Chapter 3

Words and Images

3.1 Hellenistic Logic

There is a certain head, and that head you have not. Now this being so, there is a head which you have not, therefore you are without a head.

If anyone is in Megara, he is not in Athens: now there is a man in Megara, therefore there is not a man in Athens.

If you say something, it passes through your lips: now you say wagon, consequently a wagon passes through your lips.

If you have never lost something, you have it still; but you have never lost horns, ergo you have horns.

— Chrysippus of Soli (Diogenes Laertius, 7.186–187)

A man says he is lying. Is what he says true or false?

— Chrysippus (Cicero, De divinatione, 2.108; cf. Academica, 2.96)
## 3.1.1 Modal Logic

During the Hellenistic period (third to first centuries BCE), significant progress was made in many areas of logic.\(^1\) For example, Aristotle had begun a study of modal propositions (those involving possibility, impossibility, contingency and necessity), in which he enumerated the 112 possible modal syllogisms and determined which are valid (*Pr. An.* 1.3.8–22). A different system of modal logic was developed by Theophrastus (c. 370–c. 288 BCE), his successor as head of the Peripatetic school. In the next century the Megarians and the Stoics made further advances and investigated temporal or tense logic, which deals with propositions that may be true at some times but not others. Modal and temporal logics are important components of modern artificial intelligence systems (Sowa, *CS*, pp. 173–187). Carneades (214–129 BCE), the fourth head of Plato’s Academy, developed a theory of qualitative probability, in which propositions could have three grades of probability: (1) “convincing,” i.e., merely probable, (2) “undiverted,” that is, exhibiting consistency among multiple observations, or (3) “thoroughly explored,” that is, systematically tested for consistency with the rest of experience.\(^2\) Although this is a departure from the absolutism of Plato and reflects the sceptical turn of the New Academy, it is quite consistent with a modern scientific view of truth.

## 3.1.2 Propositional Logic

Aristotle’s logic is a logic of terms, or as we might call it now a class logic, a precursor to set theory. That is, it deals with logical relations between terms, such as ‘man’ and ‘mortal’, that denote classes (*eidê*, forms, in their terminology). It is generally considered that a major advance was made over Aristotle’s logic when, in the late nineteenth century, logicians shifted from a logic of classes to a logic of propositions. In a propositional logic the variables refer to propositions (expressions that are either true or false) rather than to classes. For example, the deduction

\[
\text{If it is day, then it is light;}
\]

\[
\text{it is not light;}
\]

\(^1\)Sources for this section are Long & Sedley (1987, §§ 27, 69), Bocheński (*HFL*, Pts. 2, 3), Kneale & Kneale (*DL*, Chs. 2, 3) and Hamlyn (HoE).

therefore it is not day.

is an example of the schema:

If $A$ then $C$;
not $C$;
therefore not $A$.

in which $A = \text{the proposition 'it is day'}$ and $C = \text{‘it is light’}$.

Although propositional logic was rediscovered in the middle ages, Theophras-tus developed the first propositional logic, and the subject was well explored by the Megarian logicians Diodorus Cronus and Philo (fourth century BCE), who developed a modern, truth-functional definition of implication, and by the Stoic Chrysippus of Soli (c. 279–206 BCE), who defined schemata for a form of the “natural deduction” used by many AI systems (e.g. Cohen & Feigenbaum, *HoAI* 3, pp. 94–95); one of his rules was:

If the first then the second;
but not the second;
therefore not the first.

Here the italicized words are in effect propositional variables. In modern symbolic notation we would write:

$$F \rightarrow S, \neg S \vdash \neg F,$$

but the difference is small. This was a further step towards reducing reasoning to formal manipulation. By all accounts, Chrysippus was one of the greatest of Greek logicians; Diogenes said (7.181–182), “if the gods took to logic, they would adopt no other system than that of Chrysippus,” but none of his 705 books have survived.

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3A truth-functional definition deals only with the truth or falsity of the propositions, not with their meaning. For example, the (material) implication (or conditional) ‘if $A$ then $C$’ is considered false if $A$ is true and $C$ false, but true in all other cases (Sextus Empiricus, *Outlines of Pyrrhonism* 2.104–106). This definition still puzzles beginning logic students when they are told, for example, that “If pigs can fly then pigs like mud” is a true implication. The definition of implication was much debated in antiquity; Callimachus (2nd cent. BCE) said, “The very crows on the roofs caw about the nature of implications.” The debate continues to this day, e.g., Appiah (*A&O*), Harper & al. (*Ifs*), Jackson (*Cond*) and Traugott & al. (*OC*).
Chysippus or some other Stoic philosopher also began the study of semiotics (the scientific investigation of signs in the most general sense) by carefully distinguishing three aspects of a sign. First we have the sign itself (sēmainon), considered as a physical phenomenon (e.g., a sound or a written text). Take as an example the name ‘Socrates’. Second we have the existing object to which the name refers, for example, Socrates the person. Finally, we have the nonphysical lektōn or “significate” (sēmainomenon) of the sign, which is its abstract meaning. This is meant as the objective sense of the sign (the conceptus objectivus), rather than any subjective impression it may cause in our mind (the conceptus subjectivus), which is a fourth correlate of the sign (Bocheński, HFL, pp. 110–111). Even though some signs, such as ‘Pegasus’, refer to no existing physical object, they nevertheless signify a lektōn, since their meaning is quite definite. We know many facts about Pegasus. For example, it’s true that he has wings, but false that he has scales; indeed, one of the facts we know is that Pegasus doesn’t exist. Logic is taken to be the science of lektta.

In the lektōn theory we see again the assumption that if we can use a word meaningfully, then it must name something; since we can obviously talk about things that don’t exist, the lektōn (“thing said”) is postulated as a surrogate object of reference. A better solution is to abandon the referential assumption, and with it the denotational theory of meaning, which is the more modern approach.

3.1.3 Logical Paradoxes

Beginning at least with Eubulides (fourth century BCE) the Megarian and Stoic philosophers showed great interest in logical paradoxes; a few are shown on page 59. Although these may seem ridiculous, they are not jokes; they embody serious logical issues, some of which occupied logicians for the following two millennia. The fallacies they exemplify are still pitfalls for the designers of the knowledge representation languages used in AI (see Section 3.2 for examples). The last paradox, which is the famous Liar, is the root of Russell’s Paradox, which destroyed Frege’s mathematical system, and is the inspiration for Gödel’s proof of his famous Incompleteness Theorem, and for Turing’s proof of the uncomputability of some functions. These topics are discussed in detail in Chapters 6 and 7. The problem continues to exercise philosophers and logicians; see for example Martin (PL) and Barwise &
Etchemendy (*Liar*). After Galen (129–c. 199 CE), who was an excellent logician as well as a physician, logic died as a creative activity; there were only rehashes of prior work until the eleventh-century resurrection of Europe’s intellectual life allowed the efflorescence of scholastic logic.

### 3.2 Medieval Logic

[O]ne ought not to postulate many items when one can get by with fewer.

— Ockham (Loux, *OTT*, §12)

Ockham’s account of truth conditions will undoubtedly strike many as surprisingly contemporary, though perhaps it is fairer to say that contemporary theories should strike the reader of Ockham as surprisingly medieval.

— Freddoso & Schuurman (*OTP*, preface)

What’s a’ your jargon o’ your schools,  
Your Latin names for horns and stools;  
If honest Nature made you fools,  
What sairs your grammars?

— Robert Burns (*First Epistle to John Lapraik*)

Unlearn’d, he knew no schoolman’s subtle art,  
No language, but the language of the heart.

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*4The paradox is put in its ancient and medieval contexts in Kneale & Kneale (*DL*, pp. 16, 113–115, 227–229); van Heijenoort (*LP*) is a good overview of logical paradoxes. The Liar Paradox is commonly attributed to Epimenides, but was more likely invented by Eubulides, a fourth century BCE philosopher of Megara and teacher of Demosthenes. Although the problem is alluded to by Aristotle (*Sophistic Refutations* 25.180b2–7), the earliest clear statement is probably by Cicero (106–43 BCE), who attributes it to Chrysippus: “If you say that you are lying, and say it truly, you are lying” (Cicero, *Academica* 2.95). Eubulides probably used the paradox as an argument against the Platonic / Aristotelian correspondence theory of truth.*
By nature honest, by experience wise,  
Healthy by temp’rance, and by exercise.

