Stepping Back And Looking At Things

Current:

$$i = \frac{\Delta Q}{\Delta T} \rightarrow \frac{\text{coulombs}}{\text{second}}$$

$$i = \frac{dq}{dt}$$

Know: KCL and how to use

Voltage

$$V = \frac{\Delta W}{\Delta Q} \Rightarrow \frac{\text{Joules}}{\text{Coulomb}}$$

$$V = \frac{dW}{dq}$$

Power

$$p(t) = \frac{dW}{dt}$$

$$W = \int p\,dt$$

(This is how we pay)
Watt · Hours
KWH about 6¢ per KWH

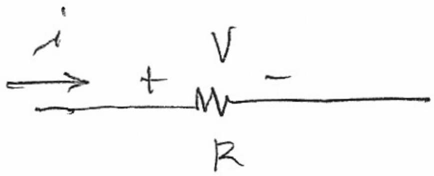
$$P = \frac{dW}{dt} = \frac{dW}{dq} \cdot \frac{dq}{dt}$$

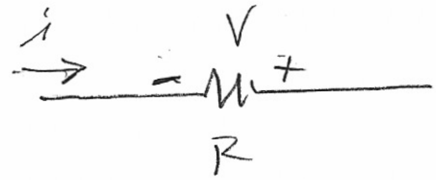
$P = Vi$

We will come back to this shortly.

Ohm's Law

$V = |iR|$





$V = +iR$ $\qquad$ $V = -iR$

Now Go Back To Power

$\boxed{P = Vi}$ $\quad$ (Sub $V = iR$)

$= iRi$

$\boxed{P = i^2 R}$

$P = Vi$ $\quad$ (Sub $i = \frac{V}{R}$)

$= V \times \frac{V}{R}$

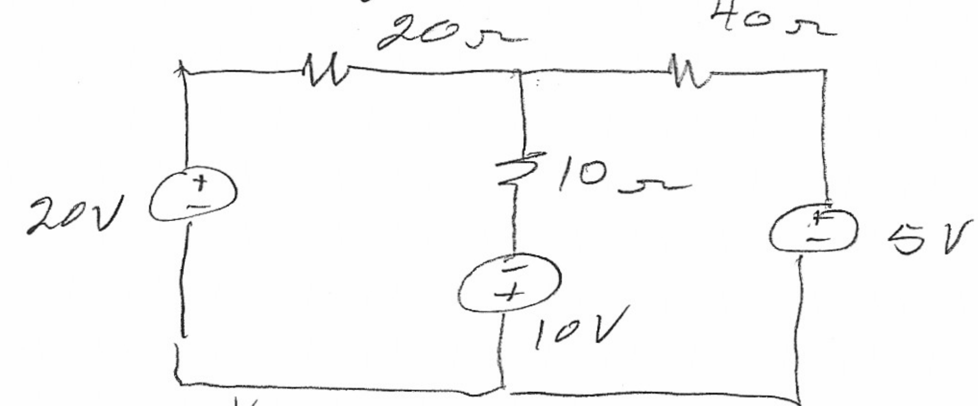
$P = \frac{V^2}{R}$

So; $\boxed{P = Vi = i^2 R = \frac{V^2}{R}}$

Know how to use

The so-called Passive Sign Convention for power.

Consider any electric circuit



We are not quit ready to solve this circuit right now but what we can say is

$$\sum P_{supplied} = \sum P_{Absorbed} \quad (1)$$

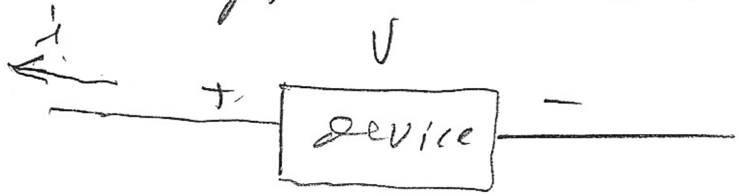
OR $\sum P_{supplied} = 0 \quad (2)$

$\sum P_{ABSORBED} \quad (3)$

(2) & (3) must accommodate both ± supplied and absorbed. How is this done? Answer, by how we define each one.

