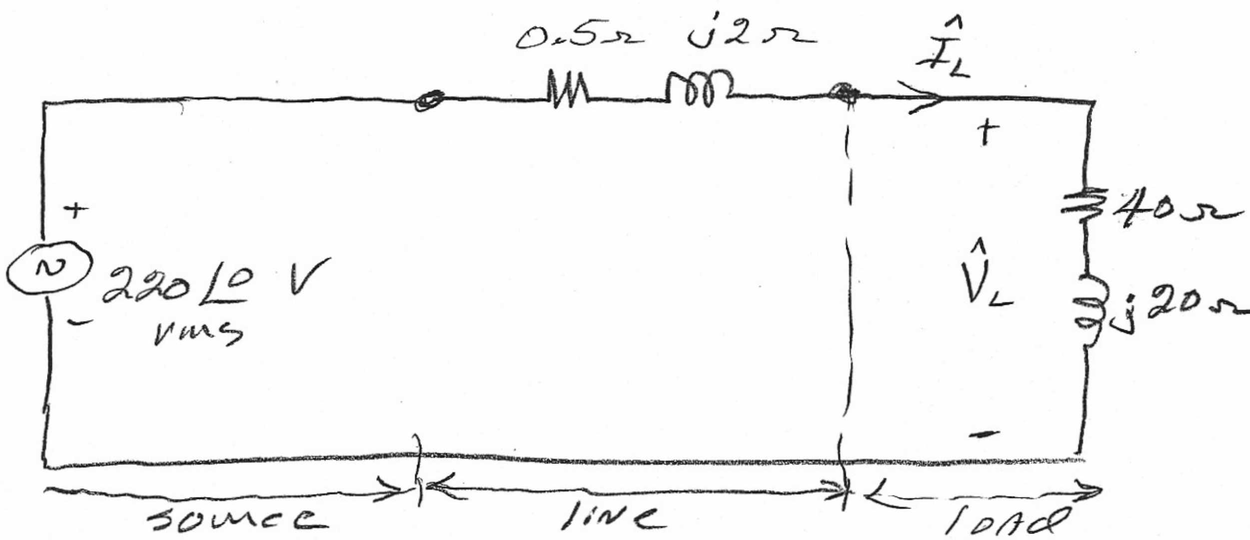


Figure 16.9: Circuit for Example 16.4.

(a)

$$\hat{I}_L = \frac{220\angle 0}{0.5 + j2 + 40 + j20} = 4.77\angle -28.5^\circ \ A_{rms}$$

$$\hat{V}_L = (40 + j20)\hat{I}_L = 213.47\angle -1.95^\circ \ V_{rms}$$

(b)

$$\hat{S}_L = \hat{V}_L \times \hat{I}_L^* = (213.47\angle -1.95)(4.77\angle 28.5$$

$$\hat{S}_L = (910.87 + j455.14) = 1.018\angle 26.6 \ KVA$$

$$P_L = 910.87W \quad Q_L = 455.14 \ VARs$$

(c)

$$\hat{S}_{Li} = |I_L|^2 Z_{Li} = (4.77)^2 \times (0.5 + j2)$$

$$\hat{S}_{Li} = (11.38 + j45.5) \ VA = 46.9\angle 76^\circ \ VA$$

$$P_{Li} = 11.38 \ W \quad Q_{Li} = 45.5 \ VARs$$

(d)

$$\hat{S}_s = \hat{V}_s \hat{I}_s^* = (220\angle 0)(4.77\angle 28.5)$$

$$\hat{S}_s = (922.23 + j500.73) \ VA$$

For a check

$$\hat{S}_s = \hat{S}_{Li} + \hat{S}_L = (1.018\angle 26.6 \ KVA + 46.9\angle 76^\circ) \ VA$$

$$\hat{S}_s = (921.6 + j501.3) \ VA \quad \text{close}$$

Example 16.5

This is problem 7.18 at the end of the chapter of Rizzoni.

A single phase motor draws 220W at a power factor of 80% (lagging) when connected across a 200V, 60 Hz source. A capacitor is connected in parallel with the load to give unity power factor. Find the required capacitance.

