Figure P1.2: (a) Pressure wave as a function of distance at $t = 0$ and (b) pressure wave as a function of time at $x = 0$.

**Problem 1.3** A harmonic wave traveling along a string is generated by an oscillator that completes 180 vibrations per minute. If it is observed that a given crest, or maximum, travels 300 cm in 10 s, what is the wavelength?

**Solution:**

\[
\begin{align*}
  f &= \frac{180}{60} = 3 \text{ Hz.} \\
  v_p &= \frac{300 \text{ cm}}{10 \text{ s}} = 0.3 \text{ m/s}. \\
  \lambda &= \frac{v_p}{f} = \frac{0.3}{3} = 0.1 \text{ m} = 10 \text{ cm}.
\end{align*}
\]

**Problem 1.4** Two waves, $y_1(t)$ and $y_2(t)$, have identical amplitudes and oscillate at the same frequency, but $y_2(t)$ leads $y_1(t)$ by a phase angle of 60°. If

\[
y_1(t) = 4 \cos(2\pi \times 10^3 t),
\]

write down the expression appropriate for $y_2(t)$ and plot both functions over the time span from 0 to 2 ms.

**Solution:**

\[
y_2(t) = 4 \cos(2\pi \times 10^3 t + 60^\circ).
\]
Problem 1.5 The height of an ocean wave is described by the function

\[ y(x,t) = 1.5 \sin(0.5t - 0.6x) \] (m).

Determine the phase velocity and the wavelength and then sketch \( y(x,t) \) at \( t = 2 \text{ s} \) over the range from \( x = 0 \) to \( x = 2\lambda \).

**Solution:** The given wave may be rewritten as a cosine function:

\[ y(x,t) = 1.5 \cos(0.5t - 0.6x - \pi/2). \]

By comparison of this wave with Eq. (1.32),

\[ y(x,t) = A \cos(\omega t - \beta x + \phi_0), \]

we deduce that

\[ \omega = 2\pi f = 0.5 \text{ rad/s}, \quad \beta = \frac{2\pi}{\lambda} = 0.6 \text{ rad/m}, \]

\[ u_p = \frac{\omega}{\beta} = \frac{0.5}{0.6} = 0.83 \text{ m/s}, \quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.6} = 10.47 \text{ m}. \]
At $t = 2$ s, $y(x, 2) = 1.5 \sin(1 - 0.6x)$ (m), with the argument of the cosine function given in radians. Plot is shown in Fig. P1.5.

Problem 1.6 A wave traveling along a string in the $+x$-direction is given by

$$y_1(x, t) = A \cos(\omega t - \beta x),$$

where $x = 0$ is the end of the string, which is tied rigidly to a wall, as shown in Fig. 1-21 (P1.6). When wave $y_1(x, t)$ arrives at the wall, a reflected wave $y_2(x, t)$ is generated. Hence, at any location on the string, the vertical displacement $y_s$ will be the sum of the incident and reflected waves:

$$y_s(x, t) = y_1(x, t) + y_2(x, t).$$

(a) Write down an expression for $y_2(x, t)$, keeping in mind its direction of travel and the fact that the end of the string cannot move.

(b) Generate plots of $y_1(x, t)$, $y_2(x, t)$ and $y_s(x, t)$ versus $x$ over the range $-2\lambda \leq x \leq 0$ at $\omega t = \pi/4$ and at $\omega t = \pi/2$.

Solution:

(a) Since wave $y_2(x, t)$ was caused by wave $y_1(x, t)$, the two waves must have the same angular frequency $\omega$, and since $y_2(x, t)$ is traveling on the same string as $y_1(x, t)$,
(b) At \( t = (\pi/50) \) s, \( y_s = y_1 + y_2 = 4[\cos(0.4\pi - 30x) - \cos(0.4\pi + 3x)] \). Using the formulas from Appendix C,

\[
2 \sin x \sin y = \cos(x - y) - (\cos x + y),
\]

we have

\[
y_s = 8 \sin(0.4\pi) \sin 30x = 7.61 \sin 30x.
\]

Hence,

\[
|y_s|_{\text{max}} = 7.61
\]

and it occurs when \( \sin 30x = 1 \), or \( 30x = \frac{\pi}{2} + 2n\pi \), or \( x = \left( \frac{\pi}{60} + \frac{2n\pi}{30} \right) \) cm, where \( n = 0, 1, 2, \ldots \).

