• From now, we ignore the superbar “-” with variables in per unit.

\[
\begin{align*}
\begin{bmatrix}
e_0 \\
e_d \\
e_F \\
e_D = 0 \\
e_Q = 0
\end{bmatrix}
= \begin{bmatrix}
R_a + 3R_n \\
R_F \\
R_D \\
R_a \\
R_Q
\end{bmatrix}
\begin{bmatrix}
-i_0 \\
-i_d \\
i_F \\
i_D \\
i_q
\end{bmatrix}
\end{align*}
\]

\[
\begin{bmatrix}
L_0 + 3L_n \\
L_l + L_{ad} \\
L_{ad} \\
L_{ad} \\
L_{aq} \\
L_{aq}
\end{bmatrix}
+ \begin{bmatrix}
L_l + L_{ad} \\
L_{ad} \\
L_{ad} \\
L_{aq} \\
L_{aq}
\end{bmatrix}
\times d \begin{bmatrix}
-i_0 \\
-i_d \\
i_F \\
i_D \\
i_q
\end{bmatrix} / dt
+ \begin{bmatrix}
-i_0 \\
-i_d \\
i_F \\
i_D \\
i_q
\end{bmatrix}
\]

\[
\begin{bmatrix}
\psi_0 \\
\psi_d \\
\psi_F \\
\psi_D \\
\psi_q \\
\psi_Q
\end{bmatrix}
= \begin{bmatrix}
L_0 \\
L_l + L_{ad} \\
L_{ad} \\
L_{ad} \\
L_{aq} \\
L_{aq}
\end{bmatrix}
\times \begin{bmatrix}
-i_0 \\
-i_d \\
i_F \\
i_D \\
i_q \\
i_Q
\end{bmatrix}
\]

\[
\begin{bmatrix}
\psi_0 \\
\psi_d \\
\psi_F \\
\psi_D \\
\psi_q \\
\psi_Q
\end{bmatrix}
= \begin{bmatrix}
L_0 \\
L_l + L_{ad} \\
L_{ad} \\
L_{ad} \\
L_{aq} \\
L_{aq}
\end{bmatrix}
\times \begin{bmatrix}
-i_0 \\
-i_d \\
i_F \\
i_D \\
i_q \\
i_Q
\end{bmatrix}
\]

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Equivalent Circuits for d- and q-axes

\[ \begin{bmatrix} e_0 \\ e_d \\ e_F \\ e_D = 0 \\ e_q \\ e_Q = 0 \end{bmatrix} = \begin{bmatrix} R_a + 3R_n \\ Ra \\ R_F \\ R_D \\ R_a \\ R_q \end{bmatrix} \begin{bmatrix} -i_0 \\ -i_d \\ i_F \\ i_d \\ -i_q \\ i_Q \end{bmatrix} + \begin{bmatrix} L_0 + 3L_n \\ L_l + L_{ad} \\ L_{ad} \\ L_F \\ M_R \\ L_D \end{bmatrix} \begin{bmatrix} -i_0 \\ -i_d \\ i_F \\ i_d \\ -i_q \\ i_Q \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega_r \psi_q \\ 0 \\ 0 \end{bmatrix} \times \frac{d}{dt} + \begin{bmatrix} \omega_r \psi_d \end{bmatrix} \]

\[ e_d = -\omega_r \psi_q - R_a i_d + p \psi_d \]
\[ = -\omega_r \psi_q - R_a i_d + L_{ad} \times d(-i_d + i_F + i_D)/dt - L_l \times di_d/dt \]

\[ e_F = R_F i_F + p \psi_F \]
\[ = R_F i_F - L_{ad} \times di_d/dt + L_F \times di_f/dt + M_R \times di_D/dt \]

\[ e_D = 0 = R_D i_D + p \psi_D \]
\[ = R_D i_D - L_{ad} \times di_d/dt + M_R \times di_F/dt + L_D \times di_D/dt \]

\[ e_q = \omega_r \psi_d - R_a i_q + p \psi_q \]
\[ = \omega_r \psi_d - R_a i_q + L_{aq} \times d(-i_q + i_Q)/dt - L_l \times di_q/dt \]

\[ e_Q = 0 = R_Q i_Q + p \psi_Q \]
\[ = R_Q i_Q - L_{aq} \times di_q/dt + L_Q \times di_Q/dt \]
Equivalent Circuits for d- and q-axes

\[
e_d = L_{ad} \frac{d(-i_d + i_F + i_D)}{dt} - L_l \frac{di_d}{dt} - \omega_r \psi_q - R_a i_d
\]

\[
e_F = -L_{ad} \frac{di_d}{dt} + L_F \frac{di_F}{dt} + M_R \frac{di_D}{dt} + R_F i_F
\]

\[
e_D = 0 = -L_{ad} \frac{di_d}{dt} + M_R \frac{di_F}{dt} + L_D \frac{di_D}{dt} + R_D i_D
\]

\[
e_q = L_{aq} \frac{d(-i_q + i_Q)}{dt} - L_l \frac{di_q}{dt} + \omega_r \psi_d - R_a i_q
\]

\[
e_Q = 0 = -L_{aq} \frac{di_q}{dt} + L_Q \frac{di_Q}{dt} + R_Q i_Q
\]
Equivalent Circuits with Multiple Damper Windings (e.g. Round-rotor Machines)

**d axis:**
\[
\begin{align*}
L_{fd} & \triangleq L_F - M_R & R_{fd} & \triangleq R_F \\
e_{fd} & \triangleq e_F & \psi_{fd} & \triangleq \psi_F \\
L_{1d} \text{ (or } L_{kd1}) & \triangleq L_D - M_R \\
R_{1d} \text{ (or } R_{kd1}) & \triangleq R_D \\
M_R - L_{ad} & \approx 0
\end{align*}
\]
(named \(L_{fkd1}\) in some literature to model rotor mutual flux leakage, i.e. the flux linking the rotor's field and damper windings but not stator windings)

**q axis:**
\[
\begin{align*}
L_{1q} \text{ (or } L_{kq1}) & \triangleq L_Q - L_{aq} \\
R_{1q} \text{ (or } R_{kq1}) & \triangleq R_Q \\
L_{2q} \text{ (or } L_{kq2}) & \triangleq L_G - L_{aq} \\
R_{2q} \text{ (or } R_{kq2}) & \triangleq R_G
\end{align*}
\]
Example: a model with 3 rotor windings in each of d- and q-axis equivalent circuits

  - The proposed equivalent circuits are expected to contain sufficient details to model all machines
  - Parameters are estimated by frequency response tests

![Diagram of equivalent circuits](image)

Figure S-2. Simulation of Nanticoke Generator Power and Field Current During a Transient Caused By Line Switching.
Synchronous Machine Model DG1S1

Model Descriptions

This model uses parameters in basic form and type 1 saturation model.

