Self consistency.

Start with the depletion approximation, to find the profiles of $\phi$, $n$, $p$, etc.

\[ n \leftarrow n_i e^{\frac{-q\phi}{kT}} \]
\[ p \leftarrow p_i n_i e^{\frac{-q\phi}{kT}} \]

Solve the exact Poisson Eq.

\[ \frac{d^2\phi}{dx^2} = -\frac{q}{\varepsilon_0 \varepsilon_r} (N_D - N_A + p - n) \]

to find a new $\phi(x)$

Easy, just take the integral twice with known boundary conditions.

$\phi(+\infty) = \phi_n$, $\phi(-\infty) = \phi_p$

How to implement the boundary conditions numerically? $+\infty >> x_n$, $-\infty << x_p$

You can then get new $n(x)$ & $p(x)$,

and another new $\phi(x)$

You can go around many times.
until you achieve self-consistency, i.e.

\[ n = n_0 \exp \left( \frac{q\phi}{kT} \right), \quad p = n_0 \exp \left( -\frac{q\phi}{kT} \right) \]

What's the numerical criterion for self-consistency?

Homework #2

A pn junction: \( N_0 = 5 \times 10^{16} \) cm\(^{-3}\), \( N_A = 2 \times 10^{16} \) cm\(^{-3}\)

Solve the equations numerically using the self-consistency method.

Visualize the results:
Plot \( n(x) \), \( p(x) \), and \( \phi(x) \), and compare with the depletion approximation results.

Use your favorite software or write your own program using your favorite language.

Device simulators are NOT allowed.
Keep in mind that we only studied the abrupt pn junction.
Real pn junctions are not made by joining two pieces.
With the methods outlined here, you can solve for any profile.

I wish I have given you a good example of rigorous analysis. From now on, we will not do the math in the classroom. We'll visualize the concepts.

- The pn junction under reverse bias
  Bias: away from equilibrium.
  The idea of an $E_F$ is no longer valid.
  But, we assume "local" equilibrium, and define local Fermi levels $E_F(x)$.

 ![Biasing a uniform sample](image)

$$\sigma = q n \mu_e + q p \mu_h$$

If $n \gg p$.
$$\sigma = q n \mu_e$$
Biasing a pn junction

The depletion region has almost no carriers ⇒ much more voltage drop there.

The barriers are decreased by $qV$.

At equilibrium, diffusion is balanced by the barriers. Now, barriers are lowered, carriers will continuously flow. — Forward bias

The barriers are increased by $-qV$. Virtually no current can flow. — Reverse bias
Under reverse bias, the depletion region widens.

For any doping profile, just replace \( \phi_i \) with \( \phi_i - V = \phi_i + |V| \) to calculate.

\[ x_d = x_n + x_p \]

For example, abrupt junction under depletion approximation:

\[
\begin{align*}
  x_d(V=0) &= \sqrt{\frac{2 \varepsilon E_r}{q \left( \frac{1}{N_D} + \frac{1}{N_A} \right)} \phi_i} \\
  x_d(V) &= \sqrt{\frac{2 \varepsilon E_r}{q \left( \frac{1}{N_D} + \frac{1}{N_A} \right)} (\phi_i - V)}
\end{align*}
\]

\( \square \) Junction capacitance (unit area)

Definition: \( C = \frac{dQ}{dV} \)

Note: this is the small signal capacitance.
Arbitrary doping profile

\[ d\varepsilon = \frac{dV}{\chi_d} \]

Gaus's law,

\[ d\varepsilon = \frac{dQ}{\varepsilon_0 \varepsilon_r} \]

\[ \therefore \frac{dQ}{\varepsilon_0 \varepsilon_r} = \frac{dV}{\chi_d} \]

\[ c = \frac{dQ}{dV} = \frac{\varepsilon_0 \varepsilon_r}{\chi_d} \]

Good! The junction cap is just like a parallel plate cap w/ \( d = \chi_d \).

Notice that this is for any doping profile.

\[ \square \text{ Junction breakdown} \]

\[ \frac{d}{J} \]

\[ V \]
1. Avalanche Breakdown

In the depletion region, carriers gain energy from the field.

An e-h pair can be generated by the energetic e.

\[ E = \int_{E_c}^{E_v} q \, F \, \text{d}x \]

The generated e & h are accelerated by \( E \), gain enough energy, and generate more e-h pairs.

On and on... — avalanche

Run the extra mile.
Read on avalanche breakdown.
Quantitative analysis.
Ionization coefficients.
2. Zener Breakdown

The bond picture

The valence electron is pulled out of the covalent bond by the electric force.

The band picture

The valence band electron tunnels into the conduction band.

The tunneling probability is calculated by quantum mechanics.

(How are you doing on homework #1?)

3. Which one dominates?

Lower doping \(\Rightarrow\) wider depletion region
\(\Rightarrow\) lower tunneling probability
\(\Rightarrow\) avalanche