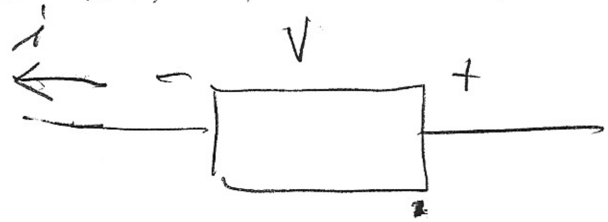
- Power supplied

Given a device and polarity assumptions as below



$P_{sup} = +Vi$

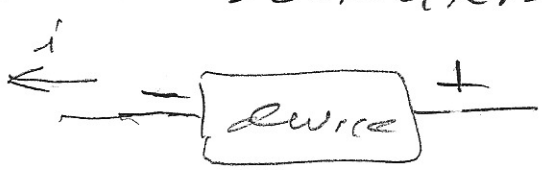
If either V or i is reversed



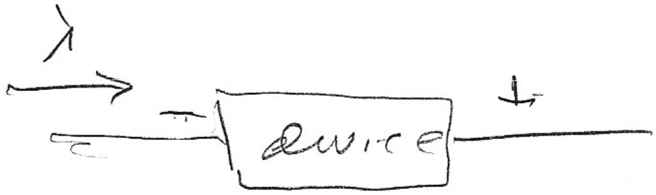
$P_{sup} = -Vi$

- Power Absorbed

Same scenario;



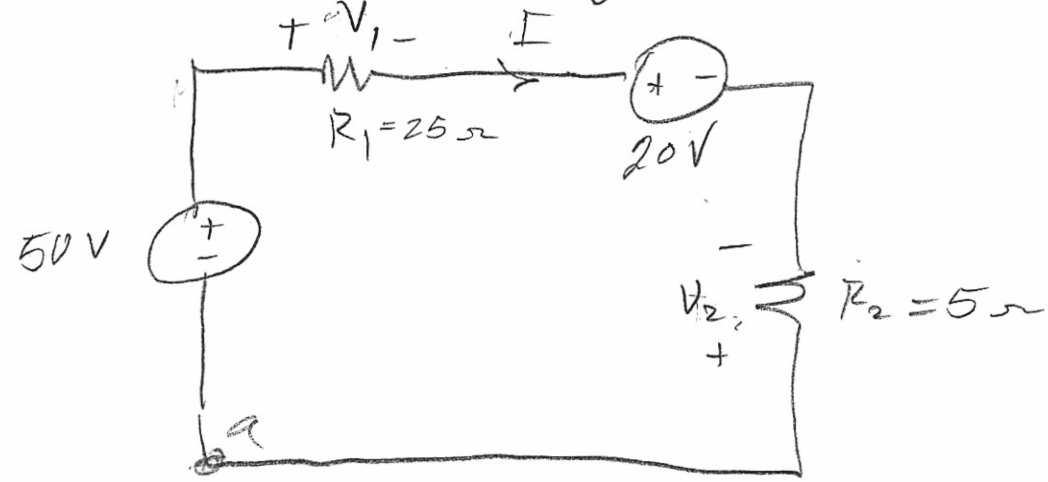
$P_{ABS} = +Vi$



$P_{ABS} = -Vi$

Example

- Solve for V_1, V_2 below.
- Find P_{sup} by each source
- Find P_{abs} by R_1, R_2



Σ drops $= 0$, start at "a", go cw.