Data Format

IBUS, ‘DG1S1’, I, MVA, Xsd, Xsd, Xd,Xd, Rd, Rd, Xq1, Xq1, Rk1, Rk1, Xkq1, Xkq1, Rkq1, Rkq1, Xkq3, Xkq3, Rkq3, Rkq3
H, Kp, α, A_sat, B_sat, vL, vQ, RS

Parameter Descriptions

IBUS - Bus number, name, or generator equipment name of the machine.
I - ID of the machine (may or may not be enclosed in single quotes).
MVA - MVA base of the machine. If not specified (i.e., no value or zero is entered), the MVA base of the matched generator in powerflow data will be used.
Xsd - Unsaturated direct axis mutual reactance in per unit on machine MVA base.
Xq1 - Unsaturated quadrature axis mutual reactance in per unit on machine MVA base.
Xd - Leakage reactance in per unit on machine MVA base.
Rd - Armature resistance in per unit on machine MVA base.
Xq2 - Field winding leakage reactance in per unit on machine MVA base.
Rq2 - Field winding resistance in per unit on machines MVA base.
Xkq1 - First quadrature axis damper winding leakage reactance in per unit on machine MVA base.
Rkq1 - First quadrature axis damper winding resistance in per unit on machine MVA base.
Xkq2 - First direct axis damper winding leakage reactance in per unit on machine MVA base.
Rkq2 - First direct axis damper winding resistance in per unit on machine MVA base.
Xkq3 - Second quadrature axis damper winding leakage reactance in per unit on machine MVA base.
Rkq3 - Second quadrature axis damper winding resistance in per unit on machine MVA base.
Xkq4 - Second direct axis damper winding leakage reactance in per unit on machine MVA base.
Rkq4 - Second direct axis damper winding resistance in per unit on machine MVA base.
Xkq5 - Third quadrature axis damper winding leakage reactance in per unit on machine MVA base.
Rkq5 - Third quadrature axis damper winding resistance in per unit on machine MVA base.
H - Inertia time constant of the machine in MW-second/MVA.
Kp - Damping coefficient in (p.u. torque)/(p.u. speed deviation).
α - This parameter is used only for synchronous motor, as the exponential in the load characteristic of the motor. Tm = K0^α (K0 is determined by TSAT based on the initial condition). It is ignored for generator model.
A_sat - Coefficient in saturation characteristic.
B_sat - Coefficient in saturation characteristic.
vL - Flux linkage on the saturation curve where the Region II characteristic starts.
vQ - Flux linkage on the saturation curve where the Region III characteristic starts.
RS - Ratio of the slopes of air-gap line and the Region III characteristic.
Example 3.1 (Kundur’s book)

A 555 MVA, 24kV, 0.9 p.f., 60Hz, 3 phase, 2 pole synchronous generator has the following inductances an resistances associated with the stator and field windings:

\[
\begin{align*}
\begin{array}{ll}
\text{laa} & = 3.2758 + 0.0458 \cos(2\theta) \quad \text{mH} \\
\text{lab} & = -1.6379 - 0.0458 \cos(2\theta + \pi/3) \quad \text{mH} \\
\text{laF} & = 40.0 \cos\theta \quad \text{mH} \\
\text{LF} & = 576.92 \quad \text{mH} \\
R_a & = 0.0031 \quad \Omega \\
R_F & = 0.0715 \quad \Omega
\end{array}
\end{align*}
\]

a. Determine \(L_d\) and \(L_q\) in H

b. If the stator leakage inductance \(L_l\) is 0.4129 mH, determine \(L_{ad}\) and \(L_{aq}\) in H

c. Using the machine rated values as the base values for the stator quantities, determine the per unit values of the following in the \(L_{ad}\) base reciprocal per unit system (assuming \(L_{ad}=M_F=M_R\) in per unit): \(L_l\), \(L_{ad}\), \(L_{aq}\), \(L_d\), \(L_q\), \(M_F\), \(L_F\), \(L_{fd}\), \(R_a\) and \(R_F\)

Solution:

\[
\begin{align*}
\text{a.} \quad \text{laa} & = L_s + L_m \cos2\theta \quad = 3.2758 + 0.0458 \cos2\theta \quad \text{mH} \\
\text{lab} & = -M_s - L_m \cos(2\theta + \pi/3) \quad = -1.6379 - 0.0458 \cos(2\theta + \pi/3) \quad \text{mH} \\
\text{laF} & = M_F \cos\theta \quad = 40.0 \cos\theta \quad \text{mH} \\
L_d & = L_s + M_s + 3L_m/2 \quad = 3.2758 + 1.6379 + \frac{3}{2} \times 0.0458 \quad = 4.9825 \quad \text{mH} \\
L_q & = L_s + M_s - 3L_m/2 \quad = 3.2758 + 1.6379 + \frac{3}{2} \times 0.0458 \quad = 4.8451 \quad \text{mH}
\end{align*}
\]

\[
\begin{align*}
\text{b.} \quad L_{ad} & = L_d - L_l \quad = 4.9825 - 0.4129 \quad = 4.5696 \quad \text{mH} \\
L_{aq} & = L_q - L_l \quad = 4.4851 - 0.4129 \quad = 4.432 \quad \text{mH}
\end{align*}
\]
c. 3-phase VA\textsubscript{base} = 555 MVA

\[ E\text{\textsubscript{RMS base}} = \frac{24}{\sqrt{3}} \]  
\[ \text{=13.856 kV} \]

\[ e_s \text{\textsubscript{base}} \text{ (peak)} = \sqrt{2} \times E\text{\textsubscript{RMS base}} = \sqrt{2} \times 13.856 \]  
\[ \text{=19.596 kV} \]

\[ I\text{\textsubscript{RMS base}} = \frac{\text{3-phase VA base}}{(3 \times E\text{\textsubscript{RMS base}})} = 555 \times 10^6 / (3 \times 13.86 \times 10^3) = 13.352 \times 10^3 \text{ A} \]

\[ i_s \text{\textsubscript{base}} \text{ (peak)} = \sqrt{2} \times I\text{\textsubscript{RMS base}} = \sqrt{2} \times 13.352 \times 10^3 \]  
\[ \text{=18.8815 \times 10^3 \text{ A}} \]

\[ Z_s \text{\textsubscript{base}} = e_s \text{\textsubscript{base}} / i_s \text{\textsubscript{base}} \]  
\[ =\frac{19.596 \times 10^3}{(18.8815 \times 10^3)} \]  
\[ \text{=1.03784 \Omega} \]

\[ \omega\text{\textsubscript{base}} = 2\pi \times 60 \]  
\[ \text{=377 elec. rad/s} \]

\[ L_s \text{\textsubscript{base}} = Z_s \text{\textsubscript{base}} / \omega\text{\textsubscript{base}} \]  
\[ =\frac{1.03784}{377 \times 10^3} \]  
\[ \text{=2.753 mH} \]

\[ i_F \text{\textsubscript{base}} = L_{ad} / M_F \times i_s \text{\textsubscript{base}} \]  
\[ =\frac{4.5696}{40 \times 18.8815 \times 10^3} \]  
\[ \text{=2158.0 A} \]

\[ e_F \text{\textsubscript{base}} = \frac{\text{3-phase VA base}}{i_F \text{\textsubscript{base}}} = 555 \times 10^6 / 2158 \]  
\[ \text{=257.183 kV} \]

\[ Z_F \text{\textsubscript{base}} = e_F \text{\textsubscript{base}} / i_F \text{\textsubscript{base}} \]  
\[ =\frac{257.183 \times 10^3}{2158} \]  
\[ \text{=119.18 \Omega} \]

\[ L_F \text{\textsubscript{base}} = Z_F \text{\textsubscript{base}} / \omega\text{\textsubscript{base}} \]  
\[ =\frac{119.18 \times 10^3}{377} \]  
\[ \text{=316.12 mH} \]

\[ M_F \text{\textsubscript{base}} = L_{S \text{\textsubscript{base}}} \times i_{S \text{\textsubscript{base}}} / i_F \text{\textsubscript{base}} \]  
\[ =\frac{2.753 \times 188815}{2158} \]  
\[ \text{=241 mH} \]

