1. A sound wave traveling in the +x-direction in water is characterized by a differential pressure \( p(x,t) \).

[Differential pressure means the deviation from the equilibrium pressure. For example, the differential pressure is 0 if the absolute pressure is 1 atmosphere pressure, for a sound wave in air].

[The unit for pressure is Newton per square meter (N/m\(^2\)].

Find an expression for \( p(x,t) \) for a sinusoidal sound wave traveling in the positive x-direction in water, given that the wave frequency (not \( \omega \)) is 1.5 kHz, the velocity of the sound is 1.5 km/s, the wave amplitude is 10 N/m\(^2\), and \( p(x,t) \) was observed to be at its maximum value at \( t = 0 \) and \( x = 0.3 \) m. Assume water to be lossless. Follow the convention in the textbook to express a traveling wave as \( p(x,t) = A \cos(\omega t - \beta x + \phi_0) \).

[Hint: The key is to get phase right. And, you need to get the units right to get full credit.] (15)

\[
\omega = 2 \pi f = 2 \pi \times 1.5 \times 10^3 \text{ s}^{-1}, \quad \beta = \frac{\omega}{v_p} = \frac{2 \pi \times 1.5 \times 10^3 \text{ s}^{-1}}{1.5 \times 10^3 \text{ m/s}} = 2 \pi \text{ m}^{-1}
\]

\[
p(x,t) = 10 \cos(2 \pi \times 1.5 \times 10^3 t - 2 \pi x + \phi_0) \text{ N/m}^2
\]

At \( x = 0.3 \text{ m} \) and \( t = 0 \),

\[
p(0.3 \text{ m}, 0) = 10 \cos(0.6 \pi + \phi_0) \text{ N/m}^2
\]

\[
= 10 \text{ N/m}^2
\]

\(-0.6 \pi + \phi_0 = 0 \quad \Rightarrow \quad \phi_0 = 0.6 \pi
\]

Actually, you could answer

\[
\phi_0 = 2n \pi + 0.6 \pi
\]

\[
= n \times 36^\circ + 108^\circ
\]

\[
\therefore \quad p(x, t) = 10 \cos(2 \pi \times 1.5 \times 10^3 t - 2 \pi x + 0.6 \pi) \text{ N/m}^2
\]
2. The dielectric medium of a slotted line is air, and its characteristic impedance is \( Z_0 = 75 \, \Omega \). When the line is terminated in a short circuit, the minima are at 16 cm, 24 cm, ... from the load on the scale. When an unknown load \( Z_L \) terminates the line, a voltage standing wave ratio \( S = 2 \) is recorded by a standing wave indicator and minima are found at 11 cm, 19 cm, ... from the load. Calculate the wavelength \( \lambda \), the frequency \( f \), and the load \( Z_L \). [Hint: what information do you get from that the slotted line is an air line?] (30)

\[
\frac{\lambda}{2} = 8 \, \text{cm} \quad \Rightarrow \quad \lambda = 16 \, \text{cm} = 0.16 \, \text{m}
\]

\[
f^2 = \frac{c}{\lambda} = \frac{3 \times 10^8 \, \text{m/s}}{0.16 \, \text{m}} = 1.88 \times 10^9 \, \text{Hz} = 1.88 \, \text{GHz}
\]

\[
|\Gamma| = \frac{S-1}{S+1} = \frac{1}{3} \quad \begin{bmatrix} \Gamma = |\Gamma| e^{j\Theta_r} \end{bmatrix}
\]

\[
\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.16} \, \text{m}^{-1}
\]

\[
d_{\text{min}} = 11 \, \text{cm} - 8 \, \text{cm} = 3 \, \text{cm} = 0.03 \, \text{m}
\]

\[
-2i\beta d_{\text{min}} + \Theta_r = -\pi
\]

\[
\Theta_r \quad 2\beta d_{\text{min}} - \pi = 2 \times \frac{2\pi}{0.16} \times 0.03 - \pi
\]

\[
= \frac{3}{4} \pi - \pi = -\frac{\pi}{4}
\]

\[
\Rightarrow \quad \Gamma = \frac{1}{3} \angle -\frac{\pi}{4}
\]

\[
Z_L = Z_o \frac{1 + \Gamma}{1 - \Gamma}
\]

\[
= 75 \, \Omega \left( 1.57 \angle -21.9^\circ \right)
\]

\[
= 104 - j55 \, \Omega
\]

Security check, [image of a circuit diagram with labeled components and annotations]
3. A voltage generator with \( v(t) = \cos(2\pi \times 10^9 t) \) V and an internal impedance \( Z_i = 50 \Omega \) is connected to a 50-\( \Omega \) lossless air-spaced transmission line. The line length is 5 cm and it is terminated in a load with impedance \( Z_l = (100-j100) \Omega \). Find the following:
   a) Reflection coefficient \( \Gamma \) at the load: \( \left( \frac{1}{2}, \frac{1}{2} \right) \)
   b) Input impedance \( Z_{in} \) at the input end of the transmission line: \( \left( \frac{1}{2}, \frac{1}{2} \right) \)
   c) The input voltage \( v(t) \) and input current \( i(t) \); [hint: simple circuit. First use phasors, then get to the time domain.] \( \left( \frac{1}{2}, \frac{1}{2} \right) \)
   d) The time averaged input power \( P_{av} \); [hint: again, simple circuit] \( \left( \frac{1}{2}, \frac{1}{2} \right) \)
   e) The incident voltage, incident current, and time averaged incident power; and \( \left( \frac{1}{2}, \frac{1}{2} \right) \)
   f) The reflected voltage, reflected current, and reflected power. [Okay to just give phasors.] \( \left( \frac{1}{2}, \frac{1}{2} \right) \)

