Lecture 7 ECE 301

Thevenin's Theorem
Norton's Theorem

Thevenin's Theorem

Consider the following

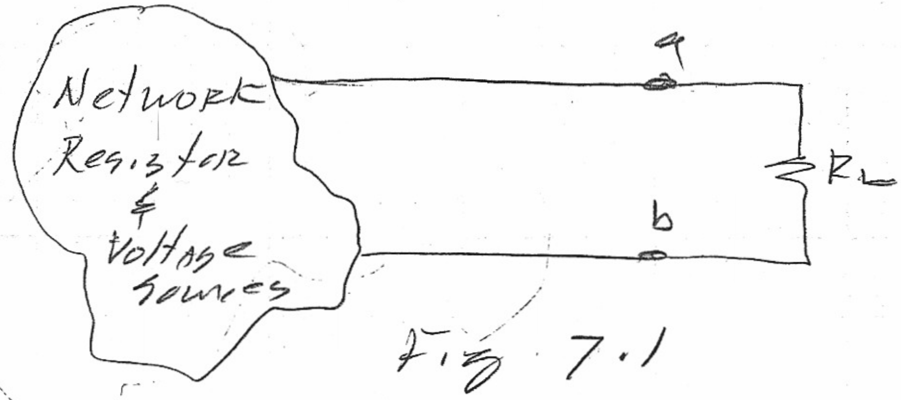


Fig 7.1

The network inside the "glob" can be very complex - structured in this case by 168 resistor and 27 independent voltage sources.

We would like to replace the network by one that has 1 resistor and 1 source that will deliver the same current and same voltage to R.

The situation is that shown in Figure 7.1

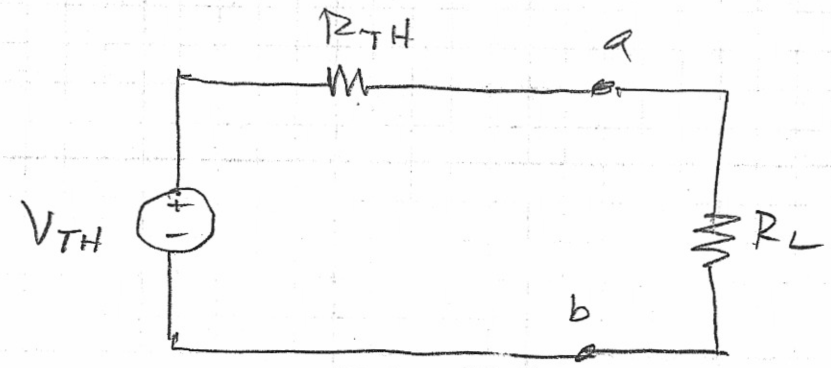


Fig. 7.2

We call $V_{TH} \rightarrow V_{THEVENIN}$ and

$R_{TH} \rightarrow R_{THEVENIN}$

If we remove R_L from the circuit in Fig 7.1 and de-activate all sources (this means replace all voltage sources with a short) if we the placed an ohmmeter across terminal a-b we would read R_{TH}.

With R_L removed, if we place a voltmeter across a-b we read the open circuit voltage.

The open circuit voltage is V_{TH}. Let's see how this works.

Example 7.1

Find the Thevenin equivalent circuit to the left of terminals a-b in the circuit of Figure 7.3

