Solution: From Eqs. (2.66) and (2.61),

\[
V_0^+ = \left( \frac{\bar{V}_g Z_m}{Z_0 + Z_m} \right) \left( \frac{1}{e^{\beta l} + Ge^{-\beta l}} \right) \\
= \left( \frac{\bar{V}_g Z_0}{Z_0 + Z_m} \left[ \frac{(1 + Ge^{-\beta l})/(1 - Ge^{-\beta l})}{1 + Ge^{-\beta l}} \right] \right) \left( \frac{e^{-\beta l}}{1 + Ge^{-\beta l}} \right) \\
= \frac{\bar{V}_g e^{-\beta l}}{(1 - Ge^{-\beta l}) + (1 + Ge^{-\beta l})} \left( \frac{1 - Ge^{-\beta l}}{1 + Ge^{-\beta l}} \right) \\
= \frac{1}{2} \bar{V}_g e^{-\beta l}.
\]

Thus, in Eq. (2.86),

\[
P_{sv} = \frac{|V_0^+|^2}{2Z_0} (1 - |\Gamma|^2) = \frac{|\bar{V}_g e^{-\beta l}|^2}{2Z_0} (1 - |\Gamma|^2) = \frac{|\bar{V}_g|^2}{8Z_0} (1 - |\Gamma|^2).
\]

Under the matched condition, |\Gamma| = 0 and \(P_L = 20\) W, so \(|\bar{V}_g|^2/8Z_0 = 20\) W. When \(Z_L = (75 + j25)\) \(\Omega\), from Eq. (2.49a),

\[
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(75 + j25) - 50}{(75 + j25) + 50} = 0.277 e^{\jmath33.6^\circ},
\]

so \(P_{sv} = 20\) W \((1 - |\Gamma|^2) = 20\) W \((1 - 0.277^2) = 18.46\) W.

Section 2-9: Smith Chart

Problem 2.35 Use the Smith chart to find the reflection coefficient corresponding to a load impedance:

(a) \(Z_L = 3Z_0\),
(b) \(Z_L = (2 - j)Z_0\),
(c) \(Z_L = -2jZ_0\),
(d) \(Z_L = 0\) (short circuit).

Solution: Refer to Fig. P2.35.

(a) Point A is \(z_L = 3 + j0\). \(\Gamma = 0.5 e^{\jmath0}\)
(b) Point B is \(z_L = 2 - j2\). \(\Gamma = 0.62 e^{-29.7^\circ}\)
(c) Point C is \(z_L = 0 - j2\). \(\Gamma = 1.0 e^{-53.1^\circ}\)
(d) Point D is \(z_L = 0 + j0\). \(\Gamma = 1.0 e^{180.0^\circ}\)
Figure P2.35: Solution of Problem 2.35.

**Problem 2.36** Use the Smith chart to find the normalized load impedance corresponding to a reflection coefficient:

(a) $\Gamma = 0.5$,
(b) $\Gamma = 0.5\angle 0^\circ$,
(c) $\Gamma = -1$,
(d) $\Gamma = 0.3\angle -30^\circ$,
(e) $\Gamma = 0$,
(f) $\Gamma = j$.

**Solution:** Refer to Fig. P2.36.
Figure P2.36: Solution of Problem 2.36.

(a) Point $A'$ is $\Gamma = 0.5$ at $z_L = 3 + j0$.
(b) Point $B'$ is $\Gamma = 0.5e^{60^\circ}$ at $z_L = 1 + j1.15$.
(c) Point $C'$ is $\Gamma = -1$ at $z_L = 0 + j0$.
(d) Point $D'$ is $\Gamma = 0.3e^{-j30^\circ}$ at $z_L = 1.60 - j0.53$.
(e) Point $E'$ is $\Gamma = 0$ at $z_L = 1 + j0$.
(f) Point $F'$ is $\Gamma = j$ at $z_L = 0 + j1$.

Problem 2.37 On a lossless transmission line terminated in a load $Z_L = 100 \, \Omega$, the standing-wave ratio was measured to be 2.5. Use the Smith chart to find the two possible values of $Z_0$. 
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Solution: Refer to Fig. P2.37. \( S = 2.5 \) is at point \( L_1 \) and the constant SWR circle is shown. \( z_L \) is real at only two places on the SWR circle, at \( L_1 \), where \( z_L = S = 2.5 \), and \( L_2 \), where \( z_L = 1/S = 0.4 \). so \( Z_{01} = z_L/z_{L1} = 100 \ \Omega/2.5 = 40 \ \Omega \) and \( Z_{02} = z_L/z_{L2} = 100 \ \Omega/0.4 = 250 \ \Omega \).