— Alexander Pope (Epistle to Dr. Arbuthnot, l. 398)

3.2.1 Debate about Universals

Medieval scholastic logic is generally considered the epitome of pedantry, yet it is relevant to contemporary approaches to knowledge representation — which are “surprisingly medieval.”

The medieval debate about the nature of universals stemmed from some problems identified by Porphyry of Tyre (232–304 CE), a Neoplatonic philosopher; among other surviving works, we have his Life of Pythagoras. Although the problem of universals had important implications for medieval theology, which are no longer very interesting to us, the question is still relevant to contemporary epistemology, philosophy of science and cognitive science: To what extent and in what ways are categories — in reality? — in the mind? — in language? As Porphyry put it (Isagoge 4.1; Kneale & Kneale, DL, p. 196):

As for genus and species, I beg to be excused from discussing at present the question whether they exist in reality or have their place simply and solely in thoughts, and if they exist, whether they are separable or exist only in sensible things and dependent upon them. For such a study is very deep and requires another and larger inquiry.

In the middle ages, there were three major positions on the question, known as realism, conceptualism and nominalism, although we will see that the differences were not so extreme as the names suggest.

Realism held that universals exist in reality; it came in two varieties, Neoplatonic (e.g., Augustine, 354–430), which placed the universals in another realm (often the mind of God), and Aristotelian (e.g., Aquinas, 1225–1274), which placed them in objective similarities in particular things. In either

Secondary sources for this section are Bocheński (HFL, Pt. 3), Kneale & Kneale (DL, Ch. 4) and Hamlyn (HoE).
case concepts in the mind are a result of it apprehending the \textit{forms} in reality.

Pierre Abélard (1079–1142)\textsuperscript{6} proposed his conceptualist theory as a third alternative to realism and the extreme form of nominalism current at that time (Roscelin’s, below). He took universals to be concepts (\textit{sermones}) in the mind, which were a kind of generic image representing the common features from the images of many particular things. When we think in terms of universals, it is these generic images that our mind manipulates. Of course, generic images would not work as representation unless there were in fact similarities between existing things, and thus in reality. So the difference between Aquinas’ and Abélard’s positions is more of emphasis than of kind. The notion of a generic image is very compatible with connectionist knowledge representation (discussed in Vol. 2).

We find the most extreme form of nominalism in Roscelin of Compiègne (c. 1050–c. 1120), who held the universals are mere \textit{flatus vocis} — puffs of breath. A more defensible position was maintained by William of Ockham (c. 1285–1349), who held that a universal was a particular kind of sign that can stand for many things.\textsuperscript{7} Ockham came from the Stoic tradition, and originally held that a universal is a \textit{logical} construct — like the Stoic \textit{lekton} — that holds an intermediate position between the mind and the thing signified. Later, in accord with his well-known “razor” (p. 63),\textsuperscript{8} he simplified his

\textsuperscript{6}Also, Peter Abailard. Credited with founding the University of Paris, he was also known for his contentiousness, and was condemned by two councils, in part because his popularity with students was considered dangerous, in part because “his thoughts seemed to lead to paganism” (McKeon, \textit{SMP}, Vol. 1, p. 207). His \textit{Sic et non} (\textit{Yes and No}) listed 158 theological questions on which the church fathers disagreed, and some of his works were considered heretical and burned. He married Héloïse, the niece of Fulbert, a canon of Paris, who had Abélard castrated for rescuing Héloïse from Fulbert’s mistreatment and hiding her in a monastery. \textit{Héloïse and Abelard} collects their subsequent correspondence.

\textsuperscript{7}Although Ockham (or Occam) is one of the most famous philosophers of the middle ages, his logical, physical and philosophical works are remarkably inaccessible. McKeon (\textit{SMP}, Vol. 2, p. 351) says that few of his works have been published since the seventeenth century, and some have not been published at all. The \textit{Quodlibeta}, selections of which he translates in his \textit{SMP} (Vol. 2, 366–421), had not been published since 1491.

Thus we are especially fortunate for the recent publication of an English translation of the first two parts of the \textit{Summa logicae} (Loux, \textit{OTT}; Freddoso & Schnurman, \textit{OTP}), but this is less than half the whole. McKeon (\textit{SMP}, Vol. 2, pp. 351–359) is a useful source, which also contains a valuable glossary of medieval philosophical terms. Ockham was at Oxford until he was accused of heresy and had to flee to Munich.

\textsuperscript{8}Ockham’s Razor exists in many forms, although few are found in his extant writings.
account by making the universal a *mental* sign referring directly to things; thus his position was not so far from conceptualism. The idea was that these signs serve a function in mental discourse analogous to the role of words in spoken discourse (more on this below).

A significant difference between Ockham’s nominalism and the realist and conceptualist theories was in the kind of information acquired by the mind from the senses. The realist and conceptualist theories claimed that the mind only knows universals (categories), which it abstracted from the images. In modern terms, cognition operates on representations constructed from general *features* computed from sense data.

Ockham denied all this. He held that the mind could be concerned directly with the particular by means of intuitions. Intuitive knowledge is a direct knowledge of a thing or its existence. (Hamlyn, HoE, p. 16)

In other words, the concrete image of a thing is available to cognition, and cognitive processes can operate on this image rather than on some intermediate representation constructed of abstract features. This view is quite compatible with connectionist theories of knowledge, which often use concrete rather than abstract representations, and is compatible with neuropsychological evidence, which does not support feature detectors in the brain (Pribram, B&P, pp. 11–16, 79–81). In this context, intuition (*intuitus*) is “that by which something is known immediately, without ratiocination” (McKeon, SMP, Vol. 2, pp. 466–467) — that is, *nonrational*, immediate cognition.

However, the directness of this intuitive awareness does not guarantee its accuracy, which varies between clear and confused. Ockham made a useful distinction, between *perfect intuitions*, which are of just the present moment, and *imperfect intuitions*, which are colored by prior experience. In other words, we may have an immediate awareness of what we are experiencing right now; regardless of whether what we perceive is real or illusory, it is an accurate image of our sensations.

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In the *Quodlibeta* (5, q.1) we find “*pluralitas non est ponenda sine necessitate*” — “a plurality must not be asserted without necessity.” However, the commonly quoted form, “*Entia non sunt multiplicanda praeter necessitatem*” (Entities are not to be multiplied beyond necessity), does not occur in his extant works. Ockham’s Razor is often claimed as the basis of the scientific preference for the simpler theory (e.g., Beveridge, ASI, p. 116; Bronowski, CSS, p. 131; Joseph, IL, p 506).

9On this topic see Ockham (*Quodlibeta* 1.13, 1.15, 5.5, 6.6: McKeon, SMP, Vol. 2, pp. 360–375).
However, when our awareness includes past experience, then inaccuracy may enter in two different ways. On one hand we may remember a prior intuition, but such recall will likely be imperfect, since it is difficult to accurately remember a (continuous) sensory image. On the other, we are better able to remember an experience if we have grasped it intellectually, since then we have analyzed it into discrete features. But this analysis is inevitably an approximation, and so an intuition incorporating it will be imperfect. The conclusion is that prior experience imposes a bias (for good or ill) on intuitive awareness.\(^\text{10}\) This will become a recurring epistemological theme.

In summary, the medieval debate over universals provides three different accounts of the objectivity of categories. Realism says they are objective because they exist in reality, either in particular things or in the mind of God; conceptualism says they are in the mind, and in this sense subjective, but get their objectivity from actual similarities of things; nominalism says that a universal is a sign that can signify any one of many particulars, and that a kind of objectivity may result either from the natural similarities of the things or from the conventions of the language.

### 3.2.2 Language of Logic

Nowadays we think of logic and mathematics as closely related disciplines (indeed one is often founded on the other), but this close association did not develop before the nineteenth century (Section 4.4). Before that, and going back to Aristotle, logic was considered one of the language arts, since its main application was in judging the validity of verbal arguments. As we have already seen (Fig. 2.8, p. 31), in the middle ages logic (under the name dialectic) was one of the three Language Arts (the Trivium), not one of the four Mathematical Arts (the Quadrivium).

Given that logic was investigated in the context of natural languages, it is not surprising that logicians became aware of the deficiencies of natural language as a logical instrument and tried to remedy them. This took two forms. One was to refine the language of science of that time — Scholastic Latin — into a tool for logical reasoning. We will look at this effort in this section, since it bears on some important issues in cognitive science.

Instead of refining natural languages, the second approach was to create artificial languages that were designed for logic or philosophical discourse.

