wlgreen

Capacitors & Inductors

In this lesson we consider capacitors and inductors. We start with the capacitor.

Capacitor:

A capacitor is a passive circuit device. In the simpliest form it can be defined as two conducting plates separated by a dielectric.

When a voltage is connected across a capacitor, charge (electrons) move through the source and collect on the opposite plate. This leaves a deficiency of electrons on the first plate. We display this as in Figure 10.1.

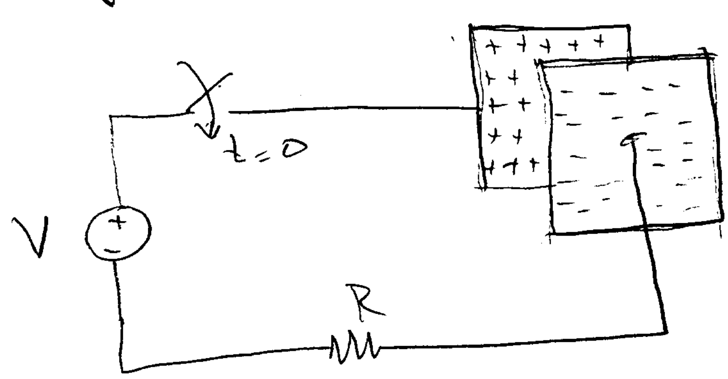


Fig. 10.1: Charging a capacitor.

There is a simple relationship

between the capacitor, the voltage and the charge;

$$q = CV \qquad \text{Eq } 10.1$$

q is the charge on one plate, V is the voltage across the capacitor, C is the value of the capacitance measured in Farads, after Faraday. Although capacitance can be expressed as $C = q/V$, the charge q is the result of the voltage and the size of the capacitor, physical parameters determine the size of the capacitor as expressed by

$$C = \frac{\varepsilon A}{l} \qquad \text{Eq } 10.2$$

A is the area of the plate, l the separation between the plates and ε the <u>permittivity</u> of the dielectric material. Typical dielectric materials are polyester, polystyrene, mica, mylar.

Like the resistor, the capacitor has a sign convention between the voltage and current.

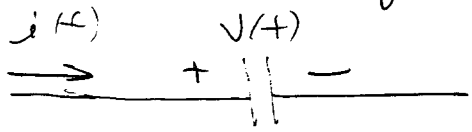
With

$$q = CV$$

we have

$$i(t) = \frac{dq}{dt} = C\frac{dv}{dt} \qquad \text{Eq } 10.3$$

We have a positive sign between $i(t)$ and $C\,dv/dt$ for the case below.



From 10.3;

$$dv = \frac{1}{C}\, i(t)\, dt$$

$$\int_{v(t_0)}^{v(t)} dv = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt$$

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt + v(t_0)} \qquad \text{Eq } 10.4$$

$$\boxed{i(t) = C\frac{dv}{dt}} \qquad \text{Eq } 10.5$$

<u>Eqs 10.4 and 10.5 are important expressions for the capacitor and should be remembered.</u> They become quite important when we consider transients.

We can write;

$$p(t) = v(t)\,i(t) \qquad \text{Eq } 10.6$$

and for the capacitor,

$$p(t) = v(t)\,C\frac{dv}{dt} \qquad \text{Eq. } 10.7$$

and energy,

$$w(t) = \int p(t)\,dt \qquad \text{Eq } 10.8$$

giving

$$w(t) = \int_{-\infty}^{t} v(t)\,C\frac{dv}{\cancel{dt}}\,\cancel{dt} = \frac{1}{2}\,C v^2(t)\Big|_{t=-\infty}^{t}$$

$$W(t) = \frac{1}{2} C V^2(t) - \frac{1}{2} C V^2(-\infty)$$

but $V(-\infty) = 0$ so

$$\boxed{W = \frac{1}{2} C V^2} \text{ Joules} \qquad \text{Eq } 10.9$$

This expresses the energy stored in the electric field of the capacitor. It is important to remember this expression.

The model of a capacitor would probably include a resistance shunting the ideal C; Normally this R is large (1 meg ohm or more) and we neglect it and use the ideal model in most cases.

Some facts to remember;

(1) The voltage across the capacitor does not change instantaneously (Eq 10.4)

(2) The capacitor acts like an open circuit to dc, in steady state. $i = C \frac{dv}{dt}$ $\frac{dv}{dt} = 0$ in steady state

(3) The ideal capacitor does not dissipate energy. Energy is stored in the electric field.

(4) Capacitors are useful circuit devices that are used
- to block dc
- to supress noise
- pass ac
- phase shift signals
- used in building analog filters
- used in analog computing
- others, not listed, less important

We have learned how to calculate equivalent resistance of resistors in series and parallel. What calculations do we make for capacitors?