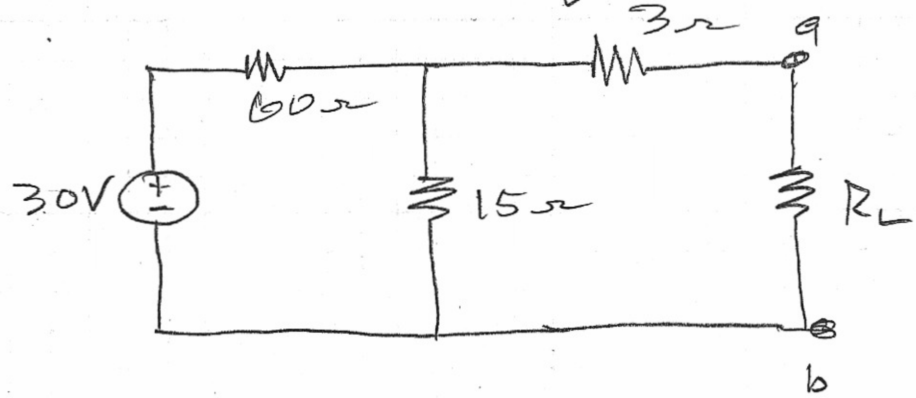


Figure 7.3

Solution

To find R_{TH}. Deactivate the 30V source, Remove R_L and determine R_{eq} below in Figure 7.4

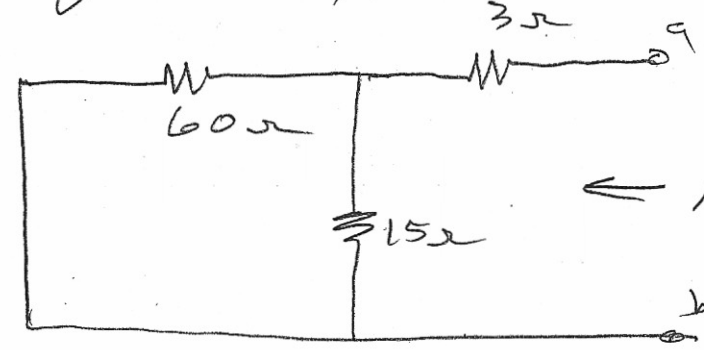
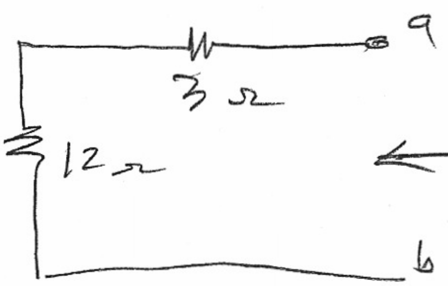


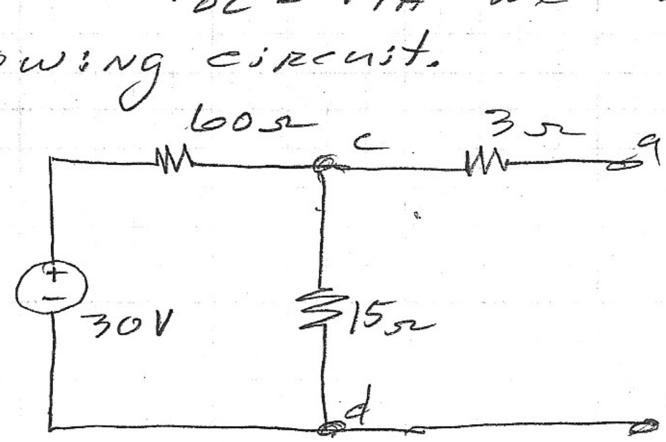
Figure 7.4

becomes



$\leftarrow R_{eq} = R_{TH} = 15\Omega$

To find $V_{OC} = V_{TH}$ we consider the following circuit.



Since terminals a-b are opened, no current flows through the 3Ω resistor so there is no voltage drop across this resistor. This means that $V_{ab} = V_{TH} = V_{cd}$. We can find V_{cd} by using the voltage division rule.