\(^{10}\)See Ockham (Quodlibeta 1.13, 1.15, 5.5, 6.6; McKeon, SMP, Vol. 2, pp. 360–375).
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This path — which goes through Lull, Wilkins, Leibniz and Boole — leads directly to symbolic logic and AI knowledge representation languages, so it will constitute the bulk of this chapter.

There are several assumptions underlying both of these approaches to an ideal language; we should be aware of them. First there is the realist assumption (coming from Plato and Aristotle) that the world has a definite categorical structure, and that a language is better to the extent that it truly reflects that structure. To illustrate the alternatives, we may take at the other extreme the Sapir-Whorf hypothesis (see also p. 107), which says that the structure of our language determines the structure of our reality. The view that an ideal language should be an accurate picture of reality was an essential part of the logical positivist movement in the twentieth century (Ch. 8).

The second assumption — which goes back at least to Plato (e.g., Sophist 263e) and is still sufficiently compelling to be considered by some “the only game in town” (Fodor, LT) — is that there is a “language of thought.” Although I will discuss criticisms of this assumption in Volume 2, here we need only observe that if there were such a language, then it would presumably be to our advantage to make our written and spoken languages as close to this as possible, since by doing so we would have eliminated all the complexities not relevant to reasoning. For this reason we have, in 1660, the famous “Port Royal Grammar” (Grammaire générale et raisonnée by Arnauld and Launcelot), which sought “to set forth what is common to all languages” and “to explain language by reference to the constitution of the human mind” (Ellegård, 1973, pp. 667b–668a); it was followed shortly by the influential “Port Royal Logic” (Logique ou l’art de penser, Arnauld & Nicole, 1662). But they are just the tip of the iceberg.

The two assumptions — a categorical structure in reality and a language of thought — together generate a pervasive, rationalistic optimism that many of our problems would be solved if we could simply use the correct language to talk about them. Thus the quest for an “ideal” language.

* * *

In presenting some relevant developments in medieval logic, I will be drawing again from the works of William of Ockham.11 We are using but

11Sources are Parts 1 and 2 of Ockham’s Summa logicae (Loux, OTT; Freddoso & Schuurman, OTP) and the Quodlibeta (McKeon, SMP, Vol. 2, pp. 360–421).
a fraction of his research in meaning and reference, temporal and modal logic, nominalism, and many other topics. Here we consider his theories of intention and supposition.

The nature of intentionality may be the central philosophical problem in cognitive science and artificial intelligence. Although it originated in the middle ages, it is still widely debated (e.g., Dennett, IS; Diamond & Teichman, I&I; Dreyfus & Harrison, HICS; Searle, Int), and many philosophers believe that intentionality is a characteristic of natural intelligence that is beyond the reach of artificial intelligence. In considering this question we must be careful to distinguish the technical term intention from its everyday meaning, “a goal, aim or objective.”

Intentionality is that property of a mental state by which it is about, or directed at, something else. It derives from Latin intendo, which means to stretch toward, to point at, or to direct one’s mind toward; and at least from Cicero’s time, the noun intention could refer to the acts of stretching, reaching, and concentrating one’s attention (Glare, OLD, s.vv. intendo, intentio). The Schoolmen adopted it as a technical term for the ability and act of the mind pointing outside of itself (McKeon, SMP, Vol. 2, pp. 465–466).

Since Ockham views thought as “mental discourse,” he treats an intention as a word in the mental language:

an intention of the soul is something in the soul capable of signifying something else... (Sum. log. 1.12; Loux, OTT, p. 73)

More specifically, an intention “is either a sign naturally signifying something else (for which it can supposit) or a potential element in a mental proposition” (Sum. log. 1.12; Loux, OTT, p. 74). (‘Supposit’ is a scholastic technical term meaning ‘stand in place of’; we’ll encounter it again.) Thus for Ockham intentionality is akin to the referential relation between a sign and the thing it signifies.

Avicenna (Ibn Sīnā, Abū’ Alī al-Husayn, 980–1037) had distinguished two kinds of intentions. In Ockham’s formulation, a first intention is a mental sign that does not signify an intention or sign. Thus most everyday concepts, such as man, blue, and fire, are first intentions.
A second intention, in contrast, is a mental sign of a first intention; examples are logical terms such as species and genus.

That is, a first intention, such as man refers to things outside the mind (e.g., Socrates, Hypatia and other people), whereas a second intention, such as concept, refers to things inside the mind (e.g., man, blue and other categories). Ockham claims that the terms of logic are of the second intention (because logic studies first intentions), whereas the terms of the other sciences are of the first intention (because they study things other than intentions).

The theory of supposition may seem a clear example of Scholastic pedantry, but it’s an important knowledge representation issue in AI. Consider the following true statements:

- Man is mortal.
- Man is a general term.
- Man means an adult human male.
- Man is an English word.
- Man is a three-letter word.
- Man is a one-syllable word.
- Man rhymes with can.
- Man is in italic type.
- Man is the first word in this line.
- Man is the tenth occurrence of Man in this display.

In each of these true statements ‘Man’ is used in a different way and in fact refers to something different. It is worthwhile to go over these sentences again and be sure that in each case you can say what ‘Man’ denotes. To help, try replacing ‘Man’ by other expressions, such as ‘Homo sapiens’, ‘Mensch’, ‘Mann’, ‘Dan’, ‘Plan’ and ‘anthrōpos’, and make a table of the truth or falsity of each proposition. Notice that normal quotation marks are not sufficient to eliminate the difficulties; try substituting ‘“Man”’ and ‘“Plan”’ for ‘Man’.

This is an important issue for knowledge representation languages, since we do not want to allow inferences such as this:

\[
\text{Man is a three-letter word;}
\]
\[
\text{Dan is a man;}
\]
\[
\text{therefore, Dan is a three-letter word.}
\]
which happens to reach a correct conclusion, or this (cf. Sowa, CS, p. 84):

\[
\begin{align*}
\text{Elephant is a general term;} \\
\text{Jumbo is an elephant;} \\
\text{therefore, Jumbo is a general term.}
\end{align*}
\]

which reaches an incorrect conclusion. Yet this is exactly what will happen if they blindly implement a simple deduction rule such as:

\[
\begin{align*}
M & \text{ is } P; \\
S & \text{ is } M; \\
\therefore S & \text{ is } P.
\end{align*}
\]

We recognize the absurdity of these inferences, but a rule-based expert system does not have an experiential basis for recognizing sense and nonsense; therefore it’s necessary to block inferences of this kind by syntactic mechanisms (such as quotation marks) or other formal devices.

These examples illustrate that there are many ways that a word can supposit, or stand, for other things. In ‘Man is mortal’, the word ‘man’ stands for any human being; in ‘Man is a general term’ it stands for the concept or category man = human being; in ‘Man is an English word’ it may stand for either the written or spoken word ‘man’. The Schoolmen called this property of standing for something else supposition: “Supposition is said to be a sort of taking the place of something else.” (Ockham, Sum. log. 1.63; Loux, OTT, p. 189) Notice that supposition is a property of a term in the context of a proposition; that is, ‘Man’ supposit in a different way in each of the examples above. Ockham distinguished three kinds of supposition (although he also recognized finer distinctions could be made).

First, Ockham says personal supposition:

occurs when a term supposits for the thing it signifies, whether this thing be an entity outside of the soul, a spoken word, or any other thing imaginable. (Sum. log. 1.64; Loux, OTT, p. 190)

An example of personal supposition is ‘man’ in ‘man is mortal’, since in the proposition ‘man’ supposit for (stands in place of) Socrates, Plato, Hypatia, or any other person.

Next he says, “Simple supposition occurs when a term supposits for an intention of the soul and is not functioning significatively” (Sum. log. 1.64; Loux, OTT, p. 190). For example, in ‘Man is a species’ the term ‘man’ does
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not stand for individual persons, as in the previous example, but instead for
the concept (first intention) man. Man is a species, but Socrates and Hypatia
are not; conversely, the category man is not mortal but Socrates and Hypatia
are. Ockham explains that a term in simple supposition is “not functioning
significatively” because ‘man’, for example, signifies persons not intentions;
as we saw it is a term of first intention, not second intention.

Therefore, when ‘man’ is in simple supposition, it indicates the intention
man indirectly. That is, a term in simple supposition stands for an intention
in much the same way a part can refer to a whole by metonymy, for example,
when we say “I don’t have any wheels” to mean “I don’t have a car.” In
everyday speech we rely on context to distinguish personal and simple suppo-
sition; in written language we might use italics to signal simple supposition,
as I have done in this paragraph.