(c) \( |y_s|_{\text{min}} = 0 \) and it occurs when \( 30x = n\pi \), or \( x = \frac{n\pi}{30} \) cm.

**Problem 1.8** Give expressions for \( y(x, t) \) for a sinusoidal wave traveling along a string in the negative \( x \)-direction, given that \( y_{\text{max}} = 40 \) cm, \( \lambda = 30 \) cm, \( f = 10 \) Hz, and

(a) \( y(x, 0) = 0 \) at \( x = 0 \),

(b) \( y(x, 0) = 0 \) at \( x = 7.5 \) cm.

**Solution:** For a wave traveling in the negative \( x \)-direction, we use Eq. (1.17) with \( \omega = 2\pi f = 20\pi \) (rad/s), \( \beta = 2\pi/\lambda = 2\pi/0.3 = 20\pi/3 \) (rad/s), \( A = 40 \) cm, and \( x \) assigned a positive sign:

\[
y(x, t) = 40 \cos \left( 20\pi t + \frac{20\pi}{3}x + \phi_0 \right) \quad (\text{cm}),
\]

with \( x \) in meters.

(a) \( y(0, 0) = 0 = 40 \cos \phi_0 \). Hence, \( \phi_0 = \pm \pi/2 \), and

\[
y(x, t) = 40 \cos \left( 20\pi t + \frac{20\pi}{3}x \pm \frac{\pi}{2} \right)
\]

\[
= \begin{cases} 
-40 \sin \left( 20\pi t + \frac{20\pi}{3}x \right) \, (\text{cm}), & \text{if } \phi_0 = \pi/2, \\
40 \sin \left( 20\pi t + \frac{20\pi}{3}x \right) \, (\text{cm}), & \text{if } \phi_0 = -\pi/2. 
\end{cases}
\]

(b) At \( x = 7.5 \) cm = \( 7.5 \times 10^{-2} \) m, \( y = 0 = 40 \cos(\pi/2 + \phi_0) \). Hence, \( \phi_0 = 0 \) or \( \pi \), and

\[
y(x, t) = \begin{cases} 
40 \cos \left( 20\pi t + \frac{20\pi}{3}x \right) \, (\text{cm}), & \text{if } \phi_0 = 0, \\
-40 \cos \left( 20\pi t + \frac{20\pi}{3}x \right) \, (\text{cm}), & \text{if } \phi_0 = \pi.
\end{cases}
\]
Hence, \( y_2(t) \) lags \( y_1(t) \) by 54°.

**Problem 1.12** The voltage of an electromagnetic wave traveling on a transmission line is given by \( v(z,t) = 5e^{-\alpha z} \sin(4\pi \times 10^9 t - 20\pi z) \) (V), where \( z \) is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.

(b) At \( z = 2 \) m, the amplitude of the wave was measured to be 1 V. Find \( \alpha \).

**Solution:**

(a) This equation is similar to that of Eq. (1.28) with \( \omega = 4\pi \times 10^9 \) rad/s and \( \beta = 20\pi \) rad/m. From Eq. (1.29a), \( f = \omega/2\pi = 2 \times 10^9 \) Hz = 2 GHz; from Eq. (1.29b), \( \lambda = 2\pi/\beta = 0.1 \) m. From Eq. (1.30),

\[ u_p = \omega/\beta = 2 \times 10^8 \text{ m/s}. \]

(b) Using just the amplitude of the wave,

\[ 1 = 5e^{-\alpha z}, \quad \alpha = -\frac{1}{2} \ln \left( \frac{1}{5} \right) = 0.81 \text{ Np/m}. \]

**Problem 1.13** A certain electromagnetic wave traveling in sea water was observed to have an amplitude of 98.02 (V/m) at a depth of 10 m and an amplitude of 81.87 (V/m) at a depth of 100 m. What is the attenuation constant of sea water?