$$V_{CD} = V_{TH} = \frac{30 \times 15}{75} = 6\ V$$

The Thevenin circuit, connected to the load becomes

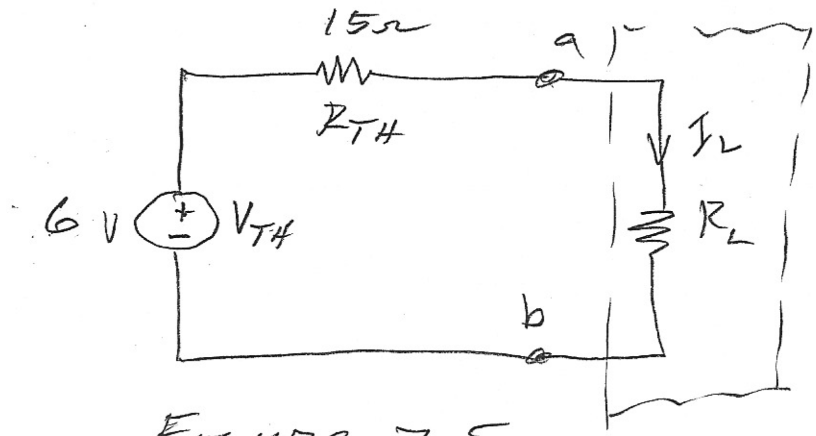


Figure 7.5

We can now find I_2 and V_{ab} for any R_L. Furthermore, R_L could be replaced by another circuit as below.

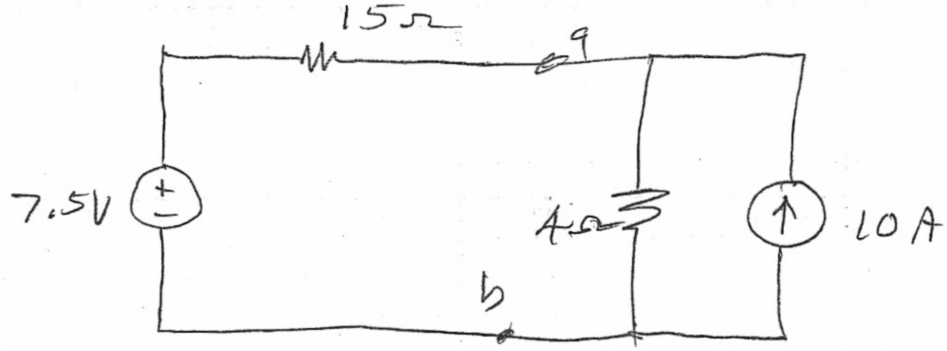


Figure 7.6

Suppose we want to find Vab. We can use source transformation to change the circuit as follows

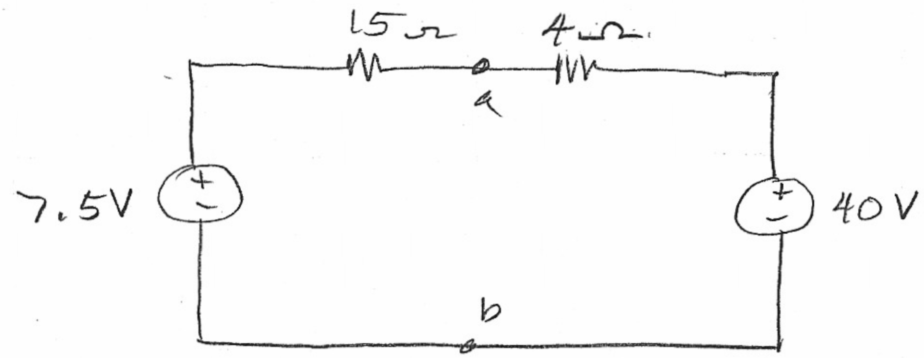
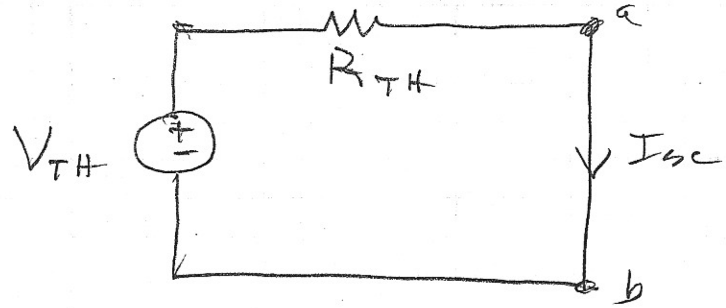