Finally, “Material supposition occurs when a term does not supposit sig-
nificatively, but supposits for a spoken word or a written word” (Sum. log.
1.64; Loux, OTT, p. 191). For example, in the proposition “Man is a name”,
the word ‘man’ refers neither to people nor to concepts, but to certain phys-
ical phenomena, either vibrations in the air (spoken words) or visible marks
on paper (written words). Once again, in spoken language, context tells
us when material supposition is intended; in written language we often use
quotation marks, as in this paragraph.  

As the preceding examples demonstrate, there are actually several kinds
of material supposition. For example, ‘man’ may refer to the spoken word
(as when we say it has one syllable), or to the written word (as when we say
it has three letters), or to a particular printed form (as when we say it is
in italics), or to a particular instance of that form (as when we say it is the
tenth occurrence on the page). For everyday purposes these are hairs that
need not be split, but they can trip up formal deductive systems.

Modern philosophy simply distinguishes between the use and the mention
of an expression. Personal supposition uses the term (significatively); simple
and material supposition mention the term, either to refer indirectly to a
mental sign (simple supposition) or to a spoken or written sign (material
supposition).  

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15 Whenever there is a chance of confusion in this book, I will follow the convention of
using italics for simple supposition and single quotes for material supposition. For example:
Man is mortal; Man is a species; but ‘Man’ is an English word. For other purposes, I use
double quotes; as usual italics are also used for emphasis.

16 Alternately, modern philosophy distinguishes reference — or extension — from mean-
3.2. MEDIEVAL LOGIC

In the theory of supposition we see the fourteenth century logicians' version of the object-language/metalanguage distinction, which is an important tool of modern logic. A metalanguage is used to talk about another language, the object-language. In fact the two may be the same language, as when we use English to talk about English. As we’ve seen, everyday language rarely makes these distinctions, but many philosophers think that a confusion of metalanguage and object-language is the root of many paradoxes and fallacies; we’ll see an example shortly. The distinction is also fundamental to the proofs of Gödel’s incompleteness theorem, Turing’s incomputability theorem, and many other important results (Chapter 7).

During the thirteenth and fourteenth centuries the study of paradoxes (sophismata) provided a vehicle for studying many important problems in meaning and reference. For example Jean Buridan (c. 1295 – after 1358) proposed paradoxes of this kind: I show you my hand and ask if you know whether the number of coins in it is odd or even. Of course you say you do not know. But then I open it and show three coins, and say, “You claimed that you didn’t know if the number of coins is odd or even, but the number of coins is three, therefore you have said that you don’t know whether three is odd or even.” The number of coins equals three, and I have simply substituted equals for equals, ‘three’ for ‘the number of coins’, in your assertion. Where is the fallacy?

In modern terms, we say that intentional contexts are referentially opaque, but this requires some explanation. An intentional context is an intention or a description of an intention, and so it is about something else. In particular, verbs of knowing, believing, hoping, fearing, etc. describe intentions, because you know that something or believe that something, etc. An expression is in a referentially transparent context when the expression can be replaced by any other with the same referent without changing the truth of the proposition; it is referentially opaque when this is not the case, that is, when equals cannot be substituted for equals. To return to Buridan’s example, ‘the number of coins is odd’ is referentially transparent since its truth value doesn’t change when ‘the number of coins’ is replaced by any other expression with the same referent. Since ‘the number of coins’ and ‘the square root of nine’ both have

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17See Moody (ML); Bocheński (HFL, §35) contains a selection of medieval sophismata.
CHAPTER 3. WORDS AND IMAGES

the same referent, three, one may be substituted for the other in this or in any referentially transparent context. For example, from three being the square root of nine we could conclude that the number of coins is the square root of nine.

The designers of knowledge representation and inference systems must be careful to distinguish transparent and opaque contexts; indeed, it is even important in reasoning about programs (MacLennan, *FP*, pp. 11–13, 100–101). Further, referential opacity is often considered a hallmark of intentional and mental states and events (Dennett, *IS*, pp. 174–175, 184–185, 240–242).

The Liar Paradox

Of course, the Schoolmen were especially interested in The Liar, and it kept them busy for two centuries. Many books were published with names such as *The Unsolvable* (*Insolubile*), which remind us of twentieth century works such as Gödel’s “Undecidable Propositions...”. However, there was no shortage of “solutions”; by 1429 Paul of Venice had published fourteen solutions from his predecessors and added a fifteenth of his own (*Logica magna*; Bocheński, *HFL*, pp. 241–251).

Ockham’s solution

Ockham was one of the first to treat The Liar as a serious logical problem, and a chapter of his *Summa logicae* (Part 3) was called “About Insolubles.” His solution was to claim that the self-contradictory statement was illegitimate, because the subject of the proposition was the proposition itself; thus it was being simultaneously used and mentioned. This solution was not considered adequate at the time, and in fact did not handle paradoxes such as:

Socrates believes this and no other: ‘Plato is deceived’... but
Plato... believes this: ‘Socrates is not deceived’. (Bocheński, *HFL*, 35.14)

Here we have a reciprocal reference between two propositions: *S* affirms *P*, but *P* affirms not-*S*, yet neither directly mentions itself.\(^{18}\)

Buridan’s Solution

Buridan devised a better solution, which is quite contemporary in approach. He observed that the semantics of the language creates a correspondence between each object-language proposition *P* and a corresponding metalanguage statement, “‘*P*’ is true”. Therefore, the following two propositions must have the same truth value:

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\(^{18}\)In computer science this is called *indirect recursion*, as opposed to the *direct recursion* of The Liar.
3.3. COMBINING IMAGES AND LETTERS

this is false.  \iff \text{‘this is false’ is true.}

The result is a contradiction. For if $Q = \text{‘this is false’}$, then the meaning of $Q$ is that $Q$ is false. But $Q$ also implies the right-hand side, which says $Q$ is true. Since $Q$ implies contradictory propositions, it must in fact be false. As Moody (ML, p. 534) puts it, “a proposition depends on the semantical structure of the language to which it belongs, so that it cannot be used to violate the conditions which give it its status as a linguistic expression.” Investigation of The Liar continues still (e.g., Barwise & Etchemendy, Liar; Martin, PL), and it is the basis of Gödel’s and Turing’s proofs of the limitations of formal logic and digital computation (Ch. 7).

We’ve seen that in the course of trying to use Latin as a logical language, the Schoolmen investigated a variety of deep semantical problems in logic and language, and developed sophisticated solutions to many of them, including the theories of intention and supposition. On the other hand, the very difficulty of working with a natural language may have opened the way for the creation of ideal, artificial languages for logic and science. In Section 3.4 we’ll look at Lull’s early, naive, but very influential approach, and later consider the much more sophisticated systems of Wilkins, Leibniz and Boole. Out of them symbolic logic developed into a powerful tool for knowledge representation and inference, which encouraged its later use in AI and cognitive science.

3.3 Combining Images and Letters

3.3.1 The Art of Memory

Before addressing Lull and his Art, I consider briefly several ideas that lurk in the background of the theories of thought and computation that are the topic of this chapter. We begin with the Art of Memory (Ars Memorativa), which has its roots in ancient Greece, where the poet Simonides of Ceos was credited with its invention.\footnote{Principal sources for this subsection are Yates (AoM), Bolzoni (GM), and Small (WTM, Pt. II). The latter deals with the cognitive science of memory techniques, as does McDaniel & Pressley (IRMP).} It was practiced, especially by orators, throughout antiquity and continued to be popular through the middle ages and Renaissance; indeed the techniques are still taught (e.g., Bellezza, IYMS).

Simonides of Ceos: 556–468 BCE

The Method of Places
One of the central techniques (credited to Simonides) was called the method of places and was used often to remember speeches. First, each topic in the speech was associated with a vivid, emotion-laden and therefore memorable image; in this way the individual topics were remembered. Active images (imaginæ agentes), incorporating motion, were especially recommended. Then the information was organized — the topics placed in the correct order — by making use of human spatial memory, for the ancient technique was to imagine some familiar place, such as the forum, with a number of distinct places (Grk. topoi, Lat. loci), for example, temples, fountains, shops, and street corners. In the imagination, the images were put in the selected places in the order that the places would be reached in a walk, and later the speaker remembered the topics in the correct order by imaging a walk from each place to the next. At each place, the image reminded the speaker of the topic to be addressed at that point in the speech.\(^{20}\)

In summary, the method of places made use of emotion-laden sensuous images located in physically real places, with the spatial organization of the places representing the logical organization of the individual topics. In modern terms the image is an encoding technique that gives a distinctive visual representation to an abstract topic, and the places constitute a unified spatial representation for the topics’ organization; they correspond to visual-processing pathways in the brain specialized to what and where.