Then per unit values are:

\[ L_l = \frac{0.4129}{2.753} = 0.15 \text{ pu} \]

\[ L_{ad} = \frac{4.5696}{2.753} = 1.66 \text{ pu} \]

\[ L_{aq} = \frac{4.432}{2.753} = 1.61 \text{ pu} \]

\[ L_d = L_l + L_{ad} = 0.15 + 1.66 = 1.81 \text{ pu} \]

\[ L_q = L_l + L_{aq} = 0.15 + 1.61 = 1.76 \text{ pu} \]

\[ M_F = \frac{M_F}{M_{F \text{\textsubscript{base}}}} = \frac{400}{241} = 1.66 \text{ pu} \]

\[ L_f = L_F - M_R = L_F - L_{ad} = 1.825 - 1.66 = 0.165 \text{ pu} \]

\[ R_a = \frac{0.0031}{1.03784} = 0.003 \text{ pu} \]

\[ R_F = \frac{0.0715}{119.18} = 0.0006 \text{ pu} \]
Steady-state Analysis

All flux linkages, voltages and currents are constant:

\[ p \psi_{1d} = 0 \rightarrow R_{1d} i_{1d} = 0 \Rightarrow i_{1d} = 0 \]
\[ p \psi_{1q} = 0 \rightarrow R_{1q} i_{1q} = 0 \Rightarrow i_{1q} = 0 \]

(Damper winding currents are all zero due to no change in the magnetic field)

\[ \psi_d = -(L_l + L_{ad}) i_d + L_{ad} i_{fd} + L_{ad} i_{1d} = -L_d i_d + L_{ad} i_{fd} \]
\[ \psi_q = -(L_l + L_{aq}) i_q + L_{aq} i_{1q} = -L_q i_q \]

\[ p \psi_d = 0 \rightarrow e_d = -\omega_r \psi_d - R_a i_d = \omega_r L_q i_q - R_a i_d \]
\[ p \psi_q = 0 \rightarrow e_q = \omega_r \psi_d - R_a i_q = \omega_r L_d i_d + \omega_r L_{ad} i_{fd} - R_a i_q \]
\[ p \psi_{fd} = 0 \rightarrow e_{fd} = R_{fd} i_{fd} \]

\( \omega_r = 1 \) and \( L = X \) in p.u.

\[ e_d = X_q i_q - R_a i_d \]
\[ e_q = -X_d i_d + X_{ad} i_{fd} - R_a i_q \]

\[ i_{fd} = (e_q + R_a i_q + X_d i_d)/X_{ad} \]

Can we have a single equivalent circuit representing both \( d \) and \( q \) axes circuits under balanced steady-state conditions?
\[ e_a = E_m \cos(\omega_s t + \alpha) \]
\[ e_b = E_m \cos(\omega_s t + \alpha - 2\pi/3) \]
\[ e_c = E_m \cos(\omega_s t + \alpha + 2\pi/3) \]

\[
P = \frac{2}{3} \begin{bmatrix}
cos\theta & cos\left(\theta - \frac{2\pi}{3}\right) & cos\left(\theta + \frac{2\pi}{3}\right) \\
-sin\theta & -sin\left(\theta - \frac{2\pi}{3}\right) & -sin\left(\theta + \frac{2\pi}{3}\right) \\
1/2 & 1/2 & 1/2
\end{bmatrix}
\]
\[ \theta = \omega t + \theta_0 \]

\[ e_d = E_m \cos(\alpha - \theta_0) = E_t \cos(\alpha - \theta_0) \]
\[ e_q = E_m \sin(\alpha - \theta_0) = E_t \sin(\alpha - \theta_0) \]

where \( E_t \) is the per unit RMS value of the armature terminal voltage, which equals the peak value \( E_m \) in per unit.

\[
\tilde{E}_t = X_q i_q - R_a i_d - j X_d i_d + j X_d i_{fd} - j R_a i_q \\
= -R_a \tilde{I}_t + X_q i_q - j X_d i_d + j X_d i_{fd}
\]

\[ \tilde{E}_t = \tilde{E} - R_a \tilde{I}_t - j X_S \tilde{I}_t \]

\[ X_S = X_q \]

\[ \tilde{E} = j [X_{ad}i_{fd} - (X_d - X_q)i_d] \overset{\text{def}}{=} \tilde{E}_q \]

\[ \tilde{E}_t = \tilde{E}_q - R_a \tilde{I}_t - j X_q \tilde{I}_t \]
Steady-state equivalent circuit

- Under no-load or open-circuit conditions, \( i_d = i_q = 0 \).
  \[
  \tilde{E}_t = \tilde{E}_q = j X_{ad} i_{fd} \\
  \delta_i = 0 \quad \text{(load angle)}
  \]

- For round rotor machines or machines with neglected saliency

  \[
  X_d = X_q = X_s \quad \text{(synchronous reactance)} \\
  \tilde{E}_t = \tilde{E}_q - (R_a + j X_s) \tilde{I}_t \\
  E_q = X_{ad} i_{fd}
  \]
Computing steady-state values

**Active and reactive power**

\[
S = \tilde{E}_t \tilde{I}_t^* \\
= (e_d^e + je_q^e)(i_d^e - ji_q^e) \\
= (e_d i_d + e_q i_q) + j(e_q i_d - e_d i_q)
\]

\[
P_t = e_d i_d + e_q i_q \\
Q_t = e_q i_d - e_d i_q
\]

Steady-state torque is given by

\[
T_e = \psi_d i_q - \psi_q i_d \\
= (e_d i_d + e_q i_q) + R_a (i_d^2 + i_q^2) \\
= P_t + R_a I_t^2
\]

Normally, terminal active power \( P_t \), reactive power \( Q_t \), and magnitude of voltage \( E_t \) are specified. The corresponding terminal current \( I_t \) and power factor angle \( \phi \) are computed as follows:

\[
I_t = \frac{\sqrt{P_t^2 + Q_t^2}}{E_t} \\
\phi = \cos^{-1}\left(\frac{P_t}{E_t I_t}\right)
\]

The next step is to compute the internal rotor angle \( \delta_i \). Since \( \tilde{E}_q \) lies along the \( q \)-axis, as illustrated in Figure 3.23, the internal angle is given by

\[
\delta_i = \tan^{-1}\left(\frac{X_q I_t \cos \phi - R_a I_t \sin \phi}{E_t + R_a I_t \cos \phi + X_q I_t \sin \phi}\right)
\]

![Figure 3.23 Steady-state phasor diagram](image-url)
Representation of Magnetic Saturation

• Assumptions for stability studies
  – The leakage fluxes are not significantly affected by saturation of the iron portion, so $L_l$ is constant and only $L_{ad}$ and $L_{aq}$ saturate in equivalent circuits

$$L_{adu} \rightarrow L_{ad} \quad L_{aqu} \rightarrow L_{aq}$$

$L_{adu}$ and $L_{aqu}$ denote the unsaturated values

– The leakage fluxes do not contribute to the iron saturation. Thus, saturation is determined only by the air-gap flux linkage

$$\psi_{at} = \sqrt{\psi_{ad}^2 + \psi_{aq}^2}$$

– The saturation relationship $\psi_{at} \sim i_{fd}$ or $\psi_{at} \sim MMF$ under loaded conditions is the same as under no-load conditions. This allows the open-circuit characteristic (OCC) to be considered

