at 0.250\lambda + 0.357\lambda − 0.500\lambda = 0.107\lambda on the WTL scale, and here

\[ Z_L = 0.82 - j0.39. \]

Therefore \( Z_L = z_L Z_0 = (0.82 - j0.39) \times 50 \, \Omega = (41.0 - j19.5) \, \Omega. \)

**Problem 2.44** At an operating frequency of 5 GHz, a 50-\Omega lossless coaxial line with insulating material having a relative permittivity \( \varepsilon_r = 2.25 \) is terminated in an antenna with an impedance \( Z_L = 150 \, \Omega. \) Use the Smith chart to find \( Z_{in}. \) The line length is 30 cm.

**Solution:** To use the Smith chart the line length must be converted into wavelengths. Since \( \beta = 2\pi/\lambda \) and \( \nu_p = \omega/\beta, \)

\[ \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\nu_p} = \frac{c}{\omega} = \frac{3 \times 10^8 \, \text{m/s}}{\sqrt{\varepsilon_r f}} = \frac{3 \times 10^8 \, \text{m/s}}{\sqrt{2.25 \times (5 \times 10^9 \, \text{Hz})}} = 0.04 \, \text{m}. \]

Hence, \( l = \frac{30 \, \text{m}}{0.04 \, \text{m}} = 7.5\lambda. \) Since this is an integral number of half wavelengths,

\[ Z_{in} = Z_L = 150 \, \Omega. \]

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**Section 2-10: Impedance Matching**

**Problem 2.45** A 50-\Omega lossless line 0.6\lambda long is terminated in a load with \( Z_L = (50 + j25) \, \Omega. \) At 0.3\lambda from the load, a resistor with resistance \( R = 30 \, \Omega \) is connected as shown in Fig. 2-43 (P2.45(a)). Use the Smith chart to find \( Z_{in}. \)

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**Figure P2.45:** (a) Circuit for Problem 2.45.
Figure P2.45: (b) Solution of Problem 2.45.

Solution: Refer to Fig. P2.45(b). Since the 30-Ω resistor is in parallel with the input impedance at that point, it is advantageous to convert all quantities to admittances.

\[ z_L = \frac{Z_L}{Z_0} = \frac{(50 + j25) \Omega}{50 \Omega} = 1 + j0.5 \]

and is located at point Z-LOAD. The corresponding normalized load admittance is at point Y-LOAD, which is at 0.394λ on the WTG scale. The input admittance of the load only at the shunt conductor is at 0.394λ + 0.300λ - 0.500λ = 0.194λ and is denoted by point A. It has a value of

\[ y_{inA} = 1.37 + j0.45. \]
CHAPTER 2

The shunt conductance has a normalized conductance
\[ g = \frac{50 \, \Omega}{30 \, \Omega} = 1.67. \]

The normalized admittance of the shunt conductance in parallel with the input admittance of the load is the sum of their admittances:
\[ y_{inR} = g + y_{inA} = 1.67 + 1.37 + j0.45 = 3.04 + j0.45 \]

and is located at point \( B \). On the WTG scale, point \( B \) is at 0.242\( \lambda \). The input admittance of the entire circuit is at 0.242\( \lambda \) + 0.300\( \lambda \) – 0.500\( \lambda \) = 0.042\( \lambda \) and is denoted by point \( Y-IN \). The corresponding normalized input impedance is at \( Z-IN \) and has a value of
\[ z_{in} = 1.9 - j1.4. \]

Thus,
\[ Z_{in} = z_{in} \cdot Z_0 = (1.9 - j1.4) \times 50 \, \Omega = (95 - j70) \, \Omega. \]

Problem 2.46 A 50-\( \Omega \) lossless line is to be matched to an antenna with
\[ Z_L = (75 - j20) \, \Omega \]

using a shorted stub. Use the Smith chart to determine the stub length and the distance between the antenna and the stub.

Solution: Refer to Fig. P2.46(a) and Fig. P2.46(b), which represent two different solutions.
\[ z_L = \frac{Z_L}{Z_0} = \frac{(75 - j20) \, \Omega}{50 \, \Omega} = 1.5 - j0.4 \]

and is located at point \( Z-LOAD \) in both figures. Since it is advantageous to work in admittance coordinates, \( y_L \) is plotted as point \( Y-LOAD \) in both figures. \( Y-LOAD \) is at 0.041\( \lambda \) on the WTG scale.