As the art of memory developed over the centuries, new techniques were developed as well as new applications. One new development was the use of “fictitious places,” such as imaginary palaces or theaters, which could represent abstract and complex relationships among the memorized ideas. There were even attempts, such as Giulio Camillo’s (c.1480–1544) Memory Theater, to construct physical models of these memory structures, in which physical images of important ideas were arranged in an architectural space. In the Middle Ages the art of memory was used for devotional practices, such as contemplating the virtues and vices, images of which were organized in an imaginary physical structure, such as a tree, tower, or ladder. In this way the ideas were impressed on the memory, which was supposed improve one’s character (a purpose of the art of memory since ancient times). In the Renaissance and Baroque periods, the art of memory was explored as a

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\(^{20}\)Yates (AoM, p. 46) remarks that the widespread use of the method of places is probably the reason that we use the word topic for the subjects located at the places (topoi). “Common places” (Lat., communes loci; Grk. koinoi topoi) were used for typical topics, which are still called “commonplaces” (Small, WTM, p. 90).
method for organizing all human knowledge, as will be discussed later.

### 3.3.2 Combinatorial Inference

Lull’s Ars Magna takes a *combinatorial* approach to inference, so it is worthwhile to consider this before looking at his system. Recall that Aristotle classified all syllogisms into three *figures* (Section 2.5.4). Furthermore, he recognized propositions in four forms, which in the middle ages were called A, E, I and O:

- **A**: Universal Affirmative
- **E**: Universal Negative
- **I**: Particular Affirmative
- **O**: Particular Negative

The A and E forms are called *universal* propositions, the I and O *particular*. For example,

- **A**: All men are mortal
- **E**: No insects are mammals
- **I**: Some men are Greek
- **O**: Some men are not Greek

In principle, each of the three propositions that constitute a syllogism could be in any of these four forms. Thus we have the $4^3 = 64$ possible *moods* of the syllogism: AAA, AAE, . . . , OOI, OOO. Since each mood can occur in three figures, there are a total of 192 possible Aristotelian syllogisms.

Certainly not all of the 192 possible Aristotelian syllogisms are valid, and Aristotle had already determined which are valid and which aren’t. In the process he had identified a number of general rules that must be obeyed by any valid syllogism. Later commentators listed the valid syllogisms by a process of elimination: enumerate all the possible syllogisms and then strike out those which are invalid. “Generate and test” procedures of this kind are still widely used in artificial intelligence and other computer applications (see also p. 135).

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21 These abbreviations can be remembered by the mnemonics AffIrmo (for the affirmations) and nEgO (for the denials).

22 For example, Ockham enumerated the 1368 modal syllogisms in his formulation, and identified nearly 1000 that are valid.
CHAPTER 3. WORDS AND IMAGES

The problem with combinatorial procedures such as these is that reasonably rich rules of combination may lead to a combinatorial explosion of possibilities. We have seen that even the simple Aristotelian syllogism leads to $3 \times 4 \times 4 \times 4 = 192$ possibilities. In general the number of items to be enumerated increases exponentially with the size of the items (e.g., $4^3$ syllogisms composed of 3 propositions). For practical sized problems, testing all the combinations may even exceed the capabilities of modern computers. Thus combinatorial explosion remains a problem in contemporary combinatorial algorithms. It is often the reason that a demonstration AI program will not “scale up” to practical sized problems.

3.3.3 Kabbalah

Kabbalah Defined

Another source of Lull’s ideas was Kabbalah (Hebrew for “tradition”), a kind of mystical Judaism that was becoming popular when Lull was active, and that includes a number of ideas and practices that are relevant to our topic.\(^{23}\)

Importance of Hebrew Alphabet

Fundamental to the Kabbalah is the idea that the structure of the universe is reflected in the Hebrew alphabet. Therefore the truths of heaven and earth can be discovered by contemplation and manipulation of the letters. This belief is based in part on the idea that the Torah (Law), written in Hebrew, reflects the logos (the rational structure) of the world, that is, the realm of archetypal forms or ideas; thus kabbalists identified the Torah with wisdom and the active intellect. As we will see, kabbalistic interpretation of the Torah suggested a process for generating knowledge though the manipulation of symbols.

The Sefirot

Kabbalists explain the creation of the world as an emanation from En Sof (the limitless and unknowable — that is, in/(de)finite — God) down through ten sefirot (Heb., spheres) of increasing degrees of determination and delimitation, terminating ultimately in the material world. These spheres correspond to the Decad, the numbers 1 – 10, an idea with Pythagorean roots

\(^{23}\)Also transcribed cabala, cabbala, kabala, qabalal etc., from Hebrew *qabalah* from יְבָלָה (receive, take); some authors use the variant spellings to distinguish the original Jewish tradition from its later use by Christians and Renaissance occultists, but there is no widespread convention. (For the most part I have used ALA-LC Romanization conventions.) The books by Gershon Scholem (e.g., *Kab*, *K&S*, *Z*) and by Moshe Idel (e.g., *K*, *AP*) are authoritative. The principal sources for this section are Eco (*SPL*, ch. 1) and Poncé (*K*).
3.3. COMBINING IMAGES AND LETTERS

(Scholem, *K&I*, p. 167), and are traditionally named Crown, Understanding, Wisdom, Power, Mercy, Beauty, Glory, Victory, Foundation, and Kingdom. As the principal determinate emanations of God, the *sefirot* are taken to be the chief divine attributes, the primary names of God.

The *sefirot* are often displayed in a geometrical arrangement called the Tree of Life (*Otz Chaim*), in which relationships between certain of these divine attributes are represented by lines connecting them. (As will be discussed later, there is a connection here with the use of trees and other “fictitious places” in the Art of Memory.) In the most common such diagram, there are 22 connections, corresponding to the 22 letters of the Hebrew alphabet, also considered divine names (see Fig. 3.1). The *Sefer Yetsirah (Book of Creation)*, one of the primary kabbalistic texts, says that the sefirot and the Hebrew letters together are the 32 “paths of wisdom,” which are the “stones” from which God created the universe, “by number, writing, and speech” (Wescott, *SY*, 1.1). Elsewhere (2.4), it says that God affixed the 22 letters to a wheel with 231 gates, which He rotated forward and backward (a process imitated, as we’ll see, by Lull); note that 231 is the number of combinations of two different Hebrew letters. In Ch. 4 the text observes that God was able to produce the inconceivable diversity of the world through the permutations of larger numbers of letters, for there are \(2! = 2\) permutations of two particular letters, \(3! = 6\) permutations of 3 letters, \(6! = 720\) of 6 letters, \(7! = 5040\) of 7 letters, and so forth. The number of permutations of the letters of the Hebrew alphabet is \(22! \approx 1.124 \times 10^{21}\), an astronomical number. Thus combinatorial explosion, the generative productivity of a finite set of symbols, accounts for the limitless multiplicity of our world.

From this kabbalistic perspective, the Hebrew names for things are not arbitrary, but reflect in the arrangements of their letters the true nature of things. Therefore one can discover truth both by investigating the arrangements of letters in a sacred text, such as the Torah (encoding the *logos* of the world), and by contemplating new arrangements generated by combinatorial processes.

For example, hidden connections between words are discovered by means of *gematria*, a practice based on the assignment of numerical values to Hebrew letters. Thus the messiah (*mashiah*) is related to the brass serpent (*nahash*) of Moses because the two words add up to the same numerical

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24Heb.: *Kether, Hokhmah, Binah, Gevurah, Hesed, Tifereth, Hod, Netsah, Yesod, Malkuth*, respectively. As usual, there are transcriptional variations.
Figure 3.1: The Kabbalistic Tree of Life. The spheres (sefirot) represent a decad of divine attributes or “emanations” corresponding to the numbers one to ten. The 22 lines, which relate certain pairs of attributes, are marked with the 22 letters of the Hebrew alphabet. The ten sefirot and 22 connections together constitute the 32 “paths of wisdom.”
value (368). As a result, underlying the surface text of the Torah, kabbalists discover hidden connections established by quantitative relationships (a conceptual precursor to the scientific identification of hidden quantitative relationships underlying perceptible reality). Other hidden meanings are discovered by other formal manipulations of the text (e.g., taking the first letters of every word in a phrase, or the last letters; exchanging corresponding letters in the first and second halves of the alphabet). Of course it was not uncommon to seek unapparent, allegorical, hidden, or esoteric meanings in sacred texts (it was also common in Christian and Islamic exegesis, for example, and even applied to the Homeric epics); what is important to notice here is the emphasis on the numerical and combinatorial manipulation of letters interpreted as archetypal ideas.

