ECE692

Small Signal Discrete Time Modeling of a DCM Buck Converter

In this problem, you will investigate the control-to-output transfer function of a 12-to-3 V Buck converter shown in Fig. 1

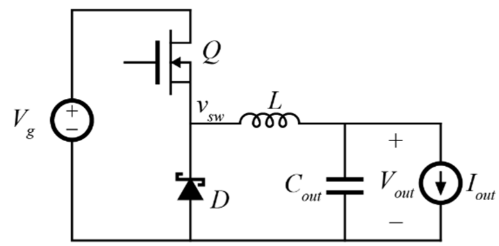

Fig. 1: Buck Converter

Table I: Converter design specification

$r_{on,Q}$	$r_{on,D}$	f_s	L	V_g	I_{load}	C_{out}
10 mΩ	10 mΩ	500 kHz	1 µH	12 V	1 A	10 µF

When conducting, the diode and MOSFET can be modeled as on-resistances of value $r_{on,D}$ and $r_{on,Q}$, respectively. When not conducting, both can be modeled as ideal open-circuits.

Derive the control-to-output transfer function $G_{vd}(z)$ from $d(t)$ to V_{out}. An ADC samples the output voltage at the instant the MOSFET turns on, with zero conversion delay.

a) Find the steady-state duty cycles D and D_2 of the converter such that the output voltage is V_{out} = 3 V and diode conduction is correctly modeled, where D is the duty cycle of MOSFET conduction and D_2 is the duty cycle of diode conduction.
b) Derive the traditional averaged model of the control-to-output transfer function $G_{vd,avg}(s)$.
c) Derive a generalized state space model of the system of the form

$$x[k+1] = f(x[k], u[k], D[k], w[k])$$
$$0 = \sigma(x[k], u[k], D[k], w[k])$$

Give full realizations of both equations, and state explicitly what auxiliary variable(s) are included in $w[k]$
d) Solve the system in (c) to obtain a complete discrete time small-signal model of the DCM buck converter. Give expressions for each matrix/vector in terms of the state space models of the converter.
e) On the same axes, produce a bode plot of the averaged model $G_{vd,avg}(s)$ and your discrete time model $G_{vd}(z)$, using the built-in discrete time conversion in the `bode()` function. Comment on any differences.