For the first solution in Fig. P2.46(a), point \( Y-LOAD-IN-1 \) represents the point at which \( g = 1 \) on the SWR circle of the load. \( Y-LOAD-IN-1 \) is at 0.145\( \lambda \) on the WTG scale, so the stub should be located at 0.145\( \lambda \) – 0.041\( \lambda \) = 0.104\( \lambda \) from the load (or some multiple of a half wavelength further). At \( Y-LOAD-IN-1 \), \( b = 0.52 \), so a stub with an input admittance of \( y_{stub} = 0 - j0.52 \) is required. This point is \( Y-STUB-IN-1 \) and is at 0.423\( \lambda \) on the WTG scale. The short circuit admittance
Figure P2.46: (a) First solution to Problem 2.46.

is denoted by point \( Y\text{-}\text{SHIT} \), located at \( 0.250\lambda \). Therefore, the short stub must be \( 0.423\lambda - 0.250\lambda = 0.173\lambda \) long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.46(b), point \( Y\text{-LOAD-IN-2} \) represents the point at which \( g = 1 \) on the SWR circle of the load. \( Y\text{-LOAD-IN-2} \) is at \( 0.355\lambda \) on the WTG scale, so the stub should be located at \( 0.355\lambda - 0.041\lambda = 0.314\lambda \) from the load (or some multiple of a half wavelength further). At \( Y\text{-LOAD-IN-2} \), \( b = -0.52 \), so a stub with an input admittance of \( y_{\text{stub}} = 0 + j0.52 \) is required. This point is \( Y\text{-STUB-IN-2} \) and is at \( 0.077\lambda \) on the WTG scale. The short circuit admittance is denoted by point \( Y\text{-SHIT} \), located at \( 0.250\lambda \). Therefore, the short stub must be \( 0.077\lambda - 0.250\lambda + 0.500\lambda = 0.327\lambda \) long (or some multiple of a half wavelength
Figure P2.46: (b) Second solution to Problem 2.46.

Problem 2.47  Repeat Problem 2.46 for a load with \( Z_L = (100 + j50) \, \Omega \).

Solution: Refer to Fig. P2.47(a) and Fig. P2.47(b), which represent two different solutions.

\[
z_L = \frac{Z_L}{Z_0} = \frac{100 + j50 \, \Omega}{50 \, \Omega} = 2 + j1
\]

and is located at point \( Z\text{-LOAD} \) in both figures. Since it is advantageous to work in admittance coordinates, \( y_L \) is plotted as point \( Y\text{-LOAD} \) in both figures. \( Y\text{-LOAD} \) is at 0.463\( \lambda \) on the WTO scale.
Figure P2.47: (a) First solution to Problem 2.47.

For the first solution in Fig. P2.47(a), point Y-LOAD-IN-1 represents the point at which \( g = 1 \) on the SWR circle of the load. Y-LOAD-IN-1 is at \( 0.162\lambda \) on the WTG scale, so the stub should be located at \( 0.162\lambda - 0.463\lambda + 0.500\lambda = 0.199\lambda \) from the load (or some multiple of a half wavelength further). At Y-LOAD-IN-1, \( b = 1 \), so a stub with an input admittance of \( y_{stub} = 0 - j1 \) is required. This point is Y-STUB-IN-1 and is at \( 0.375\lambda \) on the WTG scale. The short circuit admittance is denoted by point Y-SHT, located at \( 0.250\lambda \). Therefore, the short stub must be \( 0.375\lambda - 0.250\lambda = 0.125\lambda \) long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.47(b), point Y-LOAD-IN-2 represents the point at which \( g = 1 \) on the SWR circle of the load. Y-LOAD-IN-2 is at \( 0.338\lambda \) on the
WTG scale, so the stub should be located at $0.338\lambda - 0.463\lambda + 0.500\lambda = 0.375\lambda$ from the load (or some multiple of a half wavelength further). At $Y$-LOAD-IN-2, $b = -1$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j1$ is required. This point is $Y$-STUB-IN-2 and is at $0.125\lambda$ on the WTG scale. The short circuit admittance is denoted by point $Y$-SHT, located at $0.250\lambda$. Therefore, the short stub must be $0.125\lambda - 0.250\lambda + 0.500\lambda = 0.375\lambda$ long (or some multiple of a half wavelength longer).