$-50 + V_1 + 20 - V_2 = 0$ KVL

but ohm's law

$$V_1 = 25I$$

$$V_2 = -5I$$

$$25I - (-5I) = 50 - 20$$

$$30I = 30$$

$$I = 1 \text{ Amp}$$

$$P_{sup_{50}} = 50I = 50W$$

$$P_{sup_{20}} = -20I = -20W$$

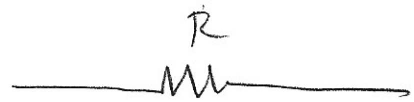
$$P_{abs_{25}} = I^2R = 25W$$

$$P_{abs_5} = 1^2 \times 5 = 5W$$

$$\Sigma P_{sup} = \Sigma P_{abs}$$

$$50 - 20 = 5 + 25 \quad \text{check}$$

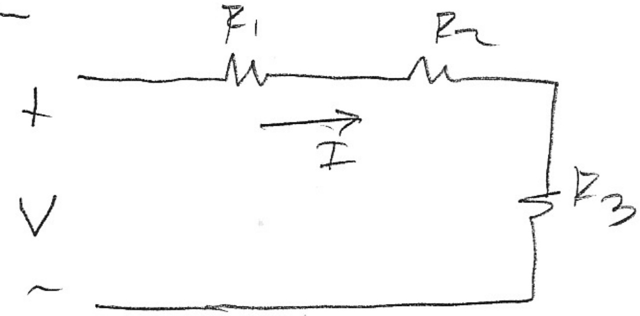
<u>Resistance</u>



ohms

G, conductance $= \frac{1}{R}$, Sieman (S)

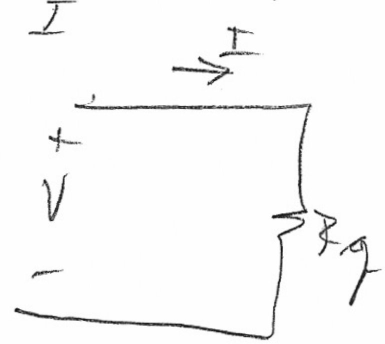
Consider



Default Sign Convention.
Assume V_R is + at the terminal where I enters; we have

$$-V + IR_1 + IR_2 + IR_3 = 0$$

$$\frac{V}{I} = R_1 + R_2 + R_3$$

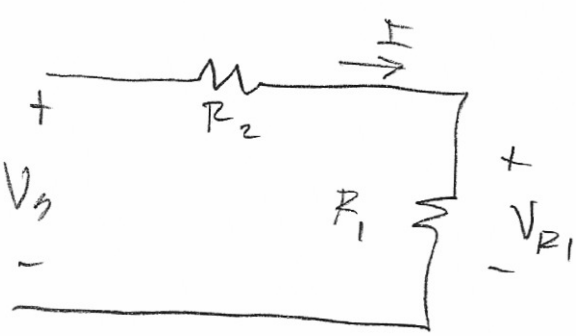


Same V,
Same I

$$\frac{V}{I} = R_q = R_1 + R_2 + R_3$$

Resistors in series add.

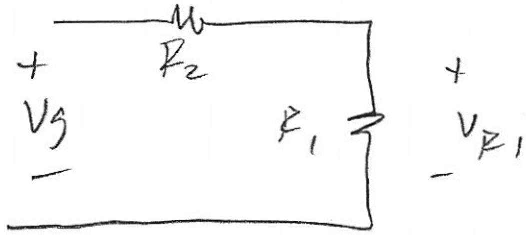
Voltage Division Rule



$$I = \frac{V_s}{R_1 + R_2}$$

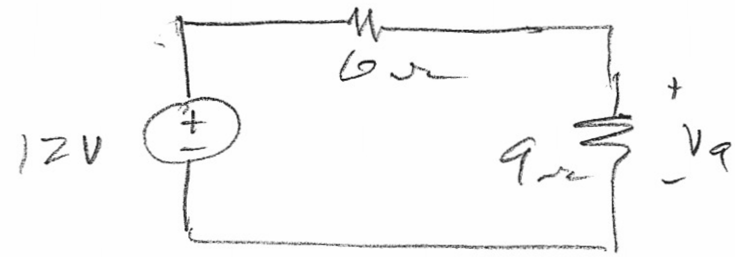
$$V_{R1} = I R_1 = \frac{V_s R_1}{R_1 + R_2}$$

- The Rule in words



V_{R1} is equal to source voltage, V_s, time R_1 divided by $R_1 + R_2$.