According to the rules of gematria, the sacred four-letter name of God, YHWH ($\text{יִהְוָה}$), has the value 72, and indeed God is said to have 72 divine names. Contemplation of the names of God was a common practice in Christianity and Islam as well as in Judaism, but in kabbalah it took an especially combinatorial form. For example, the kabbalist Abraham Abulafia (1240–91, contemporary with Lull) described a contemplative practice in which one turned the divine name “like a wheel,” generating various permutations, until it produced a word of wisdom. In effect, he was proposing a mechanical method for generating knowledge, a goal that also lies behind the attempts to mechanize thought, which we will discuss, and the development of scientific method. These techniques were supposed to work because, it will be recalled, the letters were the constituents from which God created the world.

3.4 Lull: Mechanical Reasoning

The understanding longs and strives for a universal science of all sciences, with universal principles in which the principles of the other, more special sciences would be implicit and contained as is the particular in the universal . . .

— Ramon Lull (Ars magna et ultima, 218; Bocheński, HFL, § 38.01)
The subject of the Art is the answering of all questions, assuming that one can identify them by name.

— Ramon Lull (*Ars brevis*, prologue; *SWRL*, p. 579)

Lullum, antequam Lullum noscas, ne despicias.

— A. Oliver (*Raymundi Lulli opera medica*)

**Question:** Whether God exists.

**Solution:**

| AA | being perfection | privation imperfection | SV | YZ |

— Lull (*Ars demonstrativa*; Llull, *SWRL*, p. 444)

As for Astronomy, study all the rules thereof, let passe nevertheless, the divining and judicial Astrology, and the Art of Lullius, as being nothing else but plain abuses and vanities.

— Rabelais (*Gargantua and Pantagruel*, Bk. 2, Ch. 8)

After salutation, observing me to look earnestly upon a frame, which took up the greatest part of both the length and breadth of the room; he said, perhaps I might wonder to see him employed in a project for improving speculative knowledge by practical and mechanical operations.

— Swift (*Gulliver’s Travels*, Pt. 3, Ch. 5)
3.4. **LULL: MECHANICAL REASONING**

### 3.4.1 Background

Ramon Lull would likely be a minor figure in the history of philosophy were it not for his invention in 1274 of the *Ars Magna* (Great Art), which exerted a powerful influence on later philosophers, and indirectly on AI and cognitive science.\(^{25}\)

He lived the life of a rake for some 30 years, until he saw a series of visions and became devout. After nine years of study, he spent a week contemplating God on Mt. Randa (near Palma) in 1274, and had “revealed” to him the Great Art; as a consequence he was later known as *Doctor Illuminatus*. After his illumination Lull became a prolific author, producing over 260 works (fiction as well as nonfiction), of which nearly 240 survive. Most of them were composed in Catalan or Arabic, and he effectively created literary Catalan.

After Lull’s death, *Lullism* began to grow, slowly at first in the fourteenth and fifteenth centuries, but then flourishing in the sixteenth and the first half of the seventeenth. Here we find it exerting its influence on the coinventers of the calculus, Leibniz (Section 4.3) and Newton (who owned eight volumes of Lull, including six, spurious alchemical works\(^{26}\)). It also inspired, as we’ll see, the pursuit of universal languages for representing existing knowledge and of systematic methods for generating new knowledge.

Aspects of Lullism continue in many forms to this day. For example, his Tree of Science (*Arbor scientiae*) proposed a unified hierarchy of all the sciences, which anticipates the “Unity of Science Movement” in 20th century logical positivism (Ch. 8).

### 3.4.2 Ars Magna

Like his predecessors, Lull takes a combinatorial approach to inference, but his *Ars Magna* is different from their approaches in two important respects.

First, he used simple devices for enumerating all the combinations (discussed later). Although the idea is simple, and others may have done this before him, Lull is probably the first to use mechanical aids systematically.

\(^{25}\)There is considerable variety in the spelling of Lull’s name; one finds Lull, Lully, Lulio, Lullius, Lulle and Llull, as well as Ramon, Ramón, Raimundo and Raymund. Although the modern Catalan spelling is Ramon Llull, I will use the more familiar Lull.

\(^{26}\)Spurious, because there is no evidence that Lull was an alchemist, per se.
His approach was to identify the most basic concepts in any field of study, and then to use his devices to generate all possible combinations of these concepts. He believed that by contemplating the possible relationships of these elementary concepts he could discover the primary truths of that field.

Second, Lull did not restrict his Great Art to a narrow domain, such as the science of the syllogism. Rather, he viewed his art as a means of discovering the deepest truths in every field of knowledge. Thus Lull believed he had invented a device that would allow him to generate mechanically the primary truths of all the sciences. Although he was incorrect in this belief, his vision inspired many later investigators of automated reasoning (Section 4.3 and Chapter 9) and contributed to the search for a reliable method for discovery of knowledge. Lull’s aims were similar to the kabbalists in that he thought he was exploring the archetypal ideas underlying and permeating the world.

A premiss of Lull’s approach is that there are elementary concepts in a given knowledge domain. That is, he was assuming a kind of logical atomism, the view that all categories can be analyzed into a certain number of atomic categories that cannot be further subdivided. Lull believed that by exploring all the possible relations between the atomic categories he would discover the primary truths (like those stemming from Aristotelian definitions, p. 47). These primary truths would in turn imply other truths, including relationships between composite categories (those defined in terms of the atomic categories).

The simplest Lullian device is a circle circumscribed with the basic concepts of some domain. Lines connect all the possible two-term combinations (Fig. 3.2, p. 89); some of Lull’s devices involved two or more concentric disks (Fig. 3.5, p. 92). By rotating these disks all the two (or more) term combinations could be generated (reminiscent of kabbalistic rotation through the permutations of the Hebrew alphabet). These devices have motivated some to call Lull the “father” of computer programming (Moody, ML, p. 530)! To see if this claim is at all justified, we will have to look at his Art in more detail.

In an early version of the Ars Magna (presented in *Ars demonstrativa*, c. 1260), Lull used a method of reasoning that is very similar to modern logical atomism. He believed that all knowledge could be reduced to a set of basic elements, and that by combining these elements in all possible ways, he could generate all possible truths. This approach is very similar to the work of later logicians like Leibniz, who developed a more sophisticated form of logical atomism.

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27 Leibniz pursued a more sophisticated logical atomism; see Section 4.3. Logical atomism reached the height of its popularity in the twentieth century; see Section 8.3 and Chapter 9.
Lull defines an “alphabet of thought,” which is represented by the 23 letters of the medieval alphabet (ours without J, U, W). The 16 letters B to R represent the basic terms of various subjects; the remaining seven letters (ASTVXYZ) name various figures used to structure the concepts represented by the terms. The subject domains represented by the figures are:

- A Attributes of God
- S Actions of the rational soul
- T Principles and relations
- V Virtues and vices
- X Predestination and objectification
- Y Truth
- Z Falsehood

Except for Y and Z, which represent truth values, and so are atomic, all the other figures contain the 16 terms B–R connected by colored lines in various patterns. In some cases Lull’s figures are quite simple, only showing combinations of two terms (see Fig. 3.2, which is from a later version of the Art employing only nine terms), whereas others were much more complex.

In addition to these figures, there was a Demonstrative Figure (the most complex, with six concentric moving rings), an Elemental Figure, which displayed combinations of the four elements in a square matrix, and figures containing the 16 elementary terms of various specialized disciplines, such as philosophy, theology and the law.

Bonner (Llull, SWRL, pp. 309–310) has described Lull’s Art in computer terms. The figures AVXYZ, the Elemental Figure, and the figures for the special sciences all correspond to the basic data or “knowledge base” upon which the system operates. Figure T, which is a relational figure, corresponds to the processing unit, since it establishes internal relations among the terms. Figure S is also a relational figure, but it establishes external relations, either between the terms and the operator, or between the terms and the person to be convinced by the argument. (In effect it’s a link between the object and meta levels.) Thus it corresponds to a control or input-output unit.