What we have is shown in Figure 16.10.

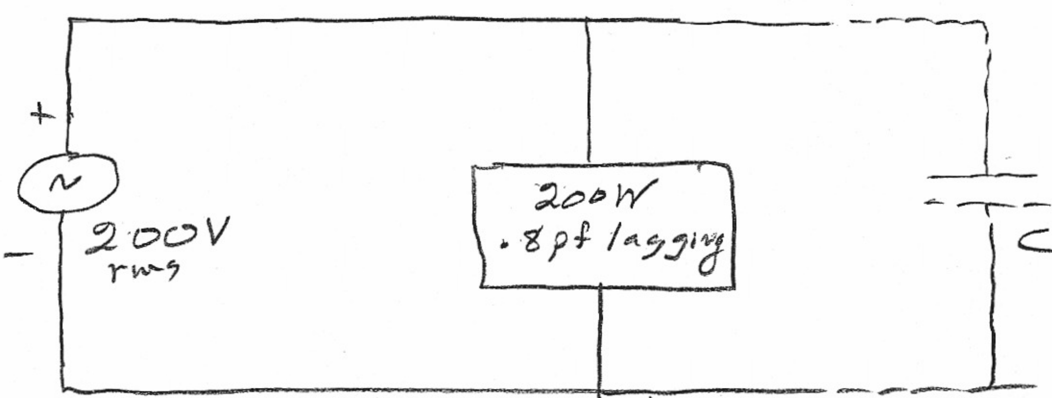
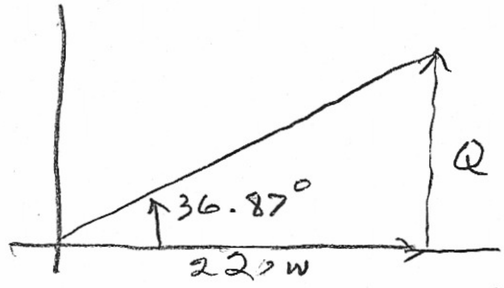


Figure 16.10: Circuit for problem 16.5

The complex power triangle is as shown below.



$\cos\Theta = .8$

$\Theta = 36.87°$

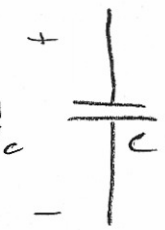
The existing load Q is

$$\frac{Q}{220} = \tan 36.87$$

$$Q = 165 \text{ VARS}$$

$$\hat{S} = 220 + j\,165 \text{ VA}$$

we want to eliminate the $+j165$ VARS with



$\hat{V}_c$ $\quad S_c = \dfrac{|V_c|^2}{Z_c^*} = \dfrac{\omega C |V_c|^2}{-j}$

$$\hat{S}_c = -j\,\omega C (200)^2 = -j\,377 \times (200)^2 C$$

so
$$377(200)^2 C = 165$$

$$C = 10.74\ \mu F$$

Example 16.6

This problem is 7.21 from Rizzoni at the end of the chapter.

The motor inside a blender can be modeled as a resistance in series with an inductor as shown in Figure 16.11.

(a) What is the average power, P_{AV}, dissipated in the load?

(b) What is the motor's power factor?

(c) What C placed in parallel with the motor will change the p.f. to 0.9 lagging?