Figure 7.7

Solving this circuit we find,

$$V_{ab} = 33.16\ V$$

Reconsider the the Thevenin equivalent circuit, with a short across the load



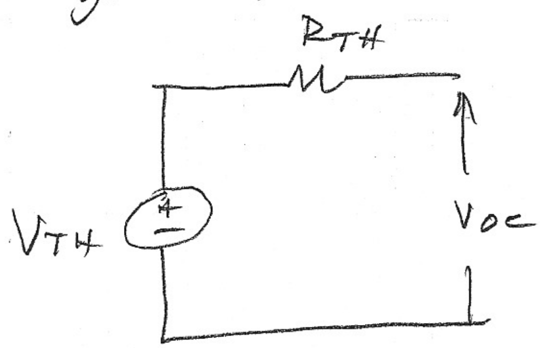
We designate the current in the short as I_{sc}. We see that

$$I_{ss} = \frac{V_{TH}}{R_{TH}}$$

Now recall that

$$V_{oc} = V_{TH}$$

which you see from the following ckt.



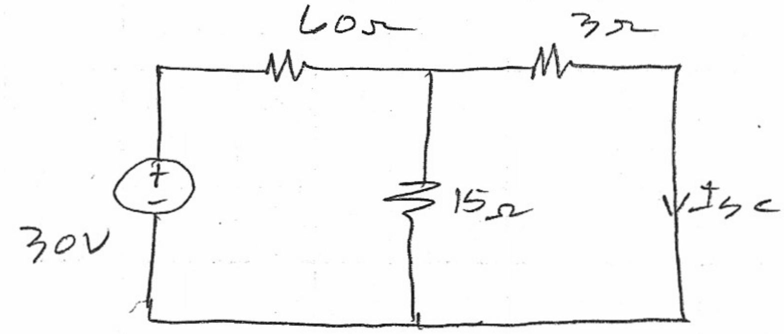
Therefore,

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{V_{TH}}{V_{TH}/R_{TH}} = R_{TH}$$

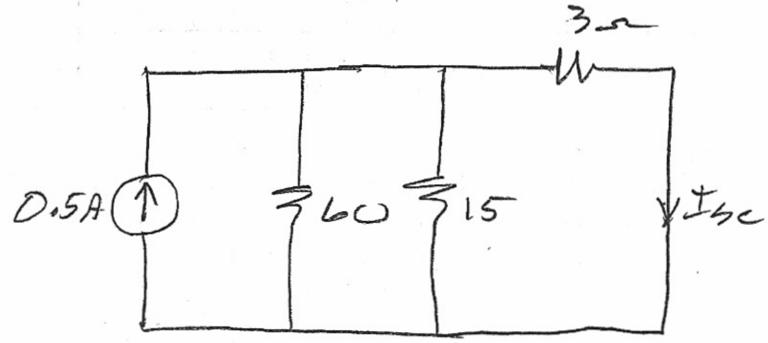
$$\therefore \; R_{TH} = \frac{V_{oc}}{I_{sc}}$$

This is an alternate way of find R_{TH}.

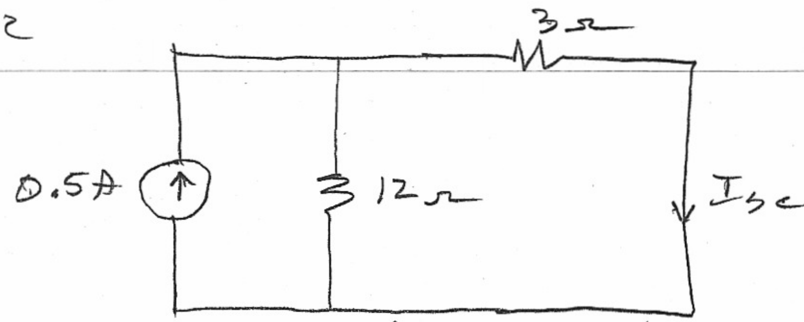
Reconsider Example 7.1 and find R_{TH} by $\frac{V_{oc}}{I_{sc}}$. Let us actually find I_{sc}.