Simple Example,

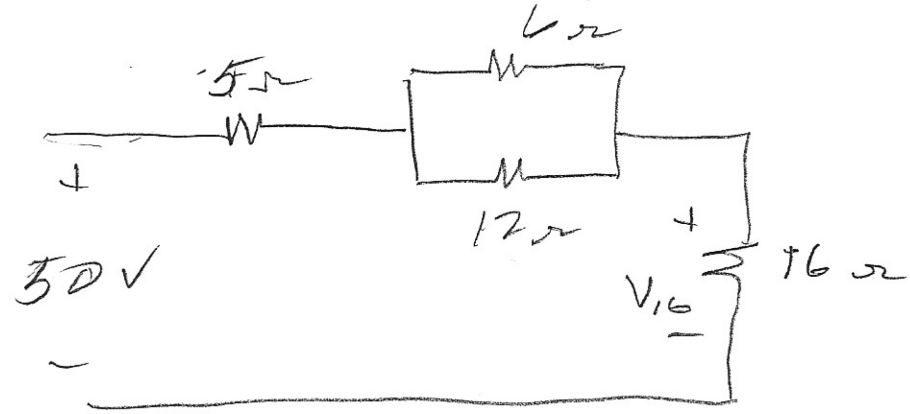


$$V_9 = \frac{12 \times 9}{9+6} = \frac{12 \times 9}{15} = 7.2\ V$$

$$V_6 = \frac{12 \times 6}{9+6} = 4.8$$

$$12 = V_6 + V_9 = 7.2 + 4.8 = \underset{ok}{12V}$$

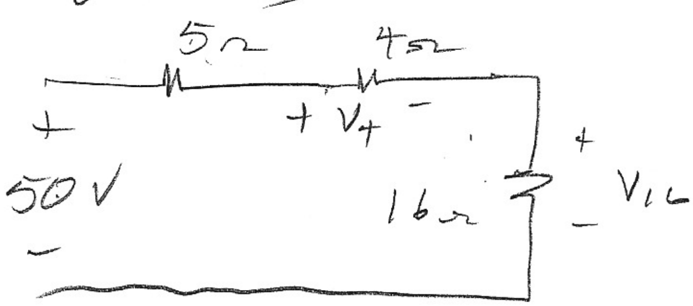
You should observe the following:



$$\frac{6 \times 12}{6+12} = 4$$

Find V_{16} using voltage division:

change to:



$$V_{16} = \frac{50 \times 16}{25} = 32V$$

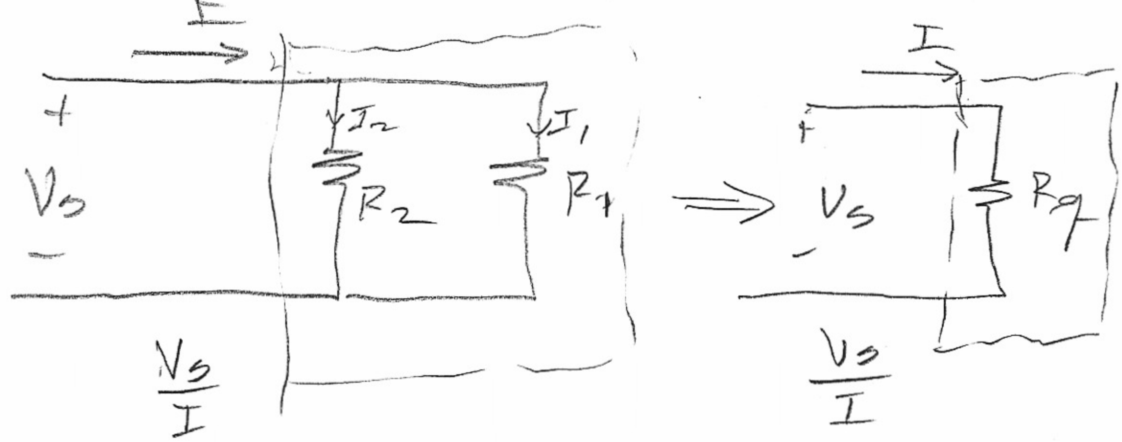
$$V_4 = \frac{50 \times 4}{25} = 8V$$

etc.

This is handy. Saves time.