**Solution:** The amplitude has the form \( Ae^{\alpha z} \). At \( z = 10 \) m,

\[ Ae^{-10\alpha} = 98.02 \]

and at \( z = 100 \) m,

\[ Ae^{-100\alpha} = 81.87 \]

The ratio gives

\[ \frac{e^{-10\alpha}}{e^{-100\alpha}} = \frac{98.02}{81.87} = 1.20 \]

or

\[ e^{-10\alpha} = 1.2e^{-100\alpha}. \]

Taking the natural log of both sides gives

\[ \ln(e^{-10\alpha}) = \ln(1.2e^{-100\alpha}), \]

\[ -10\alpha = \ln(1.2) - 100\alpha, \]

\[ 90\alpha = \ln(1.2) = 0.18. \]

Hence,

\[ \alpha = \frac{0.18}{90} = 2 \times 10^{-3} \text{ (Np/m)}. \]
CHAPTER 1

Section 1-6: Phasors

Problem 1.21 A voltage source given by \( v_s(t) = 25 \cos(2\pi \times 10^3 t - 30^\circ) \) (V) is connected to a series RC load as shown in Fig. 1-19. If \( R = 1 \) M\( \Omega \) and \( C = 200 \) pF, obtain an expression for \( v_C(t) \), the voltage across the capacitor.

Solution: In the phasor domain, the circuit is a voltage divider, and

\[
\tilde{V}_C = \tilde{V}_s \frac{1/j\omega C}{1 + 1/j\omega C} = \frac{\tilde{V}_s}{(1 + j\omega RC)}.
\]

Now \( \tilde{V}_s = 25e^{-j30^\circ} \) V with \( \omega = 2\pi \times 10^3 \) rad/s, so

\[
\tilde{V}_C = \frac{25e^{-j30^\circ} \text{ V}}{1 + j(2\pi \times 10^3 \text{ rad/s}) \times (10^6 \Omega) \times (200 \times 10^{-12} \text{ F})} = 25e^{-j30^\circ} \cdot \frac{1}{1 + j2\pi/5} = 15.57e^{-j81.5^\circ} \text{ V}.
\]

Converting back to an instantaneous value,

\[ v_C(t) = \Re(\tilde{V}_C e^{j\omega t}) = \Re(15.57e^{j(\omega t - 81.5^\circ)}) \text{ V} = 15.57 \cos(2\pi \times 10^3 t - 81.5^\circ) \text{ V}, \]

where \( t \) is expressed in seconds.

Problem 1.22 Find the phasors of the following time functions:

(a) \( v(t) = 3 \cos(\omega t - \pi/3) \) (V),
(b) \( v(t) = 12 \sin(\omega t + \pi/4) \) (V),
(c) \( i(x,t) = 2e^{-3x} \sin(\omega x + \pi/6) \) (A),
(d) \( i(t) = -2 \cos(\omega t + 3\pi/4) \) (A),
(e) \( i(t) = 4 \sin(\omega t + \pi/3) + 3 \cos(\omega t - \pi/6) \) (A).

Solution:

(a) \( \tilde{V} = 3e^{-j\pi/3} \) V.
(b) \( v(t) = 12 \sin(\omega t + \pi/4) = 12 \cos(\pi/2 - (\omega t + \pi/4)) = 12 \cos(\omega t - \pi/4) \) V, \( \tilde{V} = 12e^{-j\pi/4} \) V.
(c) \( i(t) = 2e^{-3x} \sin(\omega x + \pi/6) \text{ A} = 2e^{-3x} \cos(\pi/2 - (\omega x + \pi/6)) \text{ A} = 2e^{-3x} \cos(\omega x - \pi/3) \text{ A}, \tilde{I} = 2e^{-3x} e^{-j\pi/3} \text{ A} \).
(d)
\[ i(t) = -2 \cos(\omega t + 3\pi/4), \]
\[ I = -2e^{j3\pi/4} = 2e^{-j\pi/4}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A.} \]

