Crystal structure.

Bravais lattices.

\[ \vec{R} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3 \]

1D

The unit cell is any segment with length \( a \).

2D

The unit cell is any area \( \vec{a} \times \vec{b} \).
3D. simple cubic

2D Honeycomb, 2 sets of Bravais lattices

Conventional & primitive unit cell

BCC: \[ 8 \times \frac{1}{8} + 1 = 2 \text{ /unit cell.} \]

Fcc: \[ 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4 \text{ /unit cell.} \]
Bragg refraction and the reciprocal lattice

For those $\theta$ at which you have constructive interference:

$$2d \sin \theta = n \lambda = \frac{2n\pi}{k}$$

$$2k \sin \theta = n \frac{2\pi}{d} \equiv G$$

$$\bar{G} \cdot \hat{R} = 2\pi n$$

$$e^{i\bar{G} \cdot \hat{R}} = 1$$

The lattice of all $\bar{G}$: reciprocal lattice
1D

\[ b = \frac{2\pi}{a} \]

\[ \text{spatial frequencies} \]

\[ \frac{2\pi}{a} \]

2D

\[ \vec{b}_1 = 2\pi \frac{\hat{a}_2 \times \hat{n}}{|\hat{a}_1 \times \hat{a}_2|} \]

\[ \vec{b}_2 = 2\pi \frac{\hat{n} \times \hat{a}_1}{|\hat{a}_1 \times \hat{a}_2|} \]

Rectangular

\[ \vec{a}_1 \]

\[ \vec{a}_2 \]

\[ \vec{b}_1 = \frac{2\pi a_2}{a_1 a_2} \hat{a}_1 = \frac{2\pi}{a_1} \hat{a}_1 \]

\[ \vec{b}_2 = \frac{2\pi a_1}{a_1 a_2} \hat{a}_2 = \frac{2\pi}{a_2} \hat{a}_2 \]
Hexagonal

\[ |\vec{a}_1| = |\vec{a}_2| = a, \quad |\vec{a}_1 \times \vec{a}_2| = \frac{\sqrt{3}}{2} a^2 \]

\[ \vec{b}_1 = 2\pi \frac{a}{\frac{\sqrt{3}}{2} a^2} = \frac{2\pi}{\frac{\sqrt{3}}{2}} \hat{b}_1 \]

\[ \vec{b}_2 = \frac{2\pi}{\frac{\sqrt{3}}{2}} \hat{b}_2 \]
Referring to the origin of the definition of the reciprocal lattice, i.e., Bragg reflection, \( \vec{G} \) actually represent a plane.

Plane \((h, k, l) \Rightarrow h \vec{b}_1 + k \vec{b}_2 + l \vec{b}_3\)