Used very often in circuits

Resistors In Parallel



$$I_1 = \frac{V_s}{R_1} , \quad I_2 = \frac{V_s}{R_2}$$

$$I = I_1 + I_2$$

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \qquad I = V\left[\frac{1}{R_{eq}}\right]$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (3)$$

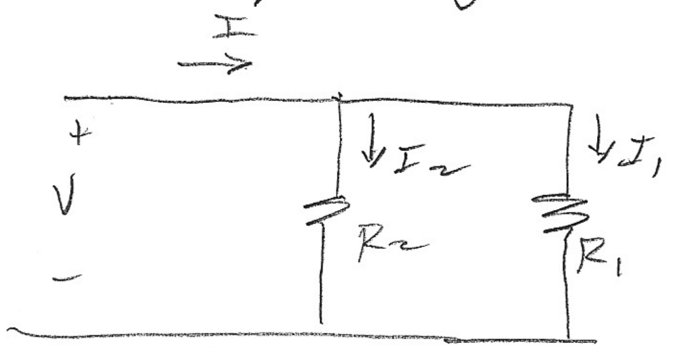
$$G_{eq} = G_1 + G_2 \qquad (4)$$

From 3

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$ (Product divided by sum)

- Current splitting or current Division.



$$I_1 = \frac{V}{R_1} = \frac{I R_{eq}}{R_1} = \frac{I\,R_1 R_2}{R_1 + R_2}$$

$$\boxed{I_1 = \frac{I \times R_2}{R_1 + R_2}}$$

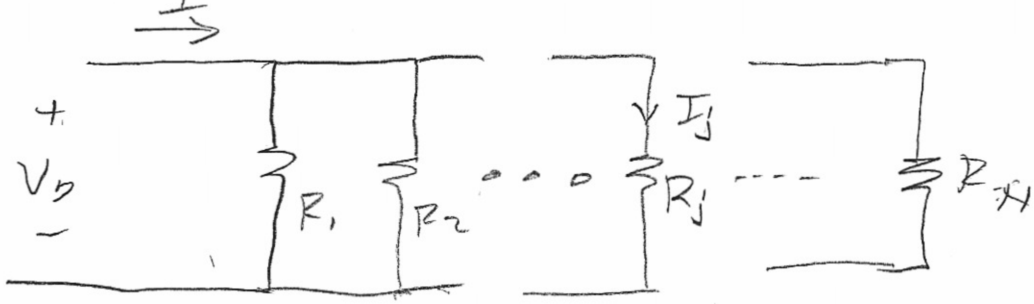
Similarly we can show

$$I_2 = \frac{I \times R_1}{R_1 + R_2}$$

Words

I_1 equals incoming current multiplied by opposite resistor divided by the sum of the resistance.

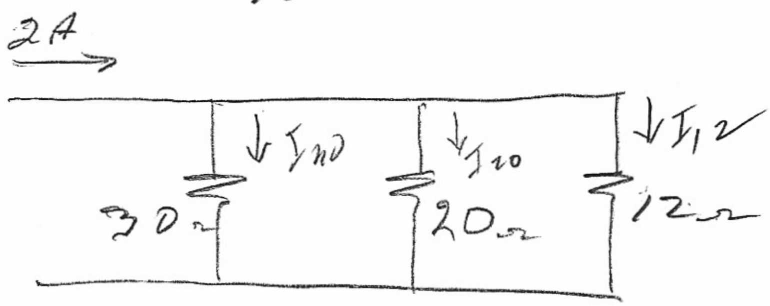
A general expression, any number of resistors,



$$I_j = \frac{I \times R_{eq}}{R_j}$$

Example

Find I_{12} below



Find R_{eq};

$$\frac{30 \times 20}{30 + 20} = \frac{600}{50} = 12\,\Omega$$

$$R_{eq} = \frac{12 \times 12}{12 + 12} = 6\,\Omega$$

$$I_{12} = \frac{2 \times R_{eq}}{12} = \frac{R_{eq}}{6} = \frac{6}{6} = 1\,A$$

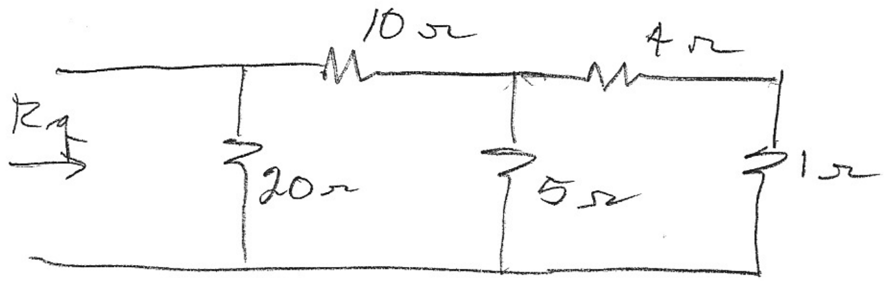
How much is I_{20}, I_{30}?