with source transformation,



or



Using current division,

$$I_{sc} = \frac{0.5 \times 12}{12+3} = 0.4A$$

We already know that $V_{oc} = V_{TH} = 16V$

$$R_{TH} = \frac{V_{os}}{I_{sc}} = \frac{16}{.4}$$

$$R_{TH} = 15\Omega$$

This agrees with previous work.

Nortons Theorem

Consider the following;

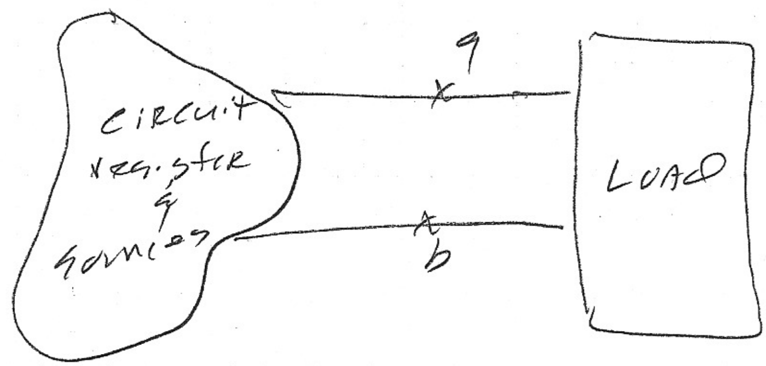


Figure 7.8

Norton's theorem states that for any linear circuit to the left of a-b we can replace by a current source in parallel with a resistor.

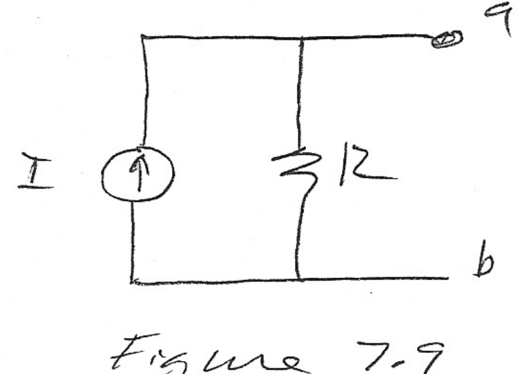
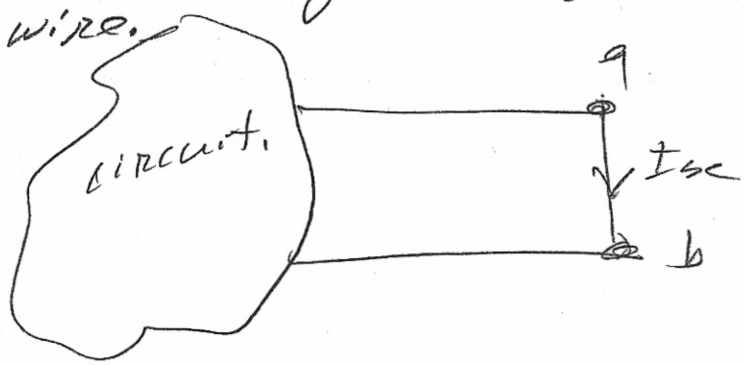


Figure 7.9

The value of R is the resistance seen looking into a-b with all independent sources dissabled.
We disable current sources by removing them. We dissable voltage sources by replacing them with a short.

The I in Figure 7.9 is the short circuit current. That is, the current that flows through a wire the load in Figure 7.8 is replaced by this wire.