(e)
\[ i(t) = 4 \sin(\omega t + \pi/3) + 3 \cos(\omega t - \pi/6) \]
\[ = 4 \cos[\pi/2 - (\omega t + \pi/3)] + 3 \cos(\omega t - \pi/6) \]
\[ = 4 \cos(-\omega t + \pi/6) + 3 \cos(\omega t - \pi/6) \]
\[ = 4 \cos(\omega t - \pi/6) + 3 \cos(\omega t - \pi/6) = 7 \cos(\omega t - \pi/6), \]
\[ I = 7e^{-j\pi/6} \text{ A.} \]

Problem 1.23
Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) \( \tilde{V} = -5e^{j\pi/3} \) (V),
(b) \( \tilde{V} = j6e^{-j\pi/4} \) (V),
(c) \( \tilde{I} = (6 + j8) \) (A),
(d) \( \tilde{I} = -3 + j2 \) (A),
(e) \( \tilde{I} = j \) (A),
(f) \( \tilde{I} = 2e^{j\pi/6} \) (A).

Solution:

(a)
\[ \tilde{V} = -5e^{j\pi/3} \text{ V} = 5e^{j(\pi/3-\pi)} \text{ V} = 5e^{-j2\pi/3} \text{ V}, \]
\[ v(t) = 5 \cos(\omega t - 2\pi/3) \text{ V}. \]

(b)
\[ \tilde{V} = j6e^{-j\pi/4} \text{ V} = 6e^{j(-\pi/4+\pi/2)} \text{ V} = 6e^{j\pi/4} \text{ V}, \]
\[ v(t) = 6 \cos(\omega t + \pi/4) \text{ V}. \]

(c)
\[ \tilde{I} = (6 + j8) \text{ A} = 10e^{j53.1^\circ} \text{ A}; \]
\[ i(t) = 10 \cos(\omega t + 53.1^\circ) \text{ A}. \]

(d)
\[ \tilde{I} = -3 + j2 = 3.61e^{j46.31^\circ}, \]
\[ i(t) = 9\Re\{3.61e^{j46.31^\circ}e^{j\omega t}\} = 3.61 \cos(\omega t + 146.31^\circ) \text{ A}. \]
(e)

\[ \tilde{I} = j e^{j\pi/2}, \]
\[ i(t) = \Re\{e^{j\pi/2} e^{j\omega t}\} = \cos(\omega t + \pi/2) = -\sin \omega t \text{ A}. \]

(f)

\[ \tilde{I} = 2e^{j\pi/6}, \]
\[ i(t) = \Re\{2e^{j\pi/6} e^{j\omega t}\} = 2\cos(\omega t + \pi/6) \text{ A}. \]

**Problem 1.24** A series RLC circuit is connected to a generator with a voltage \(v_s(t) = V_0 \cos(\omega t + \pi/3)\) (V).

(a) Write down the voltage loop equation in terms of the current \(i(t)\), \(R\), \(L\), \(C\), and \(v_s(t)\).

(b) Obtain the corresponding phasor-domain equation.

(c) Solve the equation to obtain an expression for the phasor current \(\tilde{I}\).

![RLC circuit diagram](image)

Figure P1.24: RLC circuit.

**Solution:**

(a) \(v_s(t) = Ri + L \frac{di}{dt} + \frac{1}{C} \int i \, dt\).

(b) In phasor domain: \(\tilde{V}_s = R\tilde{I} + j\omega L\tilde{I} + \frac{\tilde{I}}{j\omega C}\).

(c) \[ \tilde{I} = \frac{\tilde{V}_s}{R + j(\omega L - 1/\omega C)} = \frac{\omega CV_0 e^{j\pi/3}}{\omega RC + j(\omega L - 1)} \]

**Problem 1.25** A wave traveling along a string is given by

\[ y(x, t) = 2\sin(4\pi x + 10\pi x) \text{ (cm)} \]