![Figure P2.37: Solution of Problem 2.37.](image)

Problem 2.38 A lossless 50-\( \Omega \) transmission line is terminated in a load with \( Z_L = (50 + j25) \ \Omega \). Use the Smith chart to find the following:
(a) the reflection coefficient \( \Gamma \),
(b) the standing-wave ratio,
(c) the input impedance at 0.35\( \lambda \) from the load,
(d) the input admittance at 0.35\(\lambda\) from the load,
(e) the shortest line length for which the input impedance is purely resistive,
(f) the position of the first voltage maximum from the load.

![Solution of Problem 2.38](image_url)

**Figure P2.38: Solution of Problem 2.38.**

**Solution:** Refer to Fig. P2.38. The normalized impedance

\[ z_L = \frac{(50 + j25) \, \Omega}{50 \, \Omega} = 1 + j0.5 \]

is at point \(Z\)-\textit{LOAD}.

(a) \(\Gamma = 0.24e^{j76.0^\circ}\) The angle of the reflection coefficient is read of that scale at the point \(\theta_r\).

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(b) At the point SWR: $S = 1.64$.
(c) $Z_{in}$ is 0.350$\lambda$ from the load, which is at 0.144$\lambda$ on the wavelengths to generator scale. So point Z-IN is at 0.144$\lambda$ + 0.350$\lambda$ = 0.494$\lambda$ on the WTG scale. At point Z-IN:

$$Z_{in} = z_{in}Z_0 = (0.61 - j0.022) \times 50 \ \Omega = (30.5 - j1.09) \ \Omega.$$  

(d) At the point on the SWR circle opposite Z-IN,

$$Y_{in} = \frac{y_{in}}{Z_0} = \frac{(1.64 + j0.06)}{50 \ \Omega} = (32.7 + j1.17) \ \text{mS}.$$  

(e) Traveling from the point Z-LOAD in the direction of the generator (clockwise), the SWR circle crosses the $x_L = 0$ line first at the point SWR. To travel from Z-LOAD to SWR one must travel 0.250$\lambda$ - 0.144$\lambda$ = 0.106$\lambda$. (Readings are on the wavelengths to generator scale.) So the shortest line length would be 0.106$\lambda$.

(f) The voltage max occurs at point SWR. From the previous part, this occurs at $z = -0.106\lambda$.

---

**Problem 2.39** A lossless 50-Ω transmission line is terminated in a short circuit. Use the Smith chart to find

(a) the input impedance at a distance 2.3$\lambda$ from the load,
(b) the distance from the load at which the input admittance is $y_{in} = -j0.04$ S.

**Solution:** Refer to Fig. P2.39.

(a) For a short, $z_{in} = 0 + j0$. This is point Z-SHORT and is at 0.000$\lambda$ on the WTG scale. Since a lossless line repeats every $\lambda/2$, traveling 2.3$\lambda$ toward the generator is equivalent to traveling 0.3$\lambda$ toward the generator. This point is at A : Z-IN, and

$$Z_{in} = z_{in}Z_0 = (0 - j3.08) \times 50 \ \Omega = -j154 \ \Omega.$$  

(b) The admittance of a short is at point Y-SHORT and is at 0.250$\lambda$ on the WTG scale:

$$y_{in} = Y_{in}Z_0 = -j0.04 \ \text{S} \times 50 \ \Omega = -j2,$$

which is point B : Y-IN and is at 0.324$\lambda$ on the WTG scale. Therefore, the line length is 0.324$\lambda$ - 0.250$\lambda$ = 0.074$\lambda$. Any integer half wavelengths farther is also valid.
Problem 2.40  Use the Smith chart to find $y_L$ if $z_L = 1.5 - j0.7$.

Solution: Refer to Fig. P2.40. The point $Z$ represents $1.5 - j0.7$. The reciprocal of point $Z$ is at point $Y$, which is at $0.55 + j0.26$. 
**Problem 2.40** A lossless 100-Ω transmission line 3λ/8 in length is terminated in an unknown impedance. If the input impedance is $Z_{in} = -j2.5 \ \Omega$,

(a) use the Smith chart to find $Z_L$.

(b) What length of open-circuit line could be used to replace $Z_L$?

**Solution:** Refer to Fig. P2.41. $z_{in} = Z_{in}/Z_0 = -j2.5 \ \Omega/100 \ \Omega = 0.0 - j0.0025$ which is at point $Z-IN$ and is at 0.004λ on the wavelengths to load scale.

(a) Point $Z-LOAD$ is 0.375λ toward the load from the end of the line. Thus, on the wavelength to load scale, it is at 0.004λ + 0.375λ = 0.379λ.

$$Z_L = z_L Z_0 = (0 + j0.95) \times 100 \ \Omega = j95 \ \Omega.$$