CHAPTER 2

From Eq. (2.63)

\[ Z_{\text{in}} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \]

\[ = 100 \left( \frac{(60 + j30) + j100 \tan \left( \frac{2\pi \text{ rad} \times 0.35\lambda}{\lambda} \right)}{100 + j(60 + j30) \tan \left( \frac{2\pi \text{ rad} \times 0.35\lambda}{\lambda} \right)} \right) = (64.8 - j38.3) \, \Omega. \]

**Problem 2.19** Show that the input impedance of a quarter-wavelength long lossless line terminated in a short circuit appears as an open circuit.

**Solution:**

\[ Z_{\text{in}} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right). \]

For \( l = \frac{1}{4}, \beta l = \frac{2\pi}{4} = \frac{\pi}{2} \). With \( Z_L = 0 \), we have

\[ Z_{\text{in}} = Z_0 \left( \frac{jZ_0 \tan \pi/2}{Z_0} \right) = j\infty \, \text{(open circuit)}. \]

**Problem 2.20** Show that at the position where the magnitude of the voltage on the line is a maximum the input impedance is purely real.

**Solution:** From Eq. (2.56), \( l_{\text{max}} = (\theta_0 + 2\pi)/2\beta \), so from Eq. (2.61), using polar representation for \( \Gamma \),

\[ Z_{\text{in}}(-l_{\text{max}}) = Z_0 \left( \frac{1 + |\Gamma|e^{j\theta_0}e^{-j2\beta l_{\text{max}}}}{1 - |\Gamma|e^{j\theta_0}e^{-j2\beta l_{\text{max}}}} \right) \]

\[ = Z_0 \left( \frac{1 + |\Gamma|e^{j\theta_0}e^{-j(\theta_0 + 2\pi)}}{1 - |\Gamma|e^{j\theta_0}e^{-j(\theta_0 + 2\pi)}} \right) = Z_0 \left( \frac{1 + |\Gamma|}{1 - |\Gamma|} \right), \]

which is real, provided \( Z_0 \) is real.

**Problem 2.21** A voltage generator with \( v_g(t) = 5 \cos(2\pi \times 10^5 t) \) V and internal impedance \( Z_R = 50 \, \Omega \) is connected to a 50-\( \Omega \) lossless air-spaced transmission line. The line length is 5 cm and it is terminated in a load with impedance \( Z_L = (100 - j100) \, \Omega \). Find

(a) \( \Gamma \) at the load.
(b) \( Z_{\text{in}} \) at the input to the transmission line.
(c) the input voltage \( V_i \) and input current \( I_i \).
and \( Z_{in}^{\infty} = 1/j \omega C = 1/(j \times 10^6 \times 40 \times 10^{-12}) = -j4000 \ \Omega \).

From Eq. (2.74), \( Z_0 = \sqrt{Z_{in}^{\infty} Z_{in}^{\infty}} = \sqrt{(j0.4 \ \Omega)(-j4000 \ \Omega)} = 40 \ \Omega \). Using Eq. (2.75),

\[
\frac{u_p}{\beta} = \frac{\omega}{\tan^{-1} \sqrt{-Z_{in}^{\infty}/Z_{in}^{\infty}}} = \frac{6.28 \times 10^6 \times 0.31}{\tan^{-1} \left( \pm \sqrt{-0.4/(-j4000)} \right)} = \frac{1.95 \times 10^6}{(\pm 0.01 + n\pi)} \ \text{m/s},
\]

where \( n \geq 0 \) for the plus sign and \( n \geq 1 \) for the minus sign. For \( n = 0 \), \( u_p = 1.94 \times 10^8 \text{ m/s} = 0.65c \) and \( \varepsilon_t = (c/u_p)^2 = 1/0.65^2 = 2.4 \). For other values of \( n \), \( u_p \) is very slow and \( \varepsilon_t \) is unreasonably high.

**Problem 2.27** A 75-\( \Omega \) resistive load is preceded by a \( \lambda/4 \) section of a 50-\( \Omega \) lossless line, which itself is preceded by another \( \lambda/4 \) section of a 100-\( \Omega \) line. What is the input impedance?