Series and Parallel Capacitors

Consider the circuit shown in Fig 10.2

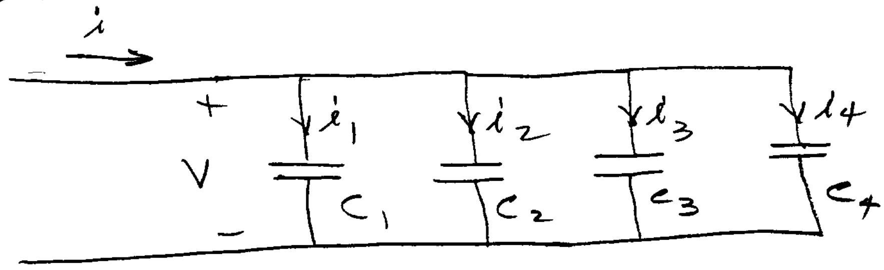


Fig 10.2 Parallel Capacitors.

From KCL we know;

$$i = i_1 + i_2 + i_3 + i_4$$

or

$$i = C_{eq}\frac{dv}{dt} = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dV}{dt} + C_4\frac{dV}{dt}$$

or

$$C_{eq}\frac{dv}{dt} = \left[C_1 + C_2 + C_3 + C_4\right]\frac{dv}{dt} \qquad \text{Eq } 10.10$$

This gives;

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + C_4} \qquad \text{Eq } 10.11$$

In general, we can say that the equivalent capacitance of capacitors in parallel is given by the sum of the individual capacitors. You might contrast this to the case of resistors. You might even anticipate how to calculate the equivalent C for capacitors in series.

Consider Fig 10.3 with series capacitors.

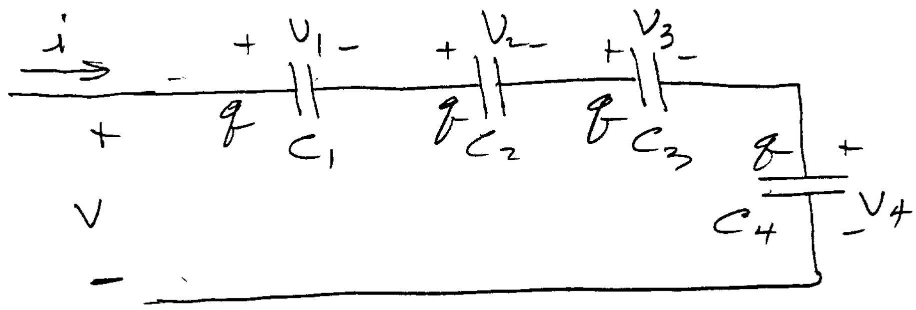


Fig 10.3: Series Capacitors.

KVL

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \qquad \text{Eq } 10.12$$

$$V(t) = \frac{1}{C_{eq}}\int i\,dt$$ (we assume 0 initial conditions)

and we also write

$$V(t) = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \frac{1}{C_3}\int i\,dt + \frac{1}{C_4}\int i\,dt$$

From the above we have

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}} \qquad \text{Eq } 10.13$$

Equivalent capacitance is calculated right opposite to the way one calculates equivalent resistance.

Inductors

In simpliest terms, an inductor is a coil of wire. In a simple case we have

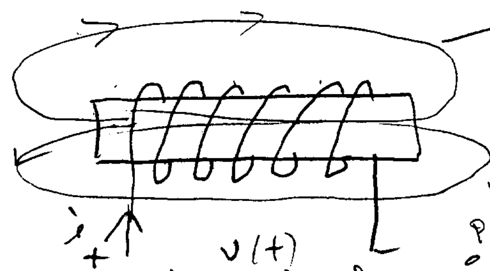


Fig 10.3: Illustrating an inductor.

We assume the current $i(t)$ into the winding of the coil causes a total flux (magnetic flux) to enclose (in sort of a donut fashion) the windings. We let this flux be $\Phi(t)$. We write, by Faraday's law,

$$V(t) = N\frac{\partial\Phi(t)}{\partial t} = N\frac{d\Phi}{\partial i}\times\frac{\partial i}{\partial t} \qquad \text{Eq } 10.14$$

And we write,

$$\boxed{v(t) = L\frac{\partial i}{\partial t}} \qquad \text{Eq } 10.15$$

with

$$L = N\frac{\partial\phi(t)}{\partial i(t)} \qquad \text{Eq } 10.16$$

An expression for determining the value of inductance from physical properties is

$$L = \frac{N^2 \mu A}{l} \qquad \text{Eq } 10.17$$

where N is the number of turns
A is the cross sectional area
l is the length of the coil
μ is the permeability

The core can be made from (a) iron, (b) steel (alloys of steel), (c) air (d) plastic. Most all transformers used in power distribution have laminated iron (lamination reduces eddy currents, which cause power loss) for the core. When analyzing transformers we use either a linear transformer model or an ideal transformer model. More consideration is later given to transformers.

Symbol for the inductor:

Recalling

$$\boxed{v(t) = L\frac{di}{dt}}$$

we separate variables and write

$$di = \frac{1}{L} v(t)\,dt$$

and integrate both sides to give

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)} \qquad \text{Eq } 10.18$$

This is an important expression. We use Eq 10.15 and 10.18 when we study transient analysis of RL and RLC circuits.

