1. A semiconductor sample is as shown in Fig. 1, with a length $L$, a width $W$, and a thickness $d$. Assume the semiconductor is n type, with an electron density $n$. A current $I$ flows through it. The electrons can be regarded as moving at an average velocity $v = \frac{I}{q}$.

The current is expressed as $I = qnvWd$, where $q$ is the elementary charge. Since $I \propto nv$, any electrical measurement alone cannot independently determine $n$ and $v$. To determine $n$, a magnetic field $B = \frac{\pi d}{2W}$ is applied.

1) A voltage, called the Hall voltage $V_H$, is measured between the two sides of the sample, as shown in the figure. Describe the origin of this voltage $V_H$, and determine its direction. Develop an expression for $V_H$ in terms of $v$, $B$ and sample dimensions. Show how to determine $n$ from this experiment (i.e. develop an expression of $n$ in terms of the measured quantity $V_H$ and known parameters $I$, $B$ and sample dimensions.)

2) If the sample is replaced with a p type semiconductor sample of the same dimensions, what would change from the above situation?

At steady state, an electric field is established, canceling the magnetic force: 

$E = vB$

$V_H = EW = vBW$ — expression for $V_H$

$I = qnvWd \Rightarrow n = \frac{I}{qWd}$

$V_H = vBW \Rightarrow v = \frac{V_H}{BW}$

(2) A hole's charge is +q.

The polarity of $V_H$ is opposite — see orange marking in figure.
2. Two infinitely long parallel wires (as in the parallel wire transmission line) of radius \( a \) carry currents of equal magnitude \( I \) but in opposite directions. The distance between the axes of the two wires is \( d \).

(1) What is the force between the two wires, assuming \( d >> a \) (meaning you can ignore the finite radii of the wires)? Derive the expression rather than copy it from the book. Do the two wires attract or repel each other? (20)

(2) What is resultant magnetic field due to the two wires at a point midway between the two wires? (10)

(3) Find the total flux per unit length threading through the area between the two lines. (10)

(1) Consider one wire in the field of the other.

\[
2\pi d B = \mu I \quad \Rightarrow \quad B = \frac{\mu I}{2\pi d}
\]

\[
F' = \frac{F}{l} = I B \frac{l}{l} = I B = \frac{\mu I^2}{2\pi d}
\]

The two wires, carrying opposite currents, repel each other.

(2) \( B_1 = B_2 = \frac{\mu I}{2\pi \left( \frac{d}{2} \right)} = \frac{\mu I}{\pi d} \)

Total field due to the two wires is

\[
B = B_1 + B_2 = \frac{2\mu I}{\pi d}
\]

(3) For any point \( x \) from wire 1.

\[
B_1 = \frac{\mu I}{2\pi x} \quad \text{and} \quad B_2 = \frac{\mu I}{2\pi (d-x)}
\]

\[
B = B_1 + B_2 = \frac{\mu I}{2\pi} \left( \frac{1}{x} + \frac{1}{d-x} \right)
\]
\[ \Phi = l \int_a^{d-a} B \, dx \]
\[ = l \int_a^{d-a} \left( \frac{1}{x} + \frac{1}{d-x} \right) \, dx \left( \frac{\mu I}{2\pi} \right) \]
\[ = 2l \int_a^{d-a} \frac{1}{x} \, dx \left( \frac{\mu I}{2\pi} \right) \]
\[ = 2l \frac{\mu I}{2\pi} \ln \frac{d-a}{a} \]
\[ \frac{\Phi}{l} = \frac{\mu I}{\pi} \ln \frac{d-a}{a} \]

Extra:
\[ L' = \frac{L}{l} = \frac{\Phi}{I} = \frac{\mu I}{\pi} \ln \frac{d-a}{a} \approx \frac{\mu I}{\pi} \ln \frac{d}{a} \]

Notice: The above is just an approximation when \( d \gg 2a \), and thus the current distribution within the wire doesn't matter.

Actually, the current in the two wires interact with each other, therefore the distribution is not uniform.
3. An inductor is formed by winding \( N \) turns of a perfectly conducting wire into a circular loop of radius \( a \). The inductor loop is in the x-y plane with its center at the origin, and connected to a resistor \( R \). In the presence of a magnetic field \( \mathbf{B} = \hat{z} B_0 (\cos \omega t) \), where \( t \) is time. Find

1) The magnetic flux linking a single turn: \( \Phi = B (\pi a^2) \)
   \[ \Phi = \pi a^2 B_0 \cos \omega t \]

2) The induced emf, given that \( N = 10 \), \( B_0 = 0.2 \text{ T} \), \( a = 5 \text{ cm} \), and \( \omega = 10^3 \text{ rad/s} \).
   \[ \text{emf} = -\frac{d\Phi}{dt} = -N \frac{d\Phi}{dt} \]
   \[ = +N \pi a^2 B_0 \omega \sin \omega t \]
   \[ = 10 \pi \times 0.05^2 \times 0.2 \times 10^3 \sin 10^3 t \]

3) \[ I = \frac{\text{emf}}{R} \]