If you know the Thevenin ckt you can easily determine the Norton because

$$I_N = \frac{V_{TH}}{R_{TH}}$$

However, we want to also be able to find I_{sc} by direct calculation from the circuit if for no other reason it makes us better at circuit analysis.

We now consider an example of finding a Norton equivalent ckt.

Example 7.2

Find the Norton equivalent circuit for the following, to the left of terminals a-b.

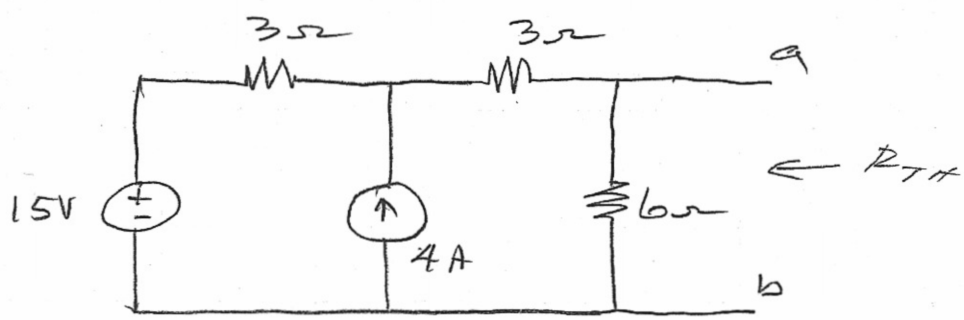
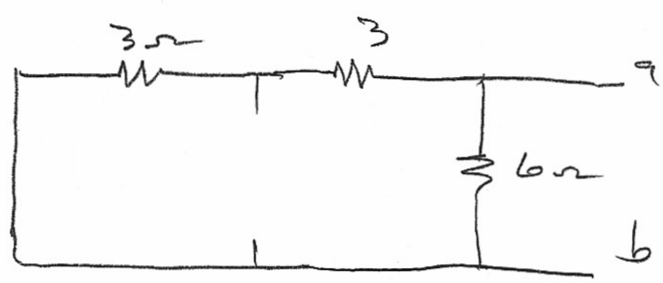


Figure 7.10

To find R_{TH}



By inspection, practically,

$$\boxed{R_{TH} = 3\Omega}$$

To find the I_{sc} we analyze the following circuit.

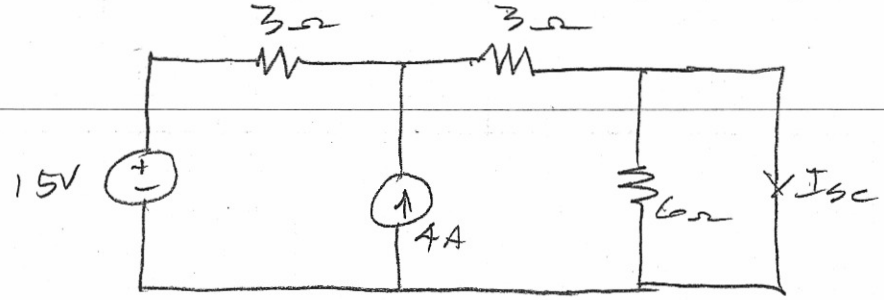


Figure 7.11

Actually, the 6Ω resistor is shorted so we can change the ckt to that of Figure 7.12

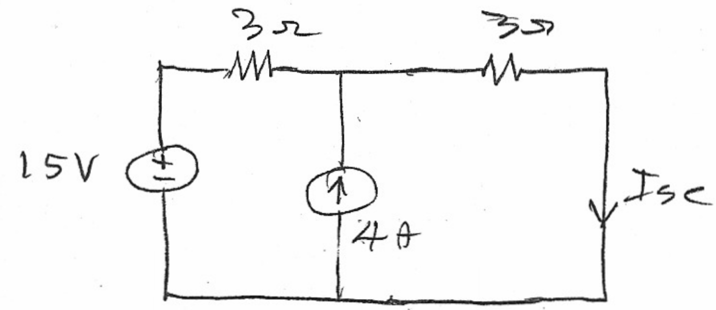


Figure 7.12

We can use nodal analysis, mesh analysis, source transformation to find I_{sc}. Source transformation is easy to apply, here. Then we have