Needless to say, the computer analogy cannot be carried too far, for not only does the Art require a human to sequence the steps (as on a hand calculator), but human interpretation is also necessary for correct execution of the individual steps (as we’ll see shortly). Nevertheless, by reducing a wide range of inquiries to a methodical generate-and-test process, the Art takes an important step toward the mechanization of reasoning.
One thing which Lull’s Art permits is systematic investigation of all the questions in a given domain; thus its combinatorial approach aids completeness. For example, one part of the *Ars demonstrativa* is devoted to systematically posing and solving 1080 questions; in 39 cases the method of solution is explained; in the remaining 1041 Lull gives only the “compartments” that show the relation of the terms — in effect, they are left as “exercises for the reader.”

To give a bit of the flavor of the Lullian Art, I will paraphrase and explain one of his “demonstrations” (Llull, *SWRL*, p. 455):

**Question:** Which is more demonstrable, truth or falsehood?

**Solution:**

```
YZ EAVY IVZ EVZ IAVY XX
```

Y [truth] accords with being and perfection, and Z [falsehood] with privation and imperfection. The first X [objectification?] of the last compartment signifies the first concordance in the second and third compartments.

In the second we have EAVY, where E = B & C & D = remembering & understanding & loving, and AVY = God & virtue & truth; in the third, IVZ, where I = F & G & H = remembering & understanding & hating, and VZ = vice & falsehood.

The second X of the last compartment signifies the second concordance in the fourth and fifth compartments: In the fourth, EVZ = remembering, understanding & loving both vice & falsehood; and in the fifth, IAVY = remembering, understanding & hating AVY, namely, God, virtue & truth.

“This being the case, the question is therefore solved by means of the above-stated signification.”

And two “exercises for the reader” (Llull, *SWRL*, pp. 476, 540):

**Question:** Whether the virtue of the soul exists in the powers in continuous or discrete quantity.

**Solution:**

```
FF EI NR AS SS ST fire fire fire earth fire air
```
Table 3.1: Meaning of Terms in Lull’s Great Art

<table>
<thead>
<tr>
<th>Terms</th>
<th>Absolute Principles</th>
<th>Relative Principles</th>
<th>Questions</th>
<th>Subjects</th>
<th>Virtues</th>
<th>Vices</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>goodness</td>
<td>difference</td>
<td>whether?</td>
<td>God</td>
<td>justice</td>
<td>avarice</td>
</tr>
<tr>
<td>C</td>
<td>greatness</td>
<td>concordance</td>
<td>what?</td>
<td>angel</td>
<td>prudence</td>
<td>gluttony</td>
</tr>
<tr>
<td>D</td>
<td>eternity</td>
<td>contrariety</td>
<td>of what?</td>
<td>heaven</td>
<td>fortitude</td>
<td>lust</td>
</tr>
<tr>
<td>E</td>
<td>power</td>
<td>beginning</td>
<td>why?</td>
<td>man</td>
<td>temperance</td>
<td>pride</td>
</tr>
<tr>
<td>F</td>
<td>wisdom</td>
<td>middle</td>
<td>how much?</td>
<td>imaginative</td>
<td>faith</td>
<td>apathy</td>
</tr>
<tr>
<td>G</td>
<td>will</td>
<td>end</td>
<td>of what kind?</td>
<td>sensitive</td>
<td>hope</td>
<td>envy</td>
</tr>
<tr>
<td>H</td>
<td>virtue</td>
<td>majority</td>
<td>when?</td>
<td>vegetative</td>
<td>charity</td>
<td>ire</td>
</tr>
<tr>
<td>I</td>
<td>truth</td>
<td>equality</td>
<td>where?</td>
<td>elementative</td>
<td>patience</td>
<td>pity</td>
</tr>
<tr>
<td>K</td>
<td>glory</td>
<td>minority</td>
<td>how?</td>
<td>instrumentative</td>
<td>inconstancy</td>
<td></td>
</tr>
</tbody>
</table>

Question: How do angels speak to one another?

Solution: [intelligence habit | AA | dignities act | form relation]

charity justice | beginning end | concordance middle

You may draw your own conclusions ...
Zohar, a principal kabbalistic text, was written in Spain while Lull was there (Yates, AoM, pp. 178). Later, in the Renaissance, the Lullian letters B–K were explicitly identified with the sefirot (Yates, AoM, p. 190), and indeed there is considerable overlap between the Lullian divine attributes (Fig. 3.2 and Table 3.1, “Absolute Principles”) and the names of the sefirot (p. 78). Lull’s notion of the divine attributes was also influenced by the Neoplatonism of John Scotus Eriigena (c.815–77), who identified the divine powers as the first causes of all things; both were influenced by the 5th-century Christian-Neoplatonic Divine Names of pseudo-Dionysius (Yates, AoM, p. 177). In addition, Lull was probably influenced by a contemporary Spanish kabbalistic practice of meditating on the names of God generated by permutation of the Hebrew alphabet, and by Sufi mystics who also recommended meditation on the names of God (Yates, AoM, pp. 178–9). Indeed, Lull thought that these divine attributes were common to Judaism and Islam as well as to Christianity, and so they could be used to convert the former to the latter (Yates, AoM, p. 178).

**Figures**

The terms B–K are the components of the propositions and questions manipulated by the “artist” using Lull’s system. The manipulation itself is facilitated by four figures (Figs. 3.2–3.5).

**First Figure**

The First Figure (Fig. 3.2) shows all the combinations of two terms, where the terms represent the “absolute principles” of Table 3.1. For example, according to context, BF can be interpreted as ‘goodness is wise’, ‘wisdom is good’, or even as ‘good wisdom’ or ‘wise goodness’.

**Second Figure**

The Second Figure (Fig. 3.3) groups the “relative principles” of Table 3.1 into three triads:

- difference, concordance, contrariety;
- beginning, middle, end;
- majority, equality, minority.

These are further divided into three subcategories, represented by the three rings of the figure.

**Third Figure**

The Third Figure (Fig. 3.4) enumerates the possible combinations of terms. The terms can be interpreted as coming from either the First or Second Figure, so a combination such as BC represents the 12 propositions we get by choosing two of:

- good(ness), great(ness), concordance(-ant), difference(-ent)
Figure 3.2: Lull’s First Figure. The edges connect all possible combinations of any two of the “absolute principles.”
Figure 3.3: Lull’s Second Figure. The terms B–K represent the “relative principles.” Each of the inner triangles connects three related principles, for example triangle BCD connects difference, concordance, contrariety. Within this are three concentric rings representing different interpretations of the terms. Thus EFG, which in general represents beginning/middle/end, can be interpreted as cause/conjunctive/privation or quantity/mensuration/termination, etc. Abbreviations: acc. = accid. = accident, intell. = intellectual, sens. = sensual, subst. = substance.
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Figure 3.4: Lull’s Third Figure. The Third Figure simply represents all combinations of two of the terms B–K.

in either order. Further, two questions are implicit in each proposition, for example:

Whether greatness is good.
What is good greatness?

Again, the method is combinatorial.

The Fourth Figure (Fig. 3.5) is the only one in the simplified Art that is an actual device, for the two inner rings can be rotated on top of the outer ring. In this way all the possible triples of the terms B–K can be generated. The triples can be used in many ways, for example, to help find the middle term in a syllogism (by trying BCD, BCE, . . . , BCK). The Fourth Figure also simplifies the exhaustive investigation of a subject. For example, since B = goodness and C = prudence, we can enumerate all the questions about goodness and prudence (BCB, BCC, BCD, . . . , BCK) as follows:

BCB: Whether prudence is good. Whether goodness is prudent.
BCC: What is good prudence? What is prudent goodness?
: : : :
BCK: How is prudence good? How is goodness prudent?

(Further, Lull specifies two to four species of each question, which I’ll pass over.)

Lull’s Art is not a system of logic, nor was it intended as one (as that would “undermine faith”). Although Lull sometimes speaks of the Art demonstrating a proposition, he also speaks of it “manifesting” or “revealing” it. Further, the system is not totally mechanical since it requires a trained operator (the “artist”) to correctly interpret the configurations of terms.
Figure 3.5: Lull’s Fourth Figure. This device allows the two inner rings to rotate over the outer ring, thus permitting all the triplets of the terms B–K to be generated. One use of this figure would be to enumerate possible syllogistic arguments.
Also, it operates on a background of shared cultural beliefs (especially about God), which are also necessary, for example, in deciding whether a result disagrees with commonly accepted truths. Reaching such an “absurd conclusion” would be the basis for rejecting a hypothesis.