**Solution:** The input impedance of the \( \lambda/4 \) section of line closest to the load is found from Eq. (2.77):

\[
Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50^2}{75} = 33.33 \ \Omega.
\]

The input impedance of the line section closest to the load can be considered as the load impedance of the next section of the line. By reapplying Eq. (2.77), the next section of \( \lambda/4 \) line is taken into account:

\[
Z_{in} = \frac{Z_0^2}{Z_L} = \frac{100^2}{33.33} = 300 \ \Omega.
\]

**Problem 2.28** A 100-MHz FM broadcast station uses a 300-\( \Omega \) transmission line between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73 \( \Omega \). You are asked to design a quarter-wave transformer to match the antenna to the line.

(a) Determine the electrical length and characteristic impedance of the quarter-wave section.

(b) If the quarter-wave section is a two-wire line with \( d = 2.5 \) cm, and the spacing between the wires is made of polystyrene with \( \varepsilon_r = 2.6 \), determine the physical length of the quarter-wave section and the radius of the two wire conductors.
Solution:
(a) For a match condition, the input impedance of a load must match that of the transmission line attached to the generator. A line of electrical length $\lambda/4$ can be used. From Eq. (2.77), the impedance of such a line should be

$$Z_0 = \sqrt{Z_0^2 Z_L} = \sqrt{300 \times 73} = 148 \, \Omega.$$  

(b)  

$$\frac{\lambda}{4} = \frac{\mu_0}{4\pi f} = \frac{c}{4\sqrt{\varepsilon_r} f} = \frac{3 \times 10^8}{4\sqrt{2.6} \times 100 \times 10^6} = 0.465 \, \text{m},$$

and, from Table 2-2,

$$Z_0 = \frac{120}{\sqrt{\varepsilon}} \ln \left[ \left( \frac{d}{2a} \right) + \sqrt{\left( \frac{d}{2a} \right)^2 - 1} \right] \Omega.$$  

Hence,

$$\ln \left[ \left( \frac{d}{2a} \right) + \sqrt{\left( \frac{d}{2a} \right)^2 - 1} \right] = \frac{148 \sqrt{2.6}}{120} = 1.99,$$

which leads to

$$\left( \frac{d}{2a} \right) + \sqrt{\left( \frac{d}{2a} \right)^2 - 1} = 7.31,$$

and whose solution is $a = d/7.44 = 25 \, \text{cm}/7.44 = 3.36 \, \text{mm}.$

Problem 2.29 A 50-MHz generator with $Z_g = 50 \, \Omega$ is connected to a load $Z_L = (50 - j25) \, \Omega$. The time-average power transferred from the generator into the load is maximum when $Z_g = Z_L^*$, where $Z_L^*$ is the complex conjugate of $Z_L$. To achieve this condition without changing $Z_g$, the effective load impedance can be modified by adding an open-circuited line in series with $Z_L$, as shown in Fig. 2-40 (P2.29). If the line’s $Z_0 = 100 \, \Omega$, determine the shortest length of line (in wavelengths) necessary for satisfying the maximum-power-transfer condition.

Solution: Since the real part of $Z_L$ is equal to $Z_g$, our task is to find $l$ such that the input impedance of the line is $Z_m = +j25 \, \Omega$, thereby cancelling the imaginary part of $Z_L$ (once $Z_L$ and the input impedance the line are added in series). Hence, using Eq. (2.73),

$$-j100 \cot \beta l = j25,$$
(c) 
\[ P_{in} = \frac{1}{2} \Re \{ \overline{V}_{i} \overline{I}_{i}^{*} \} = \frac{1}{2} \Re \{ 143.6 e^{-j11.46^\circ} \times 3.24 e^{-j10.16^\circ} \} \]
\[ = \frac{143.6 \times 3.24}{2} \cos(21.62^\circ) = 216 \ \text{(W).} \]