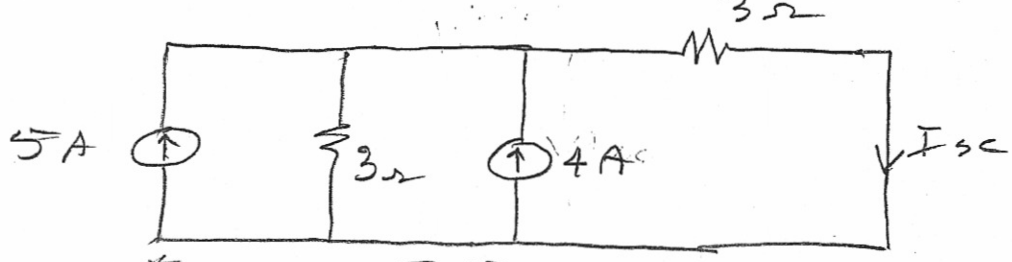


Figure 7.13

Figure 7.13 becomes

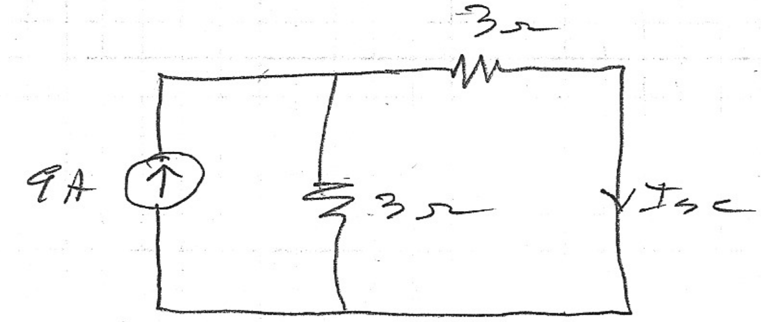


Figure 7.14

$$I_{sc} = \frac{9 \times 3}{3+3} = 4.5A$$

So the Norton circuit is

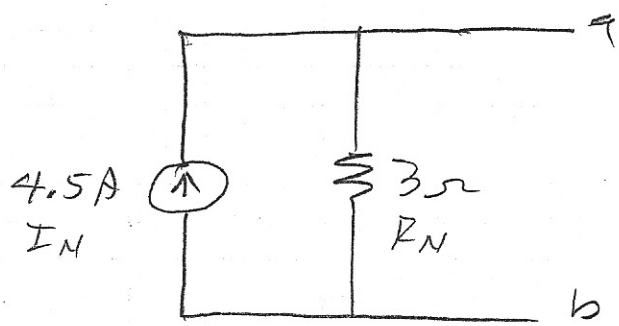


Figure 7.15

You can go back to the circuit of Figure 7.10 and find V_{TH} from the open-circuit voltage, ie, Vab. The answer should be $I_{sc} \times R_{TH}$ from above, or

$$V_{TH} = 4.5 \times 3 = 13.5^{V}$$

You might want to verify this.

Let's look at another example

Example 7.3

Find the Thevenin equivalent ckt for the following and draw the Thevenin circuit,