An interesting question is whether Lull’s Art ever produced a result that the artist did not already believe. Indeed, it can be argued that this was not its purpose, for Lull’s original goal was to find a means of “converting the infidel” by the word rather than the sword.

Thus the function of the Art was polemical rather than logical. Later, however, Lull seems to have taken the view that the Art could find new truths (see quote on p. 81 regarding the subject of the Art) — but maybe that claim was just another polemical device.

In any case, we must avoid imposing our standards of justification on Lull and his contemporaries. Lull did not want to justify Christianity by rational means, since to try to do so was a heresy, “rationalism,” because it undermined faith. Indeed, Lullism was twice condemned for this very heresy (in 1376 and 1390), though it was later exonerated (1416). (It is worth reminding ourselves that the papal inquisition began just 30 years before Lull’s birth.) In fact, the 1376 condemnation of Lullism was the direct motivation for the publication of the Directorium inquisitorum, the definitive manual of inquisitorial methods (Llull, SWRL, pp. 71–73).

### 3.4.3 From Images to Symbols

Lullism represents an important stage in the transition from imagistic representations to symbolic representations. As we’ve seen, a practitioner of the ancient Art of Memory imagined vivid, emotion-laden, sensuous and active images in real or realistic physical locations. By the Middle Ages there was increasing use of abstract geometrical spaces for organizing the images, such as the Zodiac, the Tree of Life and schematic towers, trees and ladders. In many cases iconographic images or even words were used to encode the ideas organized by these structures. (Indeed, realistic images were suspect, because they might lead the pious contemplative away from the path of virtue!)

Lull replaced images with abstract letters organized into geometrical figures, thus permitting contextual interpretation of the individual symbols and their mechanically generated combinations. (In effect, Lull exchanged the natural motion within the memory images of classical mnemotechnics for mechanical motion between lifeless letters.) In this approach we can see the
influence of the kabbalah. The movement from rich imagery to abstract symbols continued in the Renaissance with explicit attempts to combine Lullism and kabbalah and with projects to “mathematize” Lullism by replacing the letters by numbers (Yates, AoM, pp. 178–9, 364–5). This trend continued into the early modern period and influenced the birth of science and mirrored the iconoclasm of the Protestant Reformation (Yates, AoM, p. 231; see also Sec. 3.4.5 below). It underlies the development of symbolic logic and symbolic AI, although the importance of images and imagistic cognition have been rediscovered in connectionist AI and cognitive science and in studies of imagery in the philosophy of science and engineering.

3.4.4 Significance

Lull’s Art was inspired and inspirational, and many of its ideas have been adopted by mathematics, symbolic logic, programming languages and other notational systems. By representing the basic concepts by single letters Lull allows propositions to be expressed compactly enough so that their structure is easily seen and so that they can be formally (algebraically) manipulated. In particular, this permits the combinatorial generation of all formulas of a given kind. Further, the terms make explicit the basic concepts of any subject, as well as the relation of composite concepts to the basic ones. We will see that these are basic characteristics of many “ideal” or “philosophical” languages, including the knowledge representation languages used in AI (Sections 4.2, 4.3 and ??).

The figures represent potential or actual relationships between the terms. For example, the First Figure shows potential combinations of two terms; the Second Figure shows the triads of relative principles. Other figures, such as the Fourth, actually facilitate mechanical manipulation. Finally, the figures group the basic concepts of the various subjects into knowledge structures, and are mnemonic devices that help the artist to keep the basic concepts and their relationships in mind. (See Section ??.)

One of Lull’s contributions to the theory of knowledge was the idea that the primary truths of a science could be discovered by a two step process: (1) identify the elementary concepts of that science; (2) mechanically generate all possible combinations of the elementary concepts. By actually constructing

[28]Recall (p. 53) Plato’s idea that theories are constructed from elements, which are analogous to the letters from which syllables are constructed. Lull carries this out quite “literally”!
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Figure 3.6: The Tree of Porphyry. (This is the common medieval version of the Tree, slightly simplified; his was simpler yet.)

combinatorial devices he demonstrated his method (and incidentally showed its limitations). Even more important, however, than Lull’s specific Art, was his vision of a universal method, an inferential process for generating knowledge. This idea found its most direct expression in Leibniz (Section 4.3), but indirectly influenced many others.

3.4.5 Ramus and the Art of Memory

Before leaving these medieval developments, I’ll briefly discuss another one of Porphyry’s contributions to philosophy, which was important in the development of logic and symbolic artificial intelligence, the Tree of Porphyry (Fig. 3.6). It is a dichotomy, that is, a classification in which at each level of the tree we have a binary division, based on contrary predicates, \( P \) and non-\( P \). From at least the time of the Sophists a dichotomy was generally
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considered the most perfect kind of classification. The top of the tree is the *summum genus*, the most general possible class, which in medieval times was taken to be “substance” (*substantia*, that which “subsists through itself”; McKeon, *SMP*, Vol. 2, pp. 499–500). At the bottom of the tree we have individuals, such as Socates and Plato. Therefore, if the entire tree were filled out we would have a complete classification of everything in the universe — animals and people, minerals and vegetables, fire and air, earth and water, angels and devils. Further, it was believed, based on an assertion of Aristotle (*Met. Z* 1032a13–1033a24), that this classification was not arbitrary, but that there was a correct way to do it based on the *true* definition of each class. For example, from the diagram we can see that men (i.e., people) are rational animals and that animals are sensible (e.g., capable of sensation) living beings; therefore men are rational sensible living beings. Continuing this way, we can obtain a compete essential definition of a concept: man is the rational sensible animate corporeal substance. Thus the Tree was taken to be an objective matter of fact representing the actual structure of the universe, “the Great Chain of Being” (Lovejoy, *GCB*; *DHI*, v. 1, pp. 325–35). This is to be expected, for Porphyry and other Neoplatonists, in common with Platonists and Aristotelians, took a *realist* view of universals (recall Sec. 3.2.1).

Class hierarchies such as the Porphyry’s Tree are still widely assumed in AI and cognitive science, and are used in object-oriented programming languages. They permit concepts to be represented by binary features, corresponding to the yes/no directions one takes on a path from the “root” (*summum genus*) to the concept in question. Unfortunately, although class hierarchies are very tidy, there is psychological evidence that our concepts are not organized this way (discussed in Vol. 2), and they have even been found overly-restrictive for programming (MacLennan, *POPL*, pp. 418–421).

The Porphyrean Tree became important in the sixteenth century as a result of the educational reforms of Peter Ramus (Pierre de la Ramée, 1515–72). He was concerned with developing a systematic method for discovering

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29 This is the view taken by the Sophists in their Method of *Diaeresis* or Division (Kerferd, *SM*, pp. 74–75). Later philosophers, including Plato (*Phil. 16d*) and Aristotle (*Part. Anim. 642b5–644b15*), agreed that division must be into the fewest number of subkinds, but thought that this was sometimes more than two. See also Joseph (*IL*, pp. 121–128).

30 Sources for Ramus include Yates (*AoM*, ch. 10) and Rossi (*LAM*, ch. 5)
and transmitting knowledge (especially in rhetoric, logic, mathematics and geometry), and to this end he combined the arts of memory and logic. Therefore the *Ramean Tree* (or *Ramean Epitome*) proceeds by logical dichotomy from the most general term of any subject matter. In effect the Ramean Tree is an abstract geometrical diagram of the (supposed) essential structure of reality.

By organizing knowledge in memory according to this essential structure Ramus hoped to develop a method for preserving, transmitting and discovering knowledge (for classification is often the first step in a scientific investigation). Therefore Ramism contributed to the development of scientific method by Bacon, Descartes and Leibniz.

With his greater stress on logic, Ramism moved further from the vivid images of the classical art of memory and further in the direction of Lullist abstraction. Yates (*AoM*, pp. 228–9) observes that the Ramist reforms were directed against scholasticism, and therefore were attractive to Protestants. Further, “The extraordinary success of Ramism, in itself a superficial pedagogic method, in Protestant countries like England may perhaps be partly accounted for by the fact that it provided a kind of inner iconoclasm, corresponding to the outer iconoclasm” (Yates, *AoM*, p. 231).