– No magnetic coupling between $d$ and $q$ axes, such that their effects of saturation can be modeled individually
Air-gap flux and voltage

\[ \tilde{\psi}_{at} = \psi_{ad} + j\psi_{aq} \]
\[ = \psi_d + L_i i_d + j\psi_q + jL_i i_q \]
\[ = e_q + R_a i_q - j e_d - jR_a i_d + L_i \tilde{I}_t \]
\[ = -j \tilde{E}_t - jR_a \tilde{I}_t + L_i \tilde{I}_t \]
\[ = -j [\tilde{E}_t + (R_a + jX_l) \tilde{I}_t] \Delta - j \tilde{E}_{at} \]

For steady state conditions:

\[ e_d = -\psi_q - R_a i_d \rightarrow \psi_q = -e_d - R_a i_d \]
\[ e_q = \psi_d - R_a i_q \rightarrow \psi_d = e_q + R_a i_q \]

\[ \psi_{at} = E_{at} = |\tilde{E}_t + (R_a + jX_l) \tilde{I}_t| \]
Estimating Saturation Factors $K_{sd}$ and $K_{sd}$

\[ L_{ad} = K_{sd} L_{adu} \quad L_{aq} = K_{sq} L_{aqu} \]

- **Salient pole machines**
  - The path for $q$-axis flux is largely in air, so $L_{aq}$ does not vary significantly with saturation of the iron portion of the path
  - Assume $K_{sq} = 1.0$ for all loading conditions.

- **Round rotor machines**
  - There is a magnetic saturation in both axes, but $q$ axis saturation data is usually not available
  - Assume $K_{sq} = K_{sd}$

- Thus, we focus on estimating $K_{sd}$

\[ K_{sd} = \frac{L_{ad}}{L_{adu}} = \frac{\psi_{at}}{\psi_{at0}} = \frac{\psi_{at}}{(\psi_{at} + \psi_I)} = \frac{I_0}{I} \]

(See Kundur’s Example 3.3 on Estimating $K_{sd}$ for different loading conditions)
Estimating the Saturation Characteristic

- Modeled by 3 approximate functions
  - Segment I ($\psi_{at}<\psi_{T1}$):
    \[ \psi_I = \psi_{at0} - \psi_{at} = 0 \]
  - Segment II ($\psi_{T1}<\psi_{at}<\psi_{T2}$):
    \[ \psi_I = A_{sat} e^{B_{sat}} (\psi_{at} - \psi_{T1}) \]
  - Segment III ($\psi_{at}>\psi_{T2}$):
    \[ \psi_I = \psi_{G2} + L_{ratio} (\psi_{at} - \psi_{T2}) - \psi_{at} \]

- Note that segments I and II are not connected since when $\psi_{at} = \psi_{T1}$, $\psi_I = A_{sat} \neq 0$ (usually small)

- Assume that segments II and III are connected at $\psi_{at} = \psi_{T2}$ to solve $\psi_{G2}$
  \[ A_{sat} e^{B_{sat}} (\psi_{T2} - \psi_{T1}) = \psi_{G2} + L_{ratio} (\psi_{T2} - \psi_{T2}) - \psi_{T2} \]
  \[ \psi_{G2} = \psi_{T2} + A_{sat} e^{B_{sat}} (\psi_{T2} - \psi_{T1}) \]

We need to know $A_{sat}$, $B_{sat}$, $\psi_{T1}$, $\psi_{T2}$ and $L_{ratio}$
Synchronous Machine Model DG1S1

Model Descriptions
This model uses parameters in basic form and type 1 saturation model.

Data Format

IBUS, 'DG1S1', I, MVA, Xad, Xaq, XI, Ra, Xdl, Xdq, Xd1, Xq1, Xd2, Xq2, Xd3, Xq3, Rk1, Rk2, Rk3, H, X5, α, δ<sub>a</sub>, B<sub>b</sub>, Ψ<sub>a</sub>, Ψ<sub>b</sub>, RS

Parameter Descriptions

- IBUS - Bus number, name, or generator equipment name of the machine.
- I - ID of the machine (may or may not be enclosed in single quotes).
- MVA - MVA base of the machine. If not specified (i.e., no value or zero is entered), the MVA base of the matched generator in powerflow data will be used.
- Xad - Unsaturated direct axis mutual reactance in per unit on machine MVA base.
- Xaq - Unsaturated quadrature axis mutual reactance in per unit on machine MVA base.
- XI - Leakage reactance in per unit on machine MVA base.
- Ra - Armature resistance in per unit on machine MVA base.
- Xdl - Field winding leakage reactance in per unit on machine MVA base.
- Rdl - Field winding resistance in per unit on machines MVA base.
- Xk1 - First quadrature axis damper winding leakage reactance in per unit on machine MVA base.
- Rk1 - First quadrature axis damper winding resistance in per unit on machine MVA base.
- Xk1 - First direct axis damper winding leakage reactance in per unit on machine MVA base.
- Rk1 - First direct axis damper winding resistance in per unit on machine MVA base.
- Xk2 - Second quadrature axis damper winding leakage reactance in per unit on machine MVA base.
- Rk2 - Second quadrature axis damper winding resistance in per unit on machine MVA base.
- Xk2 - Second direct axis damper winding leakage reactance in per unit on machine MVA base.
- Rk2 - Second direct axis damper winding resistance in per unit on machine MVA base.
- Xk3 - Third quadrature axis damper winding leakage reactance in per unit on machine MVA base.
- Rk3 - Third quadrature axis damper winding resistance in per unit on machine MVA base.
- H - Inertia time constant of the machine in MW-second/MVA.
- KD - Damping coefficient in (p.u. torque)/(p.u. speed deviation).
- α - This parameter is used only for synchronous motor, as the exponential in the load characteristic of the motor: \( T_m = K_0 \alpha^2 \) (K is determined by TSAT based on the initial condition). It is ignored for generator model.

- A<sub>s</sub> - Coefficient in saturation characteristic.
- B<sub>s</sub> - Coefficient in saturation characteristic.
- Ψ<sub>a</sub> - Flux linkage on the saturation curve where the Region II characteristic starts.
- Ψ<sub>b</sub> - Flux linkage on the saturation curve where the Region III characteristic starts.
- RS - Ratio of the slopes of air-gap line and the Region III characteristic.
1.20 GENSAE

Salient Pole Generator Model (Exponential Saturation on Both Axes)

This model is located at system bus #_________ IBUS.
Machine identifier #_________ ID, and CONes starting with #_________ J, and STATEs starting with #_________ K.
The machine MVA is _________ for each of _________ units = _________ MBASE.
ZSOURCE for this machine is _________ on the above MBASE.

<table>
<thead>
<tr>
<th>CONs</th>
<th>#</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td></td>
<td>T_{dc} (&gt;0) (sec)</td>
<td></td>
</tr>
<tr>
<td>J+1</td>
<td></td>
<td>T_{dc} (&lt;0) (sec)</td>
<td></td>
</tr>
<tr>
<td>J+2</td>
<td></td>
<td>T_{sn} (&gt;0) (sec)</td>
<td></td>
</tr>
<tr>
<td>J+3</td>
<td></td>
<td>H, inertia</td>
<td></td>
</tr>
<tr>
<td>J+4</td>
<td></td>
<td>D, speed damping</td>
<td></td>
</tr>
<tr>
<td>J+5</td>
<td></td>
<td>X_d</td>
<td></td>
</tr>
<tr>
<td>J+6</td>
<td></td>
<td>X_q</td>
<td></td>
</tr>
<tr>
<td>J+7</td>
<td></td>
<td>X'_{d}</td>
<td></td>
</tr>
<tr>
<td>J+8</td>
<td></td>
<td>X'_{q}</td>
<td></td>
</tr>
<tr>
<td>J+9</td>
<td></td>
<td>X_h</td>
<td></td>
</tr>
<tr>
<td>J+10</td>
<td></td>
<td>S(1.0)</td>
<td></td>
</tr>
<tr>
<td>J+11</td>
<td></td>
<td>S(1.2)</td>
<td></td>
</tr>
</tbody>
</table>

Note: X_d, X_q, X'_{d}, X'_{q}, X_h, H, and D are in pu, machine MVA base.
X'_{q} must be equal to X'_{d}.