\[ a) \quad \Gamma = \frac{Z_l - Z_0}{Z_l + Z_0} = \frac{2 - 2j + 1}{2 - 2j + 1} = \frac{1 - 2j}{3 - 2j} = 0.62 \angle -29.7^\circ \]

\[ \approx 0.62 \angle -30^\circ \]

\[ \beta = \frac{\omega \sqrt{\varepsilon}}{\lambda} = \frac{2\pi \times 10^9 \times 5^{-1}}{2\pi \times 10 \times \frac{3}{2} \text{ m}} = \frac{2\pi}{3} \text{ m}^{-1} \]

\[ \beta L = 2\pi \times \frac{10}{3} \times 0.05 = \frac{\pi}{3} \text{ m} \]

\[ \tan \beta L = \sqrt{3} \]

\[ b) \quad Z_{in} = Z_0 \frac{Z_l + j \tan \beta L}{1 + j Z_l \tan \beta L} = Z_0 \frac{2 - 2j + \sqrt{3}j}{1 + j (2 - 2j) \sqrt{3}} = 50 \Omega \frac{2 - (2 - \sqrt{3})}{(1 + 2\sqrt{3}) + 2\sqrt{3}j} \]

\[ = 50 \Omega \cdot \left(0.35 \angle -45.39^\circ\right) \approx 50 \Omega \left(\frac{\sqrt{2}}{2} \angle -45^\circ\right) \]

\[ \approx 50 \Omega \left(\frac{1}{4} - \frac{1}{4}j\right) = (12.5 - j12.5) \Omega \]

\[ c) \quad \tilde{V}_{in} = \frac{\tilde{V}}{j} = \frac{Z_{in}}{Z_l + Z_{in}} = 1 \times \frac{\frac{1}{4} - j\frac{1}{4}}{1 + \frac{1}{4} - j\frac{1}{4}} = \frac{1 - j}{5 - j} \]

\[ \tilde{V}_{in} = \frac{\tilde{V}_{in}}{Z_{in}} = \frac{0.28 \angle -33.7^\circ}{0.35 \angle -45.4^\circ} \approx V \]

\[ \tilde{I}_{in} = \frac{\tilde{V}_{in}}{Z_{in}} = \frac{0.28 \angle -33.7^\circ}{0.35 \angle -45.4^\circ} \approx (0.016 \angle 11.7^\circ) A \]

\[ v_{in} = 0.28 \cos(2\pi \times 10^9 t - 33.7^\circ) V \]

\[ i_{in} = 0.016 \cos(2\pi \times 10^9 t + 11.7^\circ) A \]
d) \[ P_{av} = \frac{1}{2} \text{Re} \left[ V_{in} I_{in}^* \right] \]

\[ = \frac{1}{2} \text{Re} \left[ (0.28 \angle -33.7^\circ) (0.016 \angle -11.7^\circ) \right] \text{ V A} \]

\[ = \frac{1}{2} \text{Re} \left( 0.0045 \angle -45.4^\circ \right) \text{ W} \]

\[ \approx \frac{1}{2} \text{Re} \left( 4.5 \text{ mW} \angle -45^\circ \right) \]

\[ = \frac{1}{2} \times 4.5 \cdot \frac{1}{\sqrt{2}} \text{ mW} = 1.6 \text{ mW} \]

e) \[ V_o^+ = \frac{V_{in}}{e^{j\beta d} + \Gamma e^{-j\beta d}} = \frac{0.28 \angle -33.7^\circ}{1 \angle 60^\circ + 0.62 \angle 30^\circ - 60^\circ} \]

\[ = \frac{0.28 \angle -33.7^\circ}{0.5 + 0.87j - 0.62j} \]

\[ = \frac{0.28 \angle -33.7^\circ}{0.5 + 0.25j} \]

\[ = \frac{1.12 \angle -33.7^\circ}{2 + j} = \frac{1.12 \angle -33.7^\circ}{\sqrt{5} \tan^{-1} \frac{1}{2}} \]

\[ \approx 0.5 \angle -60^\circ \]

incident voltage = \[ V_o^+ e^{j\beta d} = 0.5 e^{j\left(\frac{3d - \frac{\pi}{3}}{3}\right)} \text{ V} \]

incident current = \[ \frac{V_o^+}{2} e^{j\beta d} = 0.01 e^{j\left(\frac{3d - \frac{\pi}{3}}{3}\right)} \text{ A} \]

(in the phasor form)
\[ P_{inc} = \frac{1}{2} \times 0.01 \, \text{A} \times 0.5 \, \text{V} = 0.0025 \, \text{W} = 2.5 \, \text{mW} \]

\[ V_0^- = \Gamma V_0^+ = (0.5 \angle -60^\circ) \times (0.62 \angle -30^\circ) \, \text{V} \]
\[ = (0.3 \angle -90^\circ) \, \text{V} = -0.3 \, j \, \text{V} \]

Reflected voltage phase \( = 0.3 \, e^{-(j \beta d + \frac{\pi}{2})} \, \text{V} \)

Current \( = 0.006 \, e^{-(j \beta d + \frac{\pi}{2})} \, \text{A} \)

|\[ P_{ref} \]| = \[ \frac{1}{2} \times 0.3 \times 0.006 = 0.0009 \, \text{W} = 0.9 \, \text{mW} \]

Sanity check.

\[ P_{av} = |P_{inc}| - |P_{ref}| = (2.5 - 0.9) \, \text{mW} = 1.6 \, \text{mW} \]