**Problem 2.48** Use the Smith chart to find $Z_{\text{in}}$ of the feed line shown in Fig. 2-44 (P2.48(a)). All lines are lossless with $Z_0 = 50 \, \Omega$. 
Figure P2.48: (a) Circuit of Problem 2.48.

Solution: Refer to Fig. P2.48(b).

\[ z_1 = \frac{Z_1}{Z_0} = \frac{50 + j50}{50} \Omega = 1 + j1 \]

and is at point \(Z\)-LOAD-1.

\[ z_2 = \frac{Z_2}{Z_0} = \frac{50 - j50}{50} \Omega = 1 - j1 \]

and is at point \(Z\)-LOAD-2. Since at the junction the lines are in parallel, it is advantageous to solve the problem using admittances. \(y_1\) is point \(Y\)-LOAD-1, which is at 0.412\(\lambda\) on the WTG scale. \(y_2\) is point \(Y\)-LOAD-2, which is at 0.088\(\lambda\) on the WTG scale. Traveling 0.300\(\lambda\) from \(Y\)-LOAD-1 toward the generator one obtains the input admittance for the upper feed line, point \(Y\)-IN-1, with a value of \(1.97 + j1.02\). Since traveling 0.700\(\lambda\) is equivalent to traveling 0.200\(\lambda\) on any transmission line, the input admittance for the lower line feed is found at point \(Y\)-IN-2, which has a value of \(1.97 - j1.02\). The admittance of the two lines together is the sum of their admittances: \(1.97 + j1.02 + 1.97 - j1.02 = 3.94 + j0\) and is denoted \(Y\)-JUNCT. 0.300\(\lambda\) from \(Y\)-JUNCT toward the generator is the input admittance of the entire feed line, point \(Y\)-IN, from which \(Z\)-IN is found.

\[ Z_{in} = z_{in}Z_0 = (1.65 - j1.79) \times 50 \Omega = (82.5 - j89.5) \Omega. \]
Problem 2.49  Repeat Problem 2.48 for the case where all three transmission lines are $\lambda/4$ in length.

Solution: Since the transmission lines are in parallel, it is advantageous to express loads in terms of admittances. In the upper branch, which is a quarter wave line,

$$Y_1_{in} = \frac{Y_0^2}{Y_1} = \frac{Z_1}{Z_0^2},$$
and similarly for the lower branch,

\[ Y_{2\text{ in}} = \frac{Y_0^2}{Y_2} = \frac{Z_2}{Z_0^2}. \]

Thus, the total load at the junction is

\[ Y_{\text{tot}} = Y_{1\text{ in}} + Y_{2\text{ in}} = \frac{Z_1 + Z_2}{Z_0^2}. \]

Therefore, since the common transmission line is also quarter-wave,

\[ Z_m = Z_0^2/Z_{\text{tot}} = Z_0^2Y_{\text{tot}} = Z_1 + Z_2 = (50 + j50) \, \Omega + (50 - j50) \, \Omega = 100 \, \Omega. \]

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Section 2.11: Transients on Transmission Lines

**Problem 2.50** Generate a bounce diagram for the voltage \( V(z, t) \) for a 1-m long lossless line characterized by \( Z_0 = 50 \, \Omega \) and \( u_p = 2c/3 \) (where \( c \) is the velocity of light) if the line is fed by a step voltage applied at \( t = 0 \) by a generator circuit with \( V_g = 60 \, \text{V} \) and \( R_g = 100 \, \Omega \). The line is terminated in a load \( Z_L = 25 \, \Omega \). Use the bounce diagram to plot \( V(i) \) at a point midway along the length of the line from \( t = 0 \) to \( t = 25 \, \text{ns} \).

**Solution:**

\[ \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3}, \]

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = -\frac{1}{3}. \]

From Eq. (2.124b),

\[ V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20 \, \text{V}. \]

Also,

\[ T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5 \, \text{ns}. \]

The bounce diagram is shown in Fig. P2.50(a) and the plot of \( V(t) \) in Fig. P2.50(b).