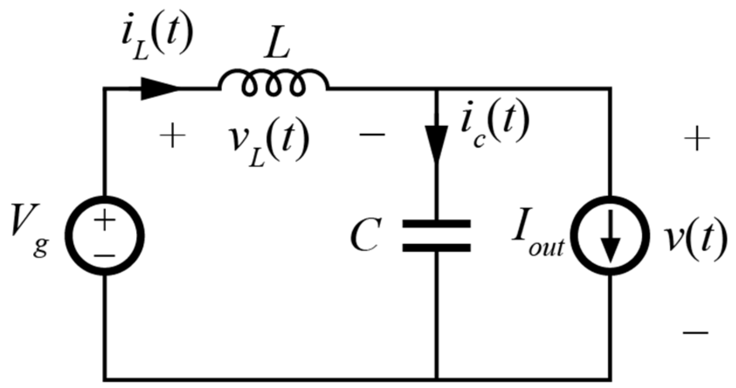
Formulation of State Space Matrices

- Network theoretic approaches exist for the formulation of state space equations
- Available commercial tools, e.g. PLECS expose solved state space descriptions to the user through command-line interface

$$
\begin{bmatrix}
B_{v_3} & B_{v_4} & B_{v_5} & B_{v_6} & B_{v_7} & B_{v_8} & B_{v_9} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & Q_{i_1} & Q_{i_2} & Q_{i_3} & Q_{i_4} & Q_{i_5} & Q_{i_6} & Q_{i_7} \\
-U_3 & \cdot & G_{35} & G_{36} & \cdot & G_{38} & \cdot & \cdot & G_{32} & \cdot & G_{34} & G_{35} & \cdot & \cdot \\
\cdot & -U_4 & G_{45} & G_{46} & \cdot & G_{48} & \cdot & \cdot & G_{42} & \cdot & G_{44} & G_{45} & \cdot & \cdot \\
\cdot & \cdot & Y_C(D) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -Z_{LR}(D) & \cdot & \cdot \\
\cdot & \cdot & G_{65} & G_{66} & \cdot & G_{68} & \cdot & \cdot & G_{62} & \cdot & G_{64} & G_{65} & -U_6 & \cdot \\
\cdot & \cdot & G_{75} & G_{76} & \cdot & G_{78} & \cdot & \cdot & G_{72} & \cdot & G_{74} & G_{75} & \cdot & -U_7
\end{bmatrix}
\begin{bmatrix} v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}
=
\begin{bmatrix}
B_{v_1} & B_{v_2} & \cdot & \cdot \\
\cdot & \cdot & Q_{i_8} & Q_{i_9} \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot
\end{bmatrix}
\begin{bmatrix} e_1 \\ e_2 \\ j_8 \\ j_9 \end{bmatrix}.
$$

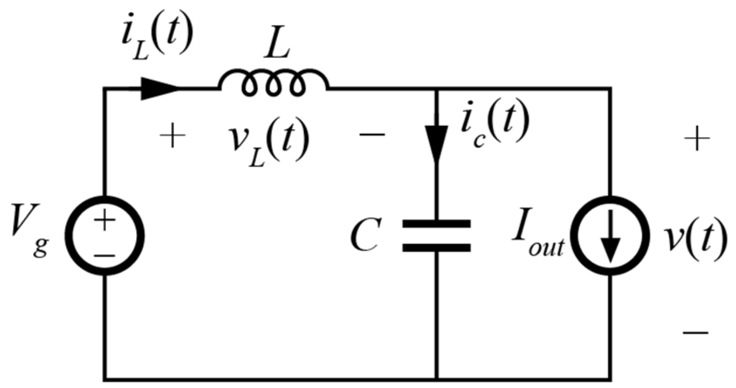
L. O. Chua and P. M. Lin, Chapter 8, *Computer-Aided Analysis of Electronic Circuits: Algorithms & Computational Techniques*, 1975
E. Purslow, "Solvability and analysis of linear active networks by use of the state equations," in *IEEE Transactions on Circuit Theory*, 1970

Example State Space Parsing

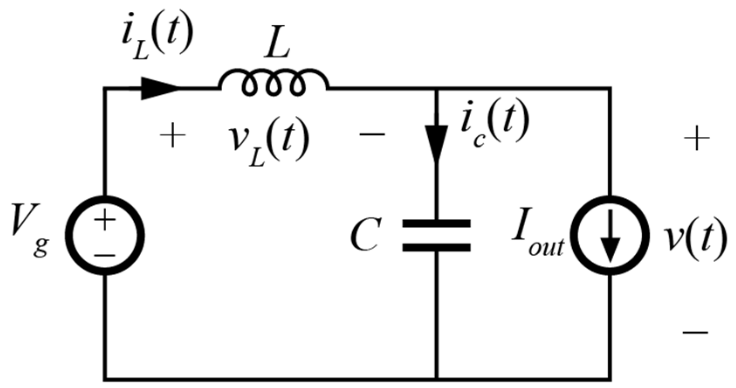


$$\frac{d}{dt}\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \cdot \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} & \\ & \end{bmatrix} [V_g \quad I_{out}]$$