We keep in mind:

(1) current through an inductor does not change instantaneously

(2) voltage across an inductor can change instantaneously.

With respect to energy, we consider

$$W = \int p\,dt = \int v(t)\,i(t)\,dt$$

$$W = \int \left(L\frac{di}{dt}\right) \times i(t)\,dt = \int L\,i(t)\,di(t)$$

$$W = L\int i(t)\,di = \frac{L}{2} i^2 \Big|_0^t = \frac{1}{2} L i^2 \qquad \text{Eq } 10.19$$

Assume zero I.C.

so

$$\boxed{W = \frac{1}{2} L i^2} \qquad \text{Eq } 10.20$$

This is an important expression.

In this case, the energy is stored in the magnetic field (associated with Φ) of the coil.

A coil does not absorb power on the average. At any instant

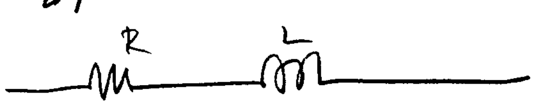
$$p(t) = v(t)\, i(t)$$

for the coil, generally, will not be zero. This $p(t)$ is present in

$$w(t) = \int p(t)\, dt$$

that determines the energy stored in the coil. The average power is zero.

Most electrical appliances present a load of



that is, an inductive load.

Series and Parallel Inductors

Consider Figure 10.4

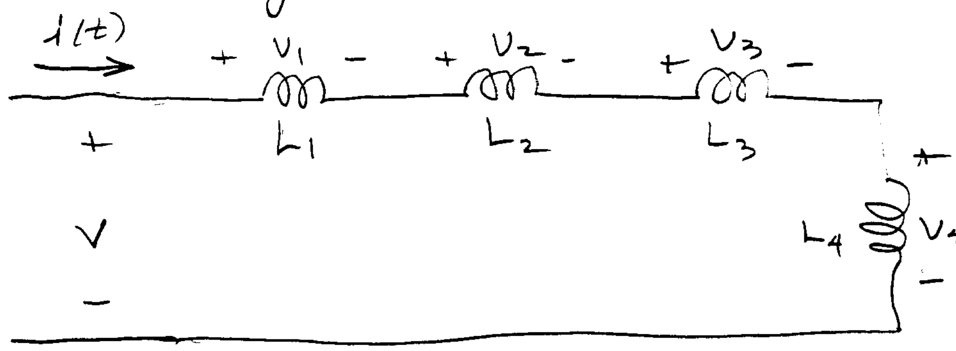


Fig 10.4; Series Inductors.

From the above diagram we can write (KVL);

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

Now we write

$$V(t) = L_{eq}\frac{di}{dt}$$

and

$$= L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + L_4\frac{di}{dt}$$

this gives

$$L_{eq}\frac{di}{dt} = (L_1 + L_2 + L_3 + L_4)\frac{di}{dt}$$

and

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + L_4} \quad \text{Eq 10.21}$$

Inductors in series

Now we consider inductors in parallel. We can already guess what the result will be.

From Figure 10.5.

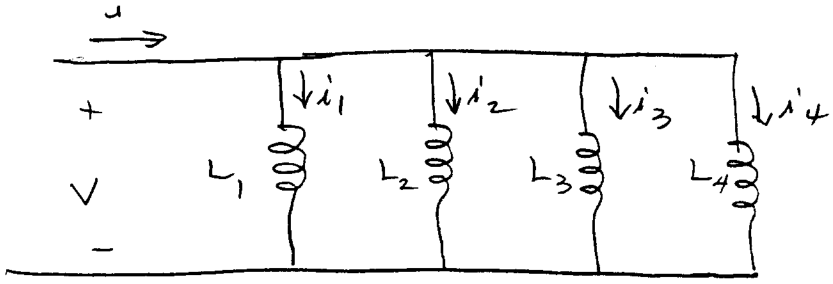


Fig 10.5:

For simplicity we assume zero initial conditions.

From above,

$$i(t) = i_1(t) + i_2(t) + i_3(t) + i_4(t)$$

and

$$i(t) = \frac{1}{L_{eq}} \int v(t)\,dt$$

so

$$\frac{1}{L_{eq}} \int v(t)\,dt = \frac{1}{L_1} \int v(t)\,dt + \frac{1}{L_2} \int v(t)\,dt + \frac{1}{L_3} \int v\,dt + \frac{1}{L_4} \int v(t)\,dt$$

Therefore,

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4}} \qquad \text{Eq 10.22}$$

Inductors in series

We see that the equivalent inductance is calculated using the same form as for resistors.

Finally, we call attention to properties of the inductor that are useful in transient analysis work.

(1) The current throught an inductor does not change instantaneously. This is why we see a spark when we open switches. See Equation 10.18.

(2) The voltage across an inductor can change instantaneously.

We will use these properties when we study transients.