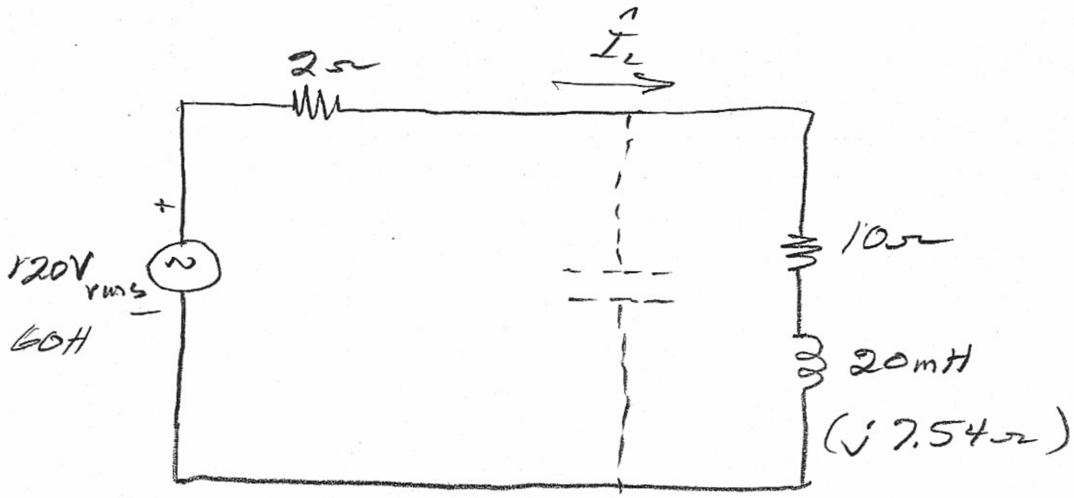


Figure 16.11: Circuit for Example 16.6.

(a) First find X_L.

$$X_L = \omega L = 2\pi \times 60 \times 20 \times 10^{-3} =$$

$$X_L = 7.54$$

$$Z_L = 10 + j\,7.54$$

$$P_{AV} = |I_L|^2 R_L$$

$$\hat{I}_L = \frac{120}{12 + j7.54} = 8.47\angle -32.1^\circ \text{ A rms}$$

$$P_{AV} = (8.47)^2 \times 10 = 717.41\text{ W}$$

$$\hat{S}_L = |I_L|^2 Z_L = (8.47)^2 (10 + j7.54)\text{ VA}$$

$$\hat{S}_L = (717.41 + j\,540.93)\text{ VA}$$

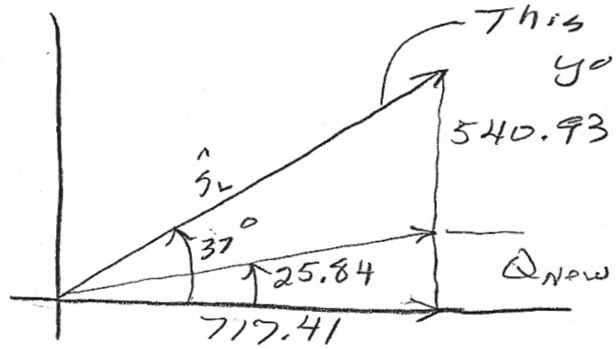
$$\hat{S}_L = 898.49\angle 37^\circ\text{ VA}$$

(b) Motor p.f.

Look at the angle of $\vec{S}_L$. It is the same as the angle of $\vec{Z}_L$. The angle of $\vec{Z}_L$ is the p.f. angle