<table>
<thead>
<tr>
<th>STATES</th>
<th>#</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td></td>
<td>E_{q}</td>
</tr>
<tr>
<td>K+1</td>
<td></td>
<td>V'_{d}</td>
</tr>
<tr>
<td>K+2</td>
<td></td>
<td>V_{d}</td>
</tr>
<tr>
<td>K+3</td>
<td></td>
<td>Δ speed (pu)</td>
</tr>
<tr>
<td>K+4</td>
<td></td>
<td>Angle (radians)</td>
</tr>
</tbody>
</table>

IBUS, 'GENSAE', ID, CON(J) to CON(J+11) /

Estimating saturation as an exponential function.

\[
S_{G1} = \frac{i_{F1} - i_{F0}}{i_{F0}}
\]

\[
S_{G2} = \frac{i_{F3} - i_{F2}}{i_{F2}} = \frac{i_{F3} - 1.2i_{F0}}{1.2i_{F0}}
\]

\[
ψ_l = A_{sat}e^{B_{sat}(ψ_{at} - ψ_{T1})} \sim S_G = A_Ge^{B_G(V_l - 0.8)}
\]

\[
A_G = \frac{S_{G1}^2}{1.2S_{G2}} \quad B_G = 5\ln(1.2S_{G2}/S_{G1})
\]
Example 3.2 in Kundur's Book

The following are the parameters in per unit on machine rating of a 555 MVA, 24 kV, 0.9 p.f., 60 Hz, 3600 RPM turbine-generator\(^1\):

\[
\begin{align*}
L_{ad} &= 1.66 & L_{aq} &= 1.61 & L_t &= 0.15 & R_a &= 0.003 \\
L_{fd} &= 0.165 & R_{fd} &= 0.0006 & L_{1d} &= 0.1713 & R_{1d} &= 0.0284 \\
L_{1q} &= 0.7252 & R_{1q} &= 0.00619 & L_{2q} &= 0.125 & R_{2q} &= 0.02368
\end{align*}
\]

\(M_R \rightarrow L_{fkd}\) is assumed to be equal to \(L_{ad}\).

(a) When the generator is delivering rated MVA at 0.9 p.f. (lag) and rated terminal voltage, compute the following:

(i) Internal angle \(\delta_i\) in electrical degrees

(ii) Per unit values of \(e_d, e_q, i_d, i_q, i_{1d}, i_{1q}, i_{2q}, e_{fd}, \psi_{fd}, \psi_{1d}, \psi_{1q}, \psi_{2q}\)

(iii) Air-gap torque \(T_e\) in per unit and in newton-meters

Assume that the effect of magnetic saturation at the given operating condition is to reduce \(L_{ad}\) and \(L_{aq}\) to 83.5% of the values given above.

(b) Compute the internal angle \(\delta_i\) and field current \(i_{fd}\) for the above operating condition, using the approximate equivalent circuit of Figure 3.22. Neglect \(R_a\).

Solution

(a) With the given operating condition, the per unit values of terminal quantities are

\[
\begin{align*}
P &= 0.9, & Q &= 0.436, & E_t &= 1.0, & I_t &= 1.0, & \phi &= 25.84^\circ
\end{align*}
\]
The saturated values of the inductances are

\[ L_{ad} = 0.835 \times 1.66 = 1.386 \]
\[ L_{aq} = 0.835 \times 1.61 = 1.344 \]
\[ L_{d} = L_{ad} + L_{l} = 1.386 + 0.15 = 1.536 \]
\[ L_{q} = L_{aq} + L_{l} = 1.344 + 0.15 = 1.494 \]

Following the procedure outlined in Section 3.6.5,

(i) \[ \delta_i = \tan^{-1}\left( \frac{1.494 \times 1.0 \times 0.9 - 0.003 \times 1.0 \times 0.436}{1.0 + 0.003 \times 1.0 \times 0.9 + 1.494 \times 1.0 \times 0.436} \right) \]
\[ = \tan^{-1}(0.812) = 39.1 \text{ electrical degrees} \]

(ii) \[ e_d = E_i \sin \delta_i = 1.0 \sin 39.1 = 0.631 \text{ pu} \]
\[ e_q = E_i \cos \delta_i = 1.0 \cos 39.1 = 0.776 \text{ pu} \]
\[ i_d = I_i \sin (\delta_i + \phi) = 1.0 \sin(39.1 + 25.84) = 0.906 \text{ pu} \]
\[ i_q = I_i \cos (\delta_i + \phi) = 1.0 \cos(39.1 + 25.84) = 0.423 \text{ pu} \]
\[ i_{fd} = \frac{e_q + R_a i_q - X_d i_d}{X_{ad}} \]
\[ = \frac{0.776 + 0.003 \times 0.423 + 1.536 \times 0.906}{1.386} \]
\[ = 1.565 \text{ pu} \]

\[ e_{fd} = R_{fd} i_{fd} = 0.0006 \times 1.565 \]
\[ = 0.000939 \text{ pu} \]
\[ \psi_{fd} = (L_{ad} + L_{fd}) i_{fd} - L_{ad} i_d \]
\[ = (1.386 + 0.165) \times 1.565 - 1.386 \times 0.907 \]
\[ = 1.17 \text{ pu} \]
\[ \psi_{1d} = L_{ad} (i_{fd} - i_d) \]
\[ = 1.386 \times (1.565 - 0.906) \]
\[ = 0.913 \text{ pu} \]
\[ \psi_{1q} = \psi_{2q} = -L_{aq} i_q = -1.344 \times 0.423 \]
\[ = -0.569 \text{ pu} \]

Under steady state,

\[ i_{1d} = i_{1q} = i_{2q} = 0 \]
(iii) Air-gap torque

\[ T_e = P_t + \frac{i_t^2 R_a}{\omega_m} \]

\[ = 0.9 + 1.0^2 \times 0.003 \]

\[ = 0.903 \text{ pu} \]

\[ T_{base} = \frac{\text{MVA}_{base} \times 10^6}{\omega_m \text{base}} \]

\[ = \frac{555 \times 10^6}{2\pi \times 60} = 1.472 \times 10^6 \text{ N\cdotm} \]

Therefore,

\[ T_e = 0.903 \times 1.472 \times 10^6 \]

\[ = 1.329 \times 10^6 \text{ N\cdotm} \]

(b) Using the saturated value of \( X_{ad} \),

\[ E_q = X_{ad} i_{fd} = 1.386 i_{fd} \]

\[ X_s = X_{ad} + X_I = 1.386 + 0.15 = 1.536 \]

