We talked about power flow on a transmission line, when we discussed impedance match, reflection, etc. Now let's look at power flow quantitatively.
We have used phasors for convenience, but let's not forget the real world quantities.

\[ v(d, t) = \text{Re} \left( \tilde{V}(d) e^{j\omega t} \right) \]

*Function of d*

For the incident wave

\[ v_{\text{inc}}(d, t) = \text{Re} \left( V_0^+ e^{j\beta d} e^{j\omega t} \right) \]

\[ = \text{Re} \left( |V_0^+| e^{j(\omega t + \beta d + \phi^+)} \right) \]

\[ = |V_0^+| \cos(\omega t + \beta d + \phi^+) \]

\[ P_{\text{inc}} = \frac{v_{\text{inc}}^2}{Z_0} = \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t + \beta d + \phi^+) \]

Incident wave only, \( v_{\text{inc}}/v_{\text{inc}} = Z_0 \)

\[ P_{\text{inc}} = v_{\text{inc}}^2 \]

Talk about equivalent impedances seen by incident, reflected, and combined waves.

For the reflected wave

\[ v_{\text{ref}}(d, t) = \text{Re} \left( \Gamma V_0^+ e^{-j\beta d} e^{j\omega t} \right) \]

\[ = \text{Re} \left[ |\Gamma| |V_0^+| e^{j(\omega t - \beta d + \phi^+ + \phi_r)} \right] \]

\[ = |\Gamma| |V_0^+| \cos(\omega t - \beta d + \phi^+ + \phi_r) \]
\[ P_{af} = -\frac{V_{af}^2}{Z_0} = -|V|^2 \frac{|V_0|^2}{Z_0} \cos^2 (\omega t - \beta d - \phi + \phi_0) \]

Remember, we are talking about a lossless line. \( Z_0 \) is real.

\[ P_{in} (d, t) > 0, \quad P_{af} (d, t) < 0 \]

\[ P_{in, av} = \frac{1}{T} \int_0^T P_{in} (d, t) \, dt = \frac{|V_0|^2}{2Z_0} \]

\[ P_{af, av} = \ldots = -|V|^2 \frac{|V_0|^2}{2Z_0} \]

\[ P_{av} = P_{in, av} + P_{af, av} = \frac{|V_0|^2}{2Z_0} (1 - |V|^2) \]

An easier way:

\[ P_{av} = \frac{1}{2} \text{Re} (\mathbf{V} \cdot \mathbf{I}^*) \]

Put \( \mathbf{V} \) & \( \mathbf{I}^* \) in, and do the algebra, you will get

\[ P_{av} = \frac{|V_0|^2}{2Z_0} (1 - |V|^2) \]

\[ |V|^2 = 1 \quad P_{av} = 0 \quad \text{No power flow} \]

Standing wave. Reactive Load!
$|I| = 0 \Rightarrow P_{av} = \frac{|V_{o}^+|^2}{2Z_0}$

What does this mean?

Actually, $P_{av} = P_{inc, av} = \frac{|V_{o}^+|^2}{2Z_0}$

$P_{ref} = -|I|^2 - \frac{|V_{o}^+|^2}{2Z_0}. = 0$

Actually, these eq's are very intuitive. You don't need to go thru the painful math.

$|I| = 0$ means perfect impedance match.

All traveling wave.