Localization of Radioactive Sources

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Outline

• Background and motivation
• Our goal and scenario
• Preliminary knowledge
• Related work
• Our approach and results
Background and Motivation

• For security purpose or searching missing radiological materials, localization of radioactive source is required.

• Many algorithms exist to perform source detection or identification. However, efforts at source localization are limited (e.g., maximum count rate, MLE).

• The detecting output may vary with angle, distance, duration time, and environment (e.g., background, shadow of obstacles).
Background and Motivation

The detector can be carried by a helicopter, truck, or human. An naïve way of radioactive source localization is base on maximum count rate.

Longer detecting time
→ more particles are captured
→ higher SNR
→ get count rate with higher confidence
Background and Motivation

Maximum count rate: 
Search every corner of the target area to find the location with the maximum count rate.

A more efficient way: 
Train a model in prior, and then estimate the location by Maximum Likelihood Estimation (MLE).
Our Goal and Scenario

**Goal**: Localize (angle $\theta$ and distance $r$) the radioactive source through human-carried detector.

**Scenario**: A person with a backpack, carrying a group of sensors with certain structure. Assume a radioactive source rotates around the person.
Our Goal and Scenario

Simulation on different distances.

Data of three sensors in different distance
Our Goal and Scenario

Final Goal:
Estimate a model or function of angle $\theta$ and distance $r$, $\mu(\theta, r)$, for each detector, so that count rate of the $i$th detector equals to $\mu_i(\theta, r)$. Assume an observation of the $i$th detector at $\theta$ and $r$ is $T_i$, thus

$$T_i \approx \mu_i(\theta, r)$$
Our Goal and Scenario

In practice, the radioactive source is fixed and the person is moving. Given $\mu(\theta, r)$ and an observation $T$, the correspond $\theta$ and $r$ can be estimated by:

- MLE: $\arg\ max_{\theta, r} P(T | \mu(\theta, r))$
- 1NN: $\arg\ min_{\theta, r} ||T - \mu(\theta, r)||_2$

(More details later ...)
Preliminary Knowledge

**Activity**: The total number of emission per second in all directions from the source. It is a constant

\[ 1 \text{Ci} = 3.7 \times 10^{10} \]

**Count rate** \((T)\): The number of emissions record by the detector. The observed count rate is always much less than the activity.

\[
T = \text{count} \times (100 \times 10^{-6}) \times (3.7 \times 10^{10}) \times 1.000
\]

Data from simulation

Recorded particles (constant)

All emitted particles per sec (constant)
Preliminary Knowledge

**Uncertainty**: Smaller count rate will result in higher uncertainty.

\[ T \sim N(T, \sqrt{T^2}) \]
Related Work

**Model-free** (sensor network):
- Angle-base (Mean of Estimates) [D. Niculescu et al., 2003]
- Distance-based (Apollonius circle) [J.C. Chin et al., 2008]
- Maximum count rate (stationary source) [D.K. Fagan et al., 2012]

**Model-based:**
Maximum Likelihood Estimation (MLE) [A. Gunatilaka et al., 2007]
- Gaussian noise model [K.D. Jarman et al., 2011]
- Poisson noise model [M. Wieneke et al., 2012]
Related Work

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Three sensors are sufficient for localizing the source
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Four sensors are sufficient for localizing the source

Apollonius’ definition of a circle:
\[\frac{d_1}{d_2} = \text{constant}\]
Related Work

**Model-free (sensor network):**

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Exhaustive search in a area

Aerial detection
Related Work

Model-based:
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1) Assume an parametric model of count rate and distance:

\[ \mu_k(x_0, y_0) = \frac{I}{(x_k - x_0)^2 + (y_k - y_0)^2} + b \]

2) Assume Gaussian noise:

\[ T_k \sim \mathcal{N}(\mu_k, \mu_k) \]

3) Maximize the likelihood:

\[ [\hat{x}_0, \hat{y}_0] = \arg\max_{x_0, y_0} P(T_1, T_2, \ldots, T_k | \mu) \]
Related Work

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The only difference is in the 2\textsuperscript{nd} step, assuming Poisson noise:

$$P(T_k; \lambda = \mu_k) = \frac{e^{-\mu_k} \cdot \mu_k^{T_k}}{T_k!}$$
Related Work

Related work:
• Scattered detectors
• Parametric model
• Gaussian noise
• Maximum likelihood

Ours approach:
• Structured detectors
• Non-parametric model
• Gaussian noise
• Maximum likelihood (1NN)
Our Approach and Results

The data we have:

- Angles:
  - 5 ~ 185 degree with increment of 5 degree.
- Distances:
  - 1 ~ 5m with increment of 0.5m;
  - 6~10m with increment of 1m;
  - 15 and 20m.
Our Approach and Results

The raw data
Our Approach and Results

The noisy data
Our Approach and Results

Step 1: Construct $\mu_i(\theta, r)$, $i = 1, 2, 3$ (assume three detectors):
1) Interpolation (regression) on both $\theta$ and $r$
2) Build 2-D lookup table (angle vs. distance)

Raw data of the $i$th detector $\mu_i(\theta, r)$ after interpolation
Our Approach and Results

**Step 2:** Assume Gaussian noise: $T_i \sim \mathcal{N}(\mu_i(\theta, r), \mu_i(\theta, r))$

$$P(T_i|\mu_i(\theta, r)) = \frac{1}{\sqrt{2\pi \mu_i(\theta, r)}} e^{-\frac{(T_i-\mu_i(\theta, r))^2}{2\mu_i(\theta, r)}}$$

**Step 3:** Maximum likelihood estimation:

$$[\hat{\theta}, \hat{r}] = \arg \max_{\theta, r} P(T_1, T_2, T_3 | \mu_1, \mu_2, \mu_3)$$
Assume the three detectors are independent,

\[
P(T_1, T_2, T_3 | \mu_1, \mu_2, \mu_3)
= P(T_1 | \mu_1, T_2 | \mu_2, T_3 | \mu_3)
= P(T_1 | \mu_1)P(T_2 | \mu_2)P(T_3 | \mu_3)
= \sum_{i=1}^{3} \frac{1}{\sqrt{2\pi\mu_i}} \exp\left(-\frac{(T_i - \mu_i)^2}{2\mu_i}\right)
\]

Log-likelihood:

\[
= -\frac{1}{2} \sum_{i=1}^{3} \log(2\pi\mu_i) - \frac{1}{2} \sum_{i=1}^{3} \frac{(T_i - \mu_i)^2}{\mu_i}
\]

Finally,

\[
\text{arg min}_{\theta, r} \left( \sum_{i=1}^{3} \log \mu_i + \sum_{i=1}^{3} \frac{(T_i - \mu_i)^2}{\mu_i} \right)
\]
Our Approach and Results

In practice, we may have only one sample for each \((\theta, r)\) pair.

\[
\arg \min_{\theta, r} \left( \sum_{i=1}^{3} \log \mu_i + \sum_{i=1}^{3} \frac{(T_i - \mu_i)^2}{\mu_i} \right)
\]

Equivalent to 1NN:

\[
\arg \min_{\theta, r} \sum_{i=1}^{3} (T_i - \mu_i)^2
\]
Our Approach and Results

Random leave-n-out cross validation, 1000 iteration:
Our Approach and Results

Random leave-n-out cross validation, apply 1NN 1000 iteration:

Angle Error vs. Distance

Distance Error vs. Angle
Our Approach and Results

If there are enough samples to estimate $\mu_i(\theta, r)$, apply MLE:
Thank You