$$I_{20} = \frac{12V}{20} = \frac{6}{10} = 0.6\,A$$

$$I_{30} = \frac{12V}{30} = \frac{6}{15} = \frac{2}{5} = 0.4A$$

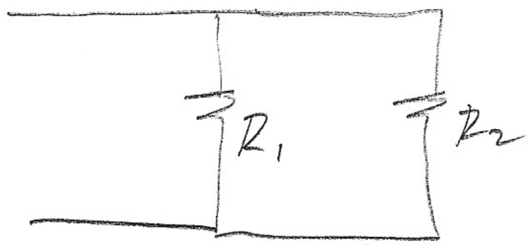
$$I_S = I_{10} + I_{20} + I_{30} = 1 + 0.6 + 0.4 = 2A$$
$$= I_S$$

Combination of Resistance



Find Req

First a comment:
The equivalent resistance of two resistors in parallel is always less than the smallest resistor.



Let $R_2 < R_1$

$$R_2 = kR_1, \quad 0 < k < 1$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(R_1)(kR_1)}{R_1 + kR_1}$$

$$R_{eq} = \frac{kR_1}{1+k} = \frac{R_1}{1+\frac{1}{k}}$$

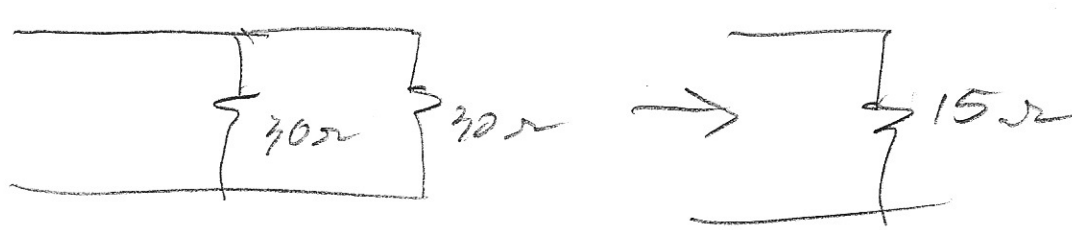
$$1 + \frac{1}{k} > 1, \text{ so}$$

$R_{eq} < R_1$ (smallest resistor)

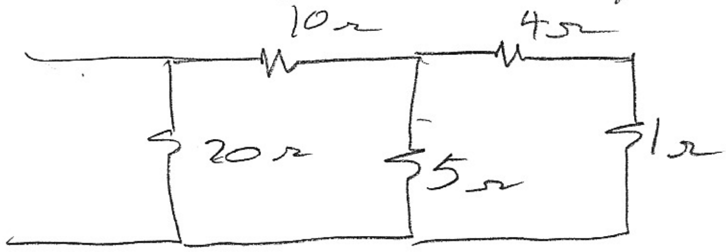
What if $R_1 = R_2$

$$R_{eq} = \frac{R_1 R_1}{R_1 + R_1} = \frac{R_1}{2}$$

R_{eq} for 2 resistor in parallel of the same value is half the resistor



For the previous problem



Generally, start at the right and work left.

