Is $n^2 + n + 1 = O(n^2)$? To prove this, we need to find a constant $c$ such that $cn^2 \geq n^2 + n + 1$. Let $c = 2$ – that should work. Now we need to find a constant $x$ such that for all $n \geq x$, $2n^2 \geq n^2 + n + 1$. We’ll try $x = 10$.

Let’s proceed by an inductive argument. To make our life simpler, let $f(n) = 2n^2$, and $g(n) = n^2 + n + 1$. When $n = 10$, $f(n) = 200$ and $g(n) = 111$, so $f(x) > g(x)$. Now, let’s assume that our statement is true for all values between 10 and $n$ for some $n$. We already know that this is true for $n = 10$. Let’s look at $n + 1$:

$$
\begin{align*}
f(n + 1) &= 2(n + 1)^2 \\
        &= 2n^2 + 4n + 2 \\
        &= f(n) + 4n + 2
\end{align*}
$$

$$
\begin{align*}
g(n + 1) &= (n + 1)^2 + (n + 1) + 1 \\
        &= n^2 + 2n + 1 + n + 1 + 1 \\
        &= n^2 + 3n + 3 \\
        &= (n^2 + n + 1) + 2n + 2 \\
        &= g(n) + 2n + 2
\end{align*}
$$

From our inductive hypothesis, we know $f(n) \geq g(n)$, thus:

$$
f(n) + 4n + 2 \geq g(n) + 4n + 2
$$

Since $n \geq 10$, $4n + 2 > 2n + 2$, and therefore:

$$
\begin{align*}
f(n) + 4n + 2 &> g(n) + 2n + 2 \\
              &> g(n + 1)
\end{align*}
$$

Therefore, for all $n \geq 10$, $2n^2 > n^2 + n + 1$, meaning $2n^2 \geq n^2 + n + 1$, and therefore $n^2 + n + 1 = O(n^2)$.