From the equivalent circuit of Figure 3.22, with \( \vec{E}_i \) as reference phasor,

\[ \vec{E}_q = \vec{E}_i + jX_s \vec{I}_t \]

\[ = 1.0 + j1.536(0.9 - j0.436) \]

\[ = 1.670 + j1.382 \]

\[ = 2.17/39.6^\circ \text{ pu} \]

\[ \delta_i = 39.6^\circ \approx 39.1^\circ \]

Therefore,

\[ i_{fd} = \frac{E_q}{X_{ad}} = \frac{2.17}{1.386} = 1.566 \text{ pu} \approx 1.565 \text{pu} \]
Sub-transient and Transient Analysis

- Following a disturbance, currents are induced in rotor circuits. Some of these induced rotor currents decay more rapidly than others.
  - **Sub-transient parameters**: influencing rapidly decaying (cycles) components
  - **Transient parameters**: influencing the slowly decaying (seconds) components
  - **Synchronous parameters**: influencing sustained (steady state) components

![Diagram showing sub-transient, transient, and steady-state periods](image)

**Figure 3.27** Fundamental frequency component of armature current
## Transient and sub-transient parameters

(a) $d$-axis equivalent circuit

(b) $q$-axis equivalent circuit

<table>
<thead>
<tr>
<th>Considered rotor windings</th>
<th>$R_{1d} \gg R_{fd}$</th>
<th>$L_{fd}/R_{fd} \gg L_{1d}/R_{1d}$</th>
<th>$L_{1q}/R_{1q} \gg L_{2q}/R_{2q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>d axis circuit</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Only field Winding</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Add the damper winding</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>q axis circuit</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Only 1$^{st}$ damper winding</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Add the 2$^{nd}$ damper winding</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Time constant

- **(open circuit)**
  - $T'_{d0} = \frac{L_{ad} + L_{fd}}{R_{fd}}$
  - $T''_{d0} = \frac{L_{ad} + L_{fd} + L_{1d}}{R_{1d}}$
  - $T'_{q0} = \frac{L_{aq} + L_{1q}}{R_{1q}}$
  - $T''_{q0} = \frac{L_{aq} + L_{1q} + L_{2q}}{R_{2q}}$

- **(short circuit)**
  - $T'_{d} = \frac{L_{ad} / L_{l} + L_{fd}}{R_{fd}}$
  - $T''_{d} = \frac{L_{ad} / L_{l} + L_{1d}}{R_{1d}}$
  - $T'_{q} = \frac{L_{aq} / L_{1q} + L_{1q}}{R_{1q}}$
  - $T''_{q} = \frac{L_{aq} / L_{1q} + L_{2q}}{R_{2q}}$

### Inductance (Reactance)

- $L'_{d} = L_{l} + L_{ad} / L_{fd}$
  - $0.30$ (pu)
- $L''_{d} = L_{l} + L_{ad} / L_{fd} / L_{1d}$
  - $0.23$ (pu)
- $L'_{q} = L_{l} + L_{aq} / L_{1q}$
  - $0.65$ (pu)
- $L''_{q} = L_{l} + L_{aq} / L_{1q} / L_{2q}$
  - $0.25$ (pu)

Based on the parameters of Example 3.2
Identifying Terminal Quantities

- Apply Laplace transform to the incremental forms of flux linkage equations (see Kundur’s Chapter 4.1 for details):

\[
\Delta \psi_d(s) = G(s) \Delta e_{fd}(s) - L_d(s) \Delta i_d(s)
\]

\[
\Delta \psi_q(s) = -L_q(s) \Delta i_q(s)
\]

\[
L_d(s) = L_d \frac{1+(T_4+T_5)s+T_4T_6s^2}{1+(T_1+T_2)s+T_1T_3s^2} \approx L_d \frac{(1+sT_4)(1+sT_6)}{(1+sT_1)(1+sT_3)}
\]

\[
= L_d \frac{(1+sT_d')(1+sT_d'')}{(1+sT_{d0}')(1+sT_{d0}'')}
\]

\[
G(s) = G_0 \frac{(1+sT_{kd})}{1+(T_1+T_2)s+T_1T_3s^2} \approx G_0 \frac{(1+sT_{kd})}{(1+sT_1)(1+sT_3)}
\]

\[
= G_0 \frac{(1+sT_{kd})}{(1+sT_{d0}')(1+sT_{d0}'')}
\]

Since \(R_{1d} \gg R_{fd}\)

\[
T_2, T_3 \ll T_1, \quad T_5, T_6 \ll T_4
\]

\[
T_1 + T_2 \approx T_1 + T_3, \quad T_4 + T_5 \approx T_4 + T_6
\]

\[
(1 + sT_d')(1 + sT_d'') \approx (1 + sT_4)(1 + sT_6)
\]

\[
(1 + sT_{d0}')(1 + sT_{d0}'') \approx (1 + sT_1)(1 + sT_3)
\]

(Classical Expressions)
### Table 4.1
Expressions for Standard Parameters of Synchronous Machine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Classical Expression</th>
<th>Accurate Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{d0}'$</td>
<td>$T_1$</td>
<td>$T_1 + T_2$</td>
</tr>
<tr>
<td>$T_d'$</td>
<td>$T_4$</td>
<td>$T_4 + T_5$</td>
</tr>
<tr>
<td>$T_{d0}''$</td>
<td>$T_3$</td>
<td>$T_3/[T_1/(T_1+T_2)]$</td>
</tr>
<tr>
<td>$T_d''$</td>
<td>$T_6$</td>
<td>$T_6/[T_4/(T_4+T_5)]$</td>
</tr>
<tr>
<td>$L_d'$</td>
<td>$L_d(T_4/T_1)$</td>
<td>$L_d(T_4+T_5)/(T_1+T_2)$</td>
</tr>
<tr>
<td>$L_d''$</td>
<td>$L_d(T_4T_6)/(T_1T_3)$</td>
<td>$L_d(T_4T_6)/(T_1T_3)$</td>
</tr>
</tbody>
</table>

with

$$T_1 = \frac{L_{ad} + L_{fjd}}{R_{fjd}}$$

$$T_2 = \frac{L_{ad} + L_{1d}}{R_{1d}}$$

$$T_3 = \frac{1}{R_{1d}} \left( L_{1d} + \frac{L_{ad}L_{fjd}}{L_{ad} + L_{fjd}} \right)$$

$$T_4 = \frac{1}{R_{fjd}} \left( L_{fjd} + \frac{L_{ad}L_l}{L_{ad} + L_l} \right)$$

$$T_5 = \frac{1}{R_{1d}} \left( L_{1d} + \frac{L_{ad}L_l}{L_{ad} + L_l} \right)$$

$$T_6 = \frac{1}{R_{1d}} \left( L_{1d} + \frac{L_{ad}L_lL_{fjd}}{L_{ad}L_{fjd} + L_{ad}L_{fjd} + L_{fjd}L_l} \right)$$
## Summary of Transient and Sub-transient Parameters (Classical Expressions)