$$5 \parallel 5 = 2.5$$

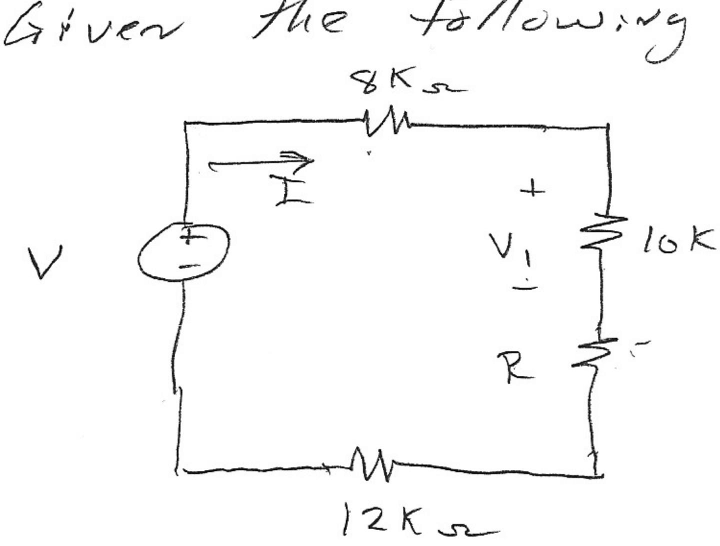
$$R_{eq} \quad (10 + 2.5) \parallel 20$$

$$R_{eq} = \frac{12.5 \times 20}{12.5 + 20}$$

$$\boxed{R_{eq} = 7.69\ \Omega}$$

Example 2.32

Given the following



Find: V_1, V, R

Given: Power supplied = 40 mW

$V_1 = \frac{V}{\ldots}$

$VI = 40\times10^{-3}$ (a)

$V_1 = \frac{V}{4} = 10\times10^{3} I$ (b)

so

$V = 40\times10^{3} I$

substitute into (a)

$40\times10^{3} I^2 = 40\times10^{-3}$

$I = 1\times10^{-3} = 1\ mA$

$\boxed{V = \frac{40\times10^{-3}}{1\times10^{-3}} = 40\ V}$

$V_1 = \frac{V}{4} = 10\ V$

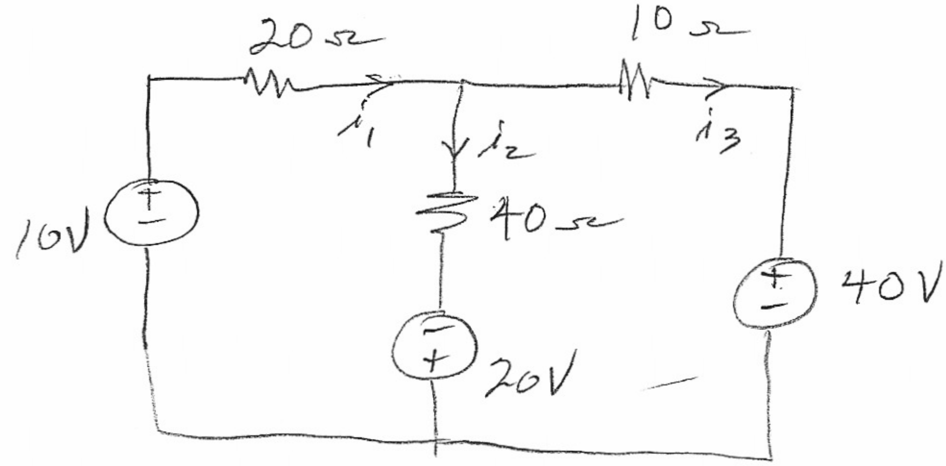
$\boxed{V_1 = 10\ V}$

$V_1 = RI = 1\times10^{-3} R = 10$

$R = \frac{10}{1\times10^{-3}} = 10k\Omega$

$\boxed{R_1 = 10k\Omega}$

Example



Find i_1, i_2, i_3 in the circuit below.
Using default convention.

$$-10 + 20i_1 + 40i_2 - 20 = 0$$

$$20i_1 + 40i_2 + 0i_3 = 30$$

$$20 - 40i_2 + 10i_3 + 40 = 0$$

$$0i_1 - 40i_2 + 10i_3 = -60$$

$$-i_1 + i_2 + i_3 = 0$$

$$\begin{bmatrix} 20 & 40 & 0 \\ 0 & -40 & 10 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 30 \\ -60 \\ 0 \end{bmatrix}$$

$i_1 = -0.643\ A \qquad i_2 = 6.0714\ A, \quad i_3 = -1.7143\ A$

By mesh

$$\begin{bmatrix} 60 & -40 \\ -40 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 30 \\ -60 \end{bmatrix}$$

$i_1 = -0.643\ A, \quad i_2 = i_3 = -1.7143\ A$

ok.