2. Two infinitely long parallel wires (as in the parallel wire transmission line) of radius \(a\) carry currents of equal magnitude \(I\) but in opposite directions. The distance between the axes of the two wires is \(d\).

(1) What is the force between the two wires, assuming \(d \gg a\) (meaning you can ignore the finite radii of the wires)? Derive the expression rather than copy it from the book. Do the two wires attract or repel each other? (20)

(2) What is resultant magnetic field due to the two wires at a point midway between the two wires? (10)

(3) Find the total flux per unit length threading through the area between the two lines. (10)

1. Consider one wire in the field of the other.

\[
2\pi dB = \mu I \quad \Rightarrow \quad B = \frac{\mu I}{2\pi d}
\]

\[
F' = \frac{F}{l} = IB \frac{l}{l} = IB = \frac{\mu I^2}{2\pi d}
\]

The two wires, carrying opposite currents, repel each other.

2. \(B_1 = B_2 = \frac{\mu I}{2\pi \left(\frac{d}{2}\right)} = \frac{\mu I}{\pi d}\)

Total field due to the two wires is

\[
B = B_1 + B_2 = \frac{2\mu I}{\pi d}
\]

3. For any point \(x\) from wire 1.

\[
B_1 = \frac{\mu I}{2\pi x} \quad \text{and} \quad B_2 = \frac{\mu I}{2\pi (d-x)}
\]

\[
B = B_1 + B_2 = \frac{\mu I}{2\pi} \left( \frac{1}{x} + \frac{1}{d-x} \right)
\]
\[
\Phi = l \int_a^{d-a} B \, dx \\
= l \int_a^{d-a} \left( \frac{1}{x} + \frac{1}{d-x} \right) \, dx \left( \frac{\mu I}{2\pi} \right) \\
= 2l \int_a^{d-a} \frac{1}{x} \, dx \left( \frac{\mu I}{2\pi} \right) \\
= 2l \frac{\mu I}{2\pi} \ln \frac{d-a}{a} \\
\frac{\Phi}{l} = \frac{\mu I}{\pi} \ln \frac{d-a}{a}
\]

Extra:
\[
L' = \frac{L}{l} = \frac{\Phi}{lI} = \frac{\mu}{\pi} \ln \frac{d-a}{a} \approx \frac{\mu}{\pi} \ln \frac{d}{a}
\]

Notice: The above is just an approximation when \( d \gg 2a \), and thus the current distribution within the wire doesn't matter.

Actually, the current in the two wires interact with each other, therefore the distribution is not uniform.