<table>
<thead>
<tr>
<th></th>
<th>d axis circuit</th>
<th>q axis circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OC Time Constant</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T'<em>d = 0 = \frac{L</em>{ad}}{R_{fd}} + L_{fd}$</td>
<td>$T''<em>d = 0 = \frac{L</em>{ad} + L_{1d}}{R_{1d}}$</td>
<td>$T'<em>q = 0 = \frac{L</em>{aq} + L_{1q}}{R_{1q}}$</td>
</tr>
<tr>
<td><strong>SC Time Constant</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T'<em>d = \frac{L</em>{ad} + L_{fd}}{L_l + L_{fd}}$</td>
<td>$T''<em>d = \frac{L</em>{ad} + L_{fd} + L_{1d}}{L_l + L_{1d}}$</td>
<td>$T'<em>q = \frac{L</em>{aq} + L_{1q}}{R_{1q}}$</td>
</tr>
<tr>
<td><strong>Inductance (Reactance)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L'<em>d = L_l + \frac{L</em>{ad} + L_{fd}}{L_{fd}}$</td>
<td>$L''<em>d = L_l + \frac{L</em>{ad} + L_{fd}}{L_{fd} + L_{1d}}$</td>
<td>$L'<em>q = L_l + \frac{L</em>{aq} + L_{1q}}{L_{1q}}$</td>
</tr>
</tbody>
</table>

- Note: time constants are all in p.u. To be converted to seconds, they have to be multiplied by $t_{base} = 1/\omega_{base}$ (i.e. 1/377 for 60Hz).
Synchronous, Transient and Sub-transient Inductances

\[ L_d(s) = L_d \frac{(1 + sT'_d)(1 + sT''_d)}{(1 + sT'_{d0})(1 + sT''_{d0})} \]

- Under steady-state condition: \( s=0 \) \( (t\rightarrow \infty) \)
  \[ L_d(0) = L_d \]
  (d-axis synchronous inductance)

- During a rapid transient: \( s\rightarrow \infty \)
  \[ L''_d = L_d(\infty) = L_d \frac{T'_dT''_d}{T'_{d0}T''_{d0}} = L_l + \frac{L_{ad}L_{fd}L_{1d}}{L_{ad}L_{fd} + L_{ad}L_{1d} + L_{fd}L_{1d}} \]
  (d-axis sub-transient inductance)

- Without the damper winding: \( s>>1/T'_d \) and \( 1/T'_{d0} \) but \( << 1/T''_d \) and \( 1/T''_{d0} \)
  \[ L'_d = L_d(\infty) = L_d \frac{T'_d}{T'_{d0}} = L_l + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}} \]
  (d-axis transient inductance)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hydraulic Units</th>
<th>Thermal Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous Reactance</td>
<td>$X_d$</td>
<td>0.6 - 1.5</td>
</tr>
<tr>
<td></td>
<td>$X_q$</td>
<td>0.4 - 1.0</td>
</tr>
<tr>
<td>Transient Reactance</td>
<td>$X'_d$</td>
<td>0.2 - 0.5</td>
</tr>
<tr>
<td></td>
<td>$X'_q$</td>
<td>-</td>
</tr>
<tr>
<td>Subtransient Reactance</td>
<td>$X''_d$</td>
<td>0.15 - 0.35</td>
</tr>
<tr>
<td></td>
<td>$X''_q$</td>
<td>0.2 - 0.45</td>
</tr>
<tr>
<td>Transient OC Time Constant</td>
<td>$T'_d0$</td>
<td>1.5 - 9.0 s</td>
</tr>
<tr>
<td></td>
<td>$T'_q0$</td>
<td>-</td>
</tr>
<tr>
<td>Subtransient OC Time Constant</td>
<td>$T''_d0$</td>
<td>0.01 - 0.05 s</td>
</tr>
<tr>
<td></td>
<td>$T''_q0$</td>
<td>0.01 - 0.09 s</td>
</tr>
<tr>
<td>Stator Leakage Inductance</td>
<td>$X_l$</td>
<td>0.1 - 0.2</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>$R_a$</td>
<td>0.002 - 0.02</td>
</tr>
</tbody>
</table>

1. Reactance values are in per unit with stator base values equal to the corresponding machine rated values.
2. Time constants are in seconds.

$$X_d \geq X_q \geq X'_q \geq X'_d \geq X''_q \geq X''_d \quad T'_d0 > T'_d > > T''_d0 > T''_d$$

$$T'_q0 > T'_q > > T''_q0 > T''_q$$
Parameter Estimation by Frequency Response Tests

\[ \Delta \psi_d(s) = G(s) \Delta e_{fd}(s) - L_d(s) \Delta i_d(s) \]

\[ L_d(s) = L_d \frac{(1+sT'_d)(1+sT''_d)}{(1+sT'_d)(1+sT''_d)} \]

\[ G(s) = G_0 \frac{(1+sT_{kd})}{(1+sT_{d_0})(1+sT''_{d_0})} \]

\[ T'_{d_0} > T'_d >> T''_{d_0} > T''_d > T_{kd} \]

\[ 1/T'_{d_0} < 1/T'_d << 1/T''_{d_0} < 1/T''_d < 1/T_{kd} \]
Figure 4.7 Lambton GS - Variation of $L_d(s)$ with frequency at standstill with field closed

Figure 4.8 Lambton GS - Variation of $sG(s)$ with frequency at standstill with field closed

Figure 4.9 Lambton GS - Variation of $L_q(s)$ with frequency at standstill
Swing Equations

\[ J \frac{d\omega_m}{dt} = T_a = T_m - T_e \]

- Define per unit inertia constant
  \[ H = \frac{1}{2} \frac{J \omega^2}{\omega_{0m}^2} \text{ (s)} \]
  \[ J = \frac{2H}{\omega_{0m}^2} \text{ VA}_\text{base} \]

Some references define \( T_M \) or \( M=2H \), called the mechanical starting time, i.e. the time required for rated torque to accelerate the rotor from standstill to rated speed.
\[ J \frac{d \omega_m}{dt} = T_a = T_m - T_e \]

\[ \frac{2H}{\omega_{0m}^2} V A_{\text{base}} \frac{d \omega_m}{dt} = T_m - T_e \]

\[ 2H \frac{d}{dt} \left( \frac{\omega_m}{\omega_{0m}} \right) = \frac{T_m - T_e}{V A_{\text{base}} / \omega_{0m}} \]

\[ 2H \frac{d}{dt} \left( \frac{\omega_m}{\omega_{0m}} \right) = \frac{T_m - T_e}{T_{\text{base}}} \]

\[ 2H \frac{d \bar{\omega}_r}{dt} = \bar{T}_m - \bar{T}_e \quad \text{(in per unit)} \]

where

\[ \omega_r \text{ (in per unit)} = \frac{\omega_m}{\omega_{0m}} = \frac{\omega_r / p_f}{\omega_0 / p_f} = \frac{\omega_r}{\omega_0} \]

Angular position of the rotor in electrical radian with respect to a synchronously rotating reference

\[ \delta = \omega_r t - \omega_0 t + \delta_0 \quad \text{in rad} \]

\[ \frac{d \delta}{dt} = \omega_r - \omega_0 = \Delta \omega_r \quad \text{in rad/s} \]

\[ \frac{d^2 \delta}{dt^2} = \frac{d \omega_r}{dt} = \frac{d(\Delta \omega_r)}{dt} \quad \text{in rad/s}^2 \]

\[ \frac{d^2 \delta}{dt^2} = \omega_0 \frac{d(\bar{\omega}_r)}{dt} = \omega_0 \frac{d(\Delta \bar{\omega}_r)}{dt} \quad \text{in rad/s}^2 \]

