and similarly for the lower branch,

\[ Y_{2\text{ in}} = \frac{V_0^2}{Y_2} = \frac{Z_2}{Z_0^2}. \]

Thus, the total load at the junction is

\[ Y_{\text{JCT}} = Y_{1\text{ in}} + Y_{2\text{ in}} = \frac{Z_1 + Z_2}{Z_0^2}. \]

Therefore, since the common transmission line is also quarter-wave,

\[ Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{JCT}}} = \frac{Z_0^2 Y_{\text{JCT}}} = \frac{Z_1 + Z_2}{Z_0} = (50 + j50) \Omega + (50 - j50) \Omega = 100 \Omega. \]

Section 2-11: Transients on Transmission Lines

**Problem 2.50** Generate a bounce diagram for the voltage \( V(z,t) \) for a 1-m long lossless line characterized by \( Z_0 = 50 \Omega \) and \( u_p = 2c/3 \) (where \( c \) is the velocity of light) if the line is fed by a step voltage applied at \( t = 0 \) by a generator circuit with \( V_g = 60 \text{ V} \) and \( R_g = 100 \Omega \). The line is terminated in a load \( Z_L = 25 \Omega \). Use the bounce diagram to plot \( V(t) \) at a point midway along the length of the line from \( t = 0 \) to \( t = 25 \text{ ns} \).

**Solution:**

\[ \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3}. \]

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = \frac{-1}{3}. \]

From Eq. (2.124b),

\[ V_i^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20 \text{ V}. \]

Also,

\[ T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5 \text{ ns}. \]

The bounce diagram is shown in Fig. P2.50(a) and the plot of \( V(t) \) in Fig. P2.50(b).
Figure P2.50: (a) Bounce diagram for Problem 2.50.

Figure P2.50: (b) Time response of voltage.
Problem 2.51  Repeat Problem 2.50 for the current $I$ on the line.

Solution:

$$\Gamma_k = \frac{R_k - Z_0}{R_k + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$$

From Eq. (2.124a),

$$I^+ = \frac{V_g}{R_g + Z_0} = \frac{60}{100 + 50} = 0.4 \text{ A}.$$  

The bounce diagram is shown in Fig. P2.51(a) and $I(t)$ in Fig. P2.51(b).

![Bounce diagram](image)

Figure P2.51: (a) Bounce diagram for Problem 2.51.
Problem 2.52  In response to a step voltage, the voltage waveform shown in Fig. 2.45 (P2.52) was observed at the sending end of a lossless transmission line with $R_g = 50 \, \Omega$, $Z_0 = 50 \, \Omega$, and $v_e = 2.25$. Determine (a) the generator voltage, (b) the length of the line, and (c) the load impedance.

Solution:
(a) From the figure, $V_1^+ = 5 \, V$. Applying Eq. (2.124b),

$$V_1^+ = \frac{V_g z_0}{R_g + Z_0} = \frac{V_g Z_0}{Z_0 + Z_0} = \frac{V_g}{2},$$

which gives $V_g = 2V_1^+ = 10 \, V$. 
Answer to Problem 2, HW6

\( \varepsilon_r = 4 \Rightarrow v_p = c/2 \)

\[ 2T = 7 \text{ } \mu\text{s} \]

\[ l = v_p T = 525 \text{ m} \]

\[ Z_L = 0 \Rightarrow \Gamma_L = -1 \]

\[ v(0, 2T) = V_1^+ - V_1^+ - \Gamma_g V_1^+ = 3 \text{ V} \]

\[ - \Gamma_g V_1^+ = 3 \text{ V} \]

\[ V_1^+ = 12 \text{ V} \]

\[ \therefore \Gamma_g = -1/4 \]

\[ R_g/Z_0 = (1+\Gamma_g)/(1-\Gamma_g) = 3/5 \]

\[ R_g = 50 \Omega \times 3/5 = 30 \Omega \]

\[ V_g = V_1^+ (R_g + Z_0)/Z_0 = 12 \text{ V} \times 8/5 = 19.2 \text{ V} \]