$$\text{p.f.} = \cos(37^\circ) = 0.7986 \text{ lagging}$$

(c) Now find C for p.f. $= .9$ lagging



$\cos\theta = .9$

$\theta = 25.84^\circ$

$$Q_{New} = 717.41 \tan 25.84 = 347.43 \text{ VARs}$$

Capacitor must provide reactive power of

$$Q_c = 540.93 - 347.43 = 193.5 \text{ V}$$

but

$$Q_c = w|\vec{V}_L|^2 C$$

$$|\hat{V}_L| = |(10 + j7.54)\vec{I}_L| = 106.08 \text{ V}_{rms}$$

$$C = \frac{193.5}{377 \times (106.08)^2} = 45.6\,\mu F$$

Example 16.7

You are given the circuit show in Figure 16.12

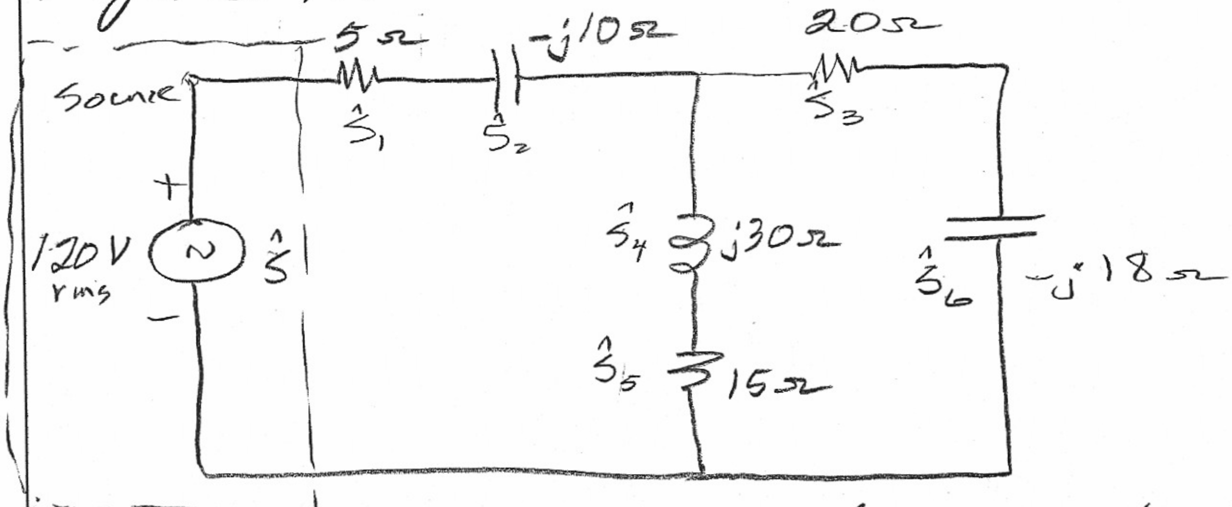


Figure 16.12: Circuit for Example 16.7.

(a) Find the complex power supplied to the circuit ($\vec{S}$) by the source

(b) Determine the power factor of the source.

(c) Determine $\vec{S}_1$, $\vec{S}_2$, $\vec{S}_3$, $\vec{S}_4$, $\vec{S}_5$ and $\vec{S}_6$

Show that

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \vec{S}_4 + \vec{S}_5 + \vec{S}_6$$

agrees with $\vec{S}$ determined in (a)

Probably the easiest way to approach this problem is to assign mesh currents $\vec{I}_1$ and $\vec{I}_2$ as indicated. Solve for $\vec{I}_1$ & $\vec{I}_2$.

By inspection

16.16

$$\begin{bmatrix} 20+j20 & -15-j30 \\ -15-j30 & 35+j12 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 120\angle 0 \\ 0 \end{bmatrix}$$

Solve with calculator

$$\vec{I}_1 = 3.74 + j1.137 = 3.91 \angle 16.91° \text{ A rms}$$

$$\vec{I}_2 = 1.696 + j3.112 = 3.544 \angle 61.42° \text{ A rms}$$

$$\vec{S} = \hat{V}_s \hat{I}_1^* = 120 (3.91 \angle -16.91)$$

$$\boxed{\vec{S} = 469.2 \angle -16.91 \text{ VA} = (448.91 - j136.48) \text{ VA}}$$

(b) Power factor

$$pf = \cos(-16.91) = 0.9568 \text{ leading}$$

(c) $\vec{S}_1 = |I_1|^2 5 = 76.44$ VA (or watt here)

$$\vec{S}_2 = |I_1|^2(-j10) = -j152.88 \text{ VA}$$

$$\vec{S}_3 = |I_2|^2 20 = 251.2 \text{ VA}$$

$$\vec{S}_4 = |(3.91\angle 16.91) - (3.544\angle 61.42)|^2 (j30)$$

$$\vec{S}_4 = 2.84^2 (j30) = j242 \text{ VA}$$

$$\vec{S}_5 = (2.84)^2 \times 15 = 120.98$$

$$\vec{S}_6 = |I_2|^2(-j18) = 3.544^2(-j18) = -j226.08$$

$$\sum_{i=1}^{6} S_i = 76.44 - j152.88 + 251.2$$
$$+ j242 + 120.98 - j226.08$$

$$S^T = (448.62 - j136.96)\ VA$$

compared to

$$S = (448.91 - j136.48)$$

in part (b) good enough