If adding a damping term proportional to speed deviation:

\[ \frac{2H}{\omega_0} \frac{d^2 \Delta \bar{\omega}_r}{dt^2} = \bar{T}_m - \bar{T}_e - K_D \Delta \bar{\omega}_r = \bar{T}_m - \bar{T}_e - \frac{K_D}{\omega_0} \frac{d \delta}{dt} \]
Block diagram representation of swing equations

\[
\frac{2H}{\omega_0} \frac{d^2 \delta}{dt^2} = T_m - T_e - K_D \Delta \bar{\omega}_r = T_m - T_e - \frac{K_D}{\omega_0} \frac{d \delta}{dt}
\]

\[
2H \frac{d (\Delta \bar{\omega}_r)}{dt} = T_m - T_e - K_D \Delta \bar{\omega}_r
\]

\[
\Delta \bar{\omega}_r = \frac{1}{\omega_0} \frac{d \delta}{dt}
\]

Figure 3.34 Block diagram representation of swing equations
State-Space Representation of a Synchronous Machine

So far, we modeled all critical dynamics about a synchronous machine:

• State variables \( (pX) \):
  – stator and rotor voltages, currents or flux linkages
  – swing equations (rotor angle and speed)

• Time constants:
  – Inertia: \( 2H \)
  – Sub-transient and transient time constants, e.g. \( T'_{d0} \) and \( T''_{d0} \)

• Other parameters
  – Stator and rotor self- or mutual-inductances and resistances
  – Rotor mechanical torque \( T_m \) and stator electromagnetic torque \( T_e \)
Differential Equations on a Salient-pole Machine

• Consider 5 windings (d, F, D, q and Q)
  – Voltage equations:

\[
\begin{bmatrix}
  e_d \\
e_F \\
e_q
\end{bmatrix} =
\begin{bmatrix}
  -R_d & R_F \\
  R_D & -R_a \\
  0 & R_Q
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_F \\
i_q
\end{bmatrix} +
\begin{bmatrix}
  -L_d & kM_F & kM_D \\
  -kM_F & L_F & M_R \\
  -kM_D & M_R & L_D
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_F \\
i_q
\end{bmatrix} +
\begin{bmatrix}
  \omega r N I
\end{bmatrix}
\]

\[e = R \times i + L \times di/dt + \omega_r N \times I\]

\[p_i = - L^{-1} (R + \omega_r N) i + L^{-1} e\]

– Swing equations:

\[
\tau p \omega_r = T_m - T_e \quad (\text{where } \tau = 2H/\omega_0)
\]

\[p \delta = \omega_r - \omega_0 = \omega_r - 1\]

\[T_e = \psi_d i_q - \psi_q i_d\]

\[
\psi_d = -L_d i_d + kM_F i_F + kM_D i_D,
\]

\[
\psi_q = -L_q i_q + kM_Q i_Q
\]

\[
T_e = -L_d i_q i_d + kM_F i_q i_F + kM_D i_q i_D + L_q i_d i_q - kM_Q i_d i_Q
\]
Thus, the state-space model of a synchronous machine:

\[
\begin{align*}
\frac{dx}{dt} &= f(x, e_d, e_q \text{ and } e_{fd}, T_m) \\
\text{where } x &= [i_d \ i_{Fd} \ i_{1d} \ i_q \ i_{1q} \ \omega_r \ \delta]^T \text{ or } [\psi_d \ \psi_{fd} \ \psi_{1d} \ \psi_q \ \psi_{1q} \ \omega_r \ \delta]^T \text{ is the state vector.}
\end{align*}
\]

(See the section 4.12 in Anderson’s book).
Simplified Models

- $[\psi_d \ \psi_{fd} \ \psi_{1d} \ \psi_q \ \psi_{1q} \ \omega_r \ \delta]^T$

  Neglect $p\psi_d$, $p\psi_q$ and variations of $\omega_r$ (i.e. $\omega_r=1$ pu) in the voltage equations.

- $[\psi_{fd} \ \psi_{1d} \ \psi_{1q} \ \omega_r \ \delta]^T$
  - Inertia $\tau \sim p\omega_r$
  - Transient $T''_{d0} \sim p\psi_{fd}$
  - Sub-transient $T''_{d0} \sim p\psi_{1d}$
  $T''_{q0} \sim p\psi_{1q}$

  Neglect damper windings, i.e. $p\psi_{1d}$ and $p\psi_{1q}$

- $[\psi_{fd} \ \omega_r \ \delta]^T$
  - Inertia $\tau \sim p\omega_r$
  - Transient $T''_{d0} \sim p\psi_{fd}$

  Constant flux linkage assumption

- $[\omega_r \ \delta]^T$ (classic model)
  - Inertia $\tau \sim p\omega_r$
Neglect of Stator $\psi$ terms

Figure 5.1 System configuration and parameters

$G$

Line #1

Line #2

Infinite bus

Disturbance:

3-phase fault at F; cleared in 0.09 s by opening line #2

Generator parameters in per unit:

$L_{rd} = 1.0$  $L_{rd} = 0.6$  $L_t = 0.18$  $L_{fd} = 0.13$

$L_{rd} = 0.11$  $L_{rd} = 0.13$  $R_g = 0.005$  $R_{fd} = 0.00075$

$R_{rd} = 0.02$  $R_{td} = 0.04$  $H = 3.5$

Figure 5.3 Effect of neglecting stator transients on speed deviation

Figure 5.2 Effect of neglecting stator transients on air-gap torque and $d-q$ components of stator currents

Figure 5.4 Effect of neglecting stator transients on rotor angle swings
Neglecting Damper Windings

\[ \psi_d = -L_d i_d + L_{ad} i_{fd} \]
\[ \psi_q = -L_q i_q \]
\[ \psi_{fd} = -L_{ad} i_d + (L_{ad} + L_{fd}) i_{fd} \]
\[ p \psi_{fd} = -e_{fd} - R_{fd} i_{fd} \]

(only differential equation left for the equivalent circuits)

\[ T_d 0 pE'_q = E_{fd} - E'_q - (L_d - L'_d) i_d \]
where \( E'_q = \frac{L_{ad}}{L_F} \psi_{fd} \) and \( E_{fd} = \frac{L_{ad}}{R_{fd}} e_{fd} \)

\[ T_q 0 pE'_d = -E'_d - (L_q - L'_q) i_q \] (added for round-rotor machines to give the so called “Two-Axis Model”)

\[ X'_d = L_l + L_{ad} / L_{fd} \]
\[ X_q = L_l + L_{aq} \]
Classic Model

- Eliminate the differential equations on flux linkages (swing equations are the only differential equations left)
- Assume \( X'_d = X'_q \)

\[
\frac{2H}{\omega_0} p \omega_r = T_m - T_e
\]

\[
p \delta = \omega_r - 1
\]

\[
\tilde{E}_t = \tilde{E}' - (R_a + jX'_d)\tilde{I}_t
\]

\[
\tilde{E}' = \tilde{E}_{t0} + (R_a + jX'_d)\tilde{I}_{t0}
\]

\( E' \) is constant and can be estimated by computing its pre-disturbance value
Simplified models neglecting $p\psi$ and saliency effects

(a) Subtransient model

$E''_0$ is the predisturbance value of internal voltage given by

$$E''_0 = E'_{t0} + jX''I_{r0}$$

$X'' = X_d'' = X_q''$

(b) Transient model

$E'_0$ is the internal voltage

$$E'_0 = E'_{t0} + jX'I_{r0}$$

$X' = X_d' = X_q'$

(c) Steady-state model

$E_q = E'_{t0} + jX_sI_{r0}$

$|E_q| = X_{sd}I_{fd} = E_I$

$X_s = X_d = X_q$
Comparison of PSS/E Generator Models

Table 1: Summary of Generator Models in Terms of Data Used

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*\( X''_q \) is assumed to be equal to \( X''_d \).*