Advanced Image Denoising Methods: TV, NLM, and BM3D

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Outline

• **TV** [Rudin, et al. 1992]:
  
  *Total Variation minimization*

• **NLM** [Buades, et al. 2005]:
  
  *Non-Local Means*

• **BM3D** [Dabov, et al. 2006/2007]:
  
  *Block-Matching and 3D filter*
**Total Variation minimization (TV)**

\[ g(x, y) = f(x, y) + n(x, y) \]

- \( f \) is the clean image.
- \( g \) is the observed image.
- \( n \) is the additive noise.

\[ \hat{f} = g \otimes h \]

- \( \hat{f} \) is the reconstructed image.
- \( h \) is the smooth kernel.

Diagram:
- Observed image \( g \)
- Clean image \( f \)
- Additive noise \( n \)
- Smooth kernel (local method) \( h \)
**Total Variation minimization (TV)**

Global method:

\[
TV(\hat{f}) = \sum_{x,y} \sqrt{\left( \hat{f}(x+1, y) - \hat{f}(x, y) \right)^2 + \left( \hat{f}(x, y+1) - \hat{f}(x, y) \right)^2}
\]

\[
TV(\hat{f}) = \sum_{x,y} \sqrt{\hat{f}(x+1, y) - \hat{f}(x, y)}^2 + \sqrt{\hat{f}(x, y+1) - \hat{f}(x, y)}^2
\]

\[
\begin{bmatrix}
124 & 100 & 30 \\
69  & 80  & 200 \\
66  & 92  & 211 \\
\vdots & & \\
\end{bmatrix}
\]

Lower TV
Total Variation minimization (TV)

\[
\begin{aligned}
\arg \min_{\hat{f}} \frac{1}{2} \| g - \hat{f} \|_2^2 + \lambda \cdot TV(\hat{f})
\end{aligned}
\]

Fidelity term
Total Variation minimization (TV)

• Straight edges are maintained.
• Details and texture can be over smoothed if $\lambda$ is too large.
Non-Local Means (NLM)

\[ NLM(x, y) = \sum_{i,j} w_{xy}(i, j)g(i, j) \]

\[ 0 \leq w_{xy}(i, j) \leq 1, \quad \sum_{i,j} w_{xy}(i, j) = 1 \]

\[ w_{xy}(i, j) = \frac{\exp \left( -\frac{\|g(N_{xy}) - g(N_{ij})\|_2^2}{\sigma^2} \right)}{\sum_{ij} \exp \left( -\frac{\|g(N_{xy}) - g(N_{ij})\|_2^2}{\sigma^2} \right)} \]
Non-Local Means (NLM)

- Preserve straight edges, as well as details and texture.

Noisy image
\[ \sigma = 20 \]

Gaussian kernel

TV

NLM
Block-Matching and 3D filter (BM3D)

Block matching + 3D transform
Block-Matching and 3D filter (BM3D)

Block matching + 3D transform

- Element-wise averaging
- Identical blocks
- Multiple blocks
- 3D transform (e.g., DWT, DFT, DCT)
**Block-Matching and 3D filter (BM3D)**

- Using the basic estimate instead of the noisy image allows to improve the grouping by block-matching.
- Using the basic estimate as the pilot signal for the empirical Wiener filtering is much more effective and accurate than the simple hard-thresholding.
Block-Matching and 3D filter (BM3D)

Noisy image $\sigma = 40$  
Basic estimate  
Final estimate
Block-Matching and 3D filter (BM3D)
Thank You