1. An antenna with an impedance $Z_L = 30 + j22.5 \, \Omega$ is to be matched to a $Z_0 = 75 \, \Omega$ lossless line with a shorted stub. Determine

a) The required stub admittance.

b) The distance between the stub and the antenna in terms of wavelengths ($d/\lambda$).

c) The length of the stub ($l/\lambda$), and

d) The standing wave ratios of the transmission line between the stub and the load, that of the stub, and that of the transmission line before the stub (i.e. between the generator and the stub, no matter how far the generator is away from the stub). **Caution:** you need to find 3 SWRs for 3 segments.

\[
Z_L = \frac{0 + j22.5}{75} = 0.4 + j0.3
\]

By using the Smith chart, or finding $y_L = \frac{1}{Z_L}$, one gets $y_L = 1.6 - j1.2$

On the Smith Chart, rotate $y_L$ to hit the $y=1$ circle at $y = 1 - j1.06$

Thus: Need $y_{stub} = +1.06$ to cancel the imaginary part of $y$.

\[
Y_0 = \frac{1}{75 \, \Omega}
\]

a) $Y_{stub} = j1.06 \times \frac{1}{75 \, \Omega} = j14.1 \, mS$

b) Using the chart, you get

\[
0.336 \lambda - 0.304 \lambda = 0.032 \lambda
\]

c) $0.13 \lambda + 0.25 \lambda = 0.38 \lambda$
d) Between the stub & the load:
You use the scale bars at the bottom of the chart to find directly $S = 2.7$

Or, you could use the chart to find $|\Gamma| \approx 0.46$
and calculate $S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$.

You could calculate $\Gamma$ from $y_2$ or $\beta_2$, and then use the above equation.

- The stub:
  It's terminated in a short
  \[ S = \infty \]

- Before the stub:
  Matching is achieved.
  \[ S = 1 \]
The Complete Smith Chart
Black Magic Design

0.13λ

y_{stub} = + j 0.06

short 3 = 0 \Rightarrow y = 0

0.304λ

0.336λ

2.8 - 2.7
2. Generate a bounce diagram for a 1.875 m long lossless line characterized by $Z_0 = 150$ Ω and phase velocity $v_p = c/1.6$ (where $c$ is the speed of light), if the line is fed by a step voltage applied at $t = 0$ by a generator with $V_g = 1$ V and $R_g = 450$ Ω. The line is terminated in a load $Z_L = 60$ Ω.

Use the bounce diagram to plot $v(t)$:

a) Midway of the line, and
b) The generator.

Then, find

c) The steady state voltage and current.

\[
T = \frac{L}{v_p} = \frac{1.6 \times 1.875 \text{ m}}{3 \times 10^8 \text{ m/s}} = 10^{-8} \text{ s} = 10 \text{ ns}
\]

\[
\Gamma_L = \frac{60 - 150}{60 + 150} = -\frac{9}{21} = -\frac{3}{7} \approx -0.429
\]

\[
\Gamma_g = \frac{450 - 150}{145 + 150} = \frac{1}{2}
\]

\[
V_i^+ = \frac{Z_0}{Z_0 + R_g} V_g = \frac{150}{150 + 450} \times 1\text{ V}
\]

\[
= 0.25 \text{ V}
\]

See next page for a) & b)

C) \[
V(\infty) = \frac{V_g R_L}{R_g + R_L} = \frac{60\text{ V}}{450 + 60} = \frac{6}{51} \text{ V} = \frac{2}{17} \text{ V}
\]

\[
\approx 0.118 \text{ V}
\]

\[
i(\infty) = \frac{V(\infty)}{R_L} = \frac{118 \text{ mV}}{60 \text{Ω}} \approx 2 \text{ mA}
\]
\[ V_i^* = \frac{Z_0}{Z_0 + R_g}, \quad V_y^* = \frac{150}{150 + 450} \cdot 1V = 0.25V \]

\[ v(t) (V) \]

\[ 0.25 \]

\[ 0.14 \]

\[ 0.089 \]

\[ 0.112 \]

\[ 0.123 \]

\[ t (ns) \]

\[ a) \]

\[ b) \]

\[ T_L = \frac{2}{7} \]