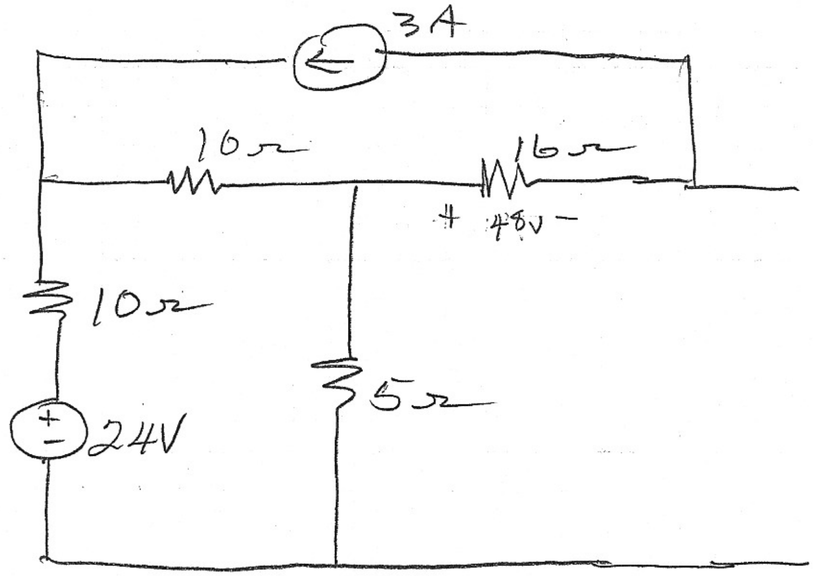


Figure 7.16

We use the following circuit to find R_{TH}.

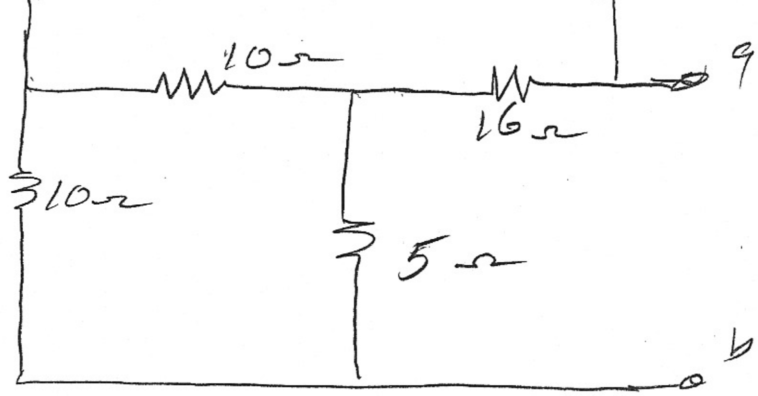


Figure 7.17

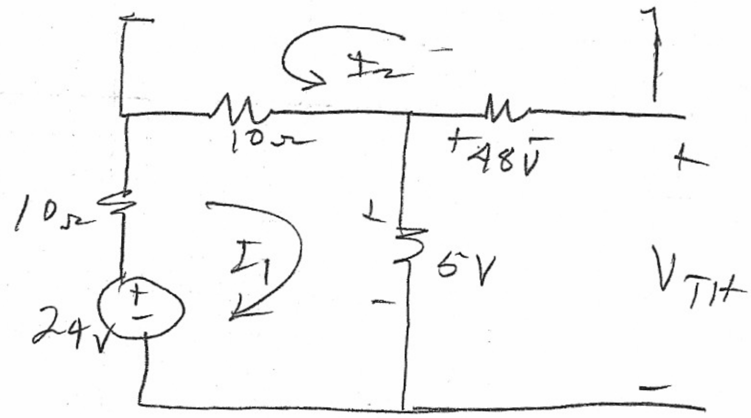
So we have

$$R_{TH} = 16 + 5 \| 20$$

$$R_{TH} = 16 + 4$$

$$R_{TH} = 20\Omega$$

We can use mesh to find V_{TH}



$$-24 + 10I_1 + 10(I_1 + I_2) + 5I_1 = 0$$

$$I_2 = 3A$$

$$-24 + 10I_1 + 10I_1 + 30 + 5I_1 = 0$$

$$25I_1 = -6$$

$$I_1 = I_1' = \frac{-6}{25}$$

$$V_{TH} = \frac{-6}{25} \times 5 - 48 = -49.2V$$

Thevenin ckt