(d) 
\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 - 50}{75 + 50} = 0.2, \]
\[ V_0 = \frac{1}{\sqrt{1 + \Gamma e^{-j\beta L}}} = \frac{143.6 e^{-j11.46^\circ}}{\sqrt{0.2 e^{-j54^\circ} + 0.2 e^{-j54^\circ}}} = 150 e^{-j54^\circ} \ \text{(V)}, \]
\[ \tilde{V}_L = V_0(1 + \Gamma) = 150 e^{-j54^\circ}(1 + 0.2) = 180 e^{-j54^\circ} \ \text{(V)}, \]
\[ \tilde{I}_L = \frac{V_0}{Z_0} (1 - \Gamma) = \frac{150 e^{-j54^\circ}}{50} (1 - 0.2) = 2.4 e^{-j54^\circ} \ \text{(A)}, \]
\[ P_L = \frac{1}{2} \Re \{ \overline{V}_{i} \overline{I}_{i}^{*} \} = \frac{1}{2} \Re \{ 180 e^{-j54^\circ} \times 2.4 e^{-j54^\circ} \} = 216 \ \text{(W).} \]

\( P_L = P_{in}, \) which is as expected because the line is lossless; power input to the line ends up in the load.

(e) Power delivered by generator:
\[ P_g = \frac{1}{2} \Re \{ \overline{V}_g \overline{I}_g \} = \frac{1}{2} \Re \{ 300 \times 3.24 e^{j10.16^\circ} \} = 486 \cos(10.16^\circ) = 478.4 \ \text{(W).} \]

Power dissipated in \( Z_g : \)
\[ P_Z = \frac{1}{2} \Re \{ \overline{I}_g^{2} \overline{Z}_g \} = \frac{1}{2} \Re \{ (2.4 e^{-j54^\circ})^2 \} = \frac{1}{2} (3.24)^2 \times 50 = 262.4 \ \text{(W).} \]

Note 1: \( P_e = P_Z = 478.4 \ \text{W.} \)

**Problem 2.32**: If the two-antenna configuration shown in Fig. 2.41 (P2.32) is connected to a generator with \( \tilde{V}_g = 250 \ \text{V} \) and \( Z_g = 50 \ \text{\Omega} \), how much average power is delivered to each antenna?

**Solution**: Since line 2 is \( \lambda/2 \) in length, the input impedance is the same as \( Z_{L_4} = 75 \ \text{\Omega} \). The same is true for line 3. At junction C–D, we now have two 75-\( \text{\Omega} \) impedances in parallel, whose combination is \( 75/2 = 37.5 \ \text{\Omega} \). Line 1 is \( \lambda/2 \) long. Hence at A–C, input impedance of line 1 is 37.5 \( \text{\Omega} \), and
\[ \tilde{I}_1 = \frac{\tilde{V}_g}{Z_g + Z_{in}} = \frac{250}{50 + 37.5} = 2.86 \ \text{(A)}, \]
Figure P2.32: Antenna configuration for Problem 2.32.

\[
P_{in} = \frac{1}{2} \text{Re}[h_i V_i^*] = \frac{1}{2} \text{Re}[h_i^* Z_{in}] = \frac{(2.86)^2 \times 37.5}{2} = 153.37 \text{ (W)}. \]

This is divided equally between the two antennas. Hence, each antenna receives \( \frac{153.37}{2} = 76.68 \text{ (W)}. \)

**Problem 2.33** For the circuit shown in Fig. 2-42 (P2.33), calculate the average incident power, the average reflected power, and the average power transmitted into the infinite 100-Ω line. The \( \lambda/2 \) line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as \( \alpha \neq 0 \).)

**Solution:** Considering the semi-infinite transmission line as equivalent to a load (since all power sent down the line is lost to the rest of the circuit), \( Z_L = Z_1 = 100 \) Ω. Since the feed line is \( \lambda/2 \) in length, Eq. (2.74) gives \( Z_{in} = Z_L = 100 \) Ω and \( \beta L = (2\pi/\lambda) (\lambda/2) = \pi \), so \( e^{\pm i\beta L} = -1 \). From Eq. (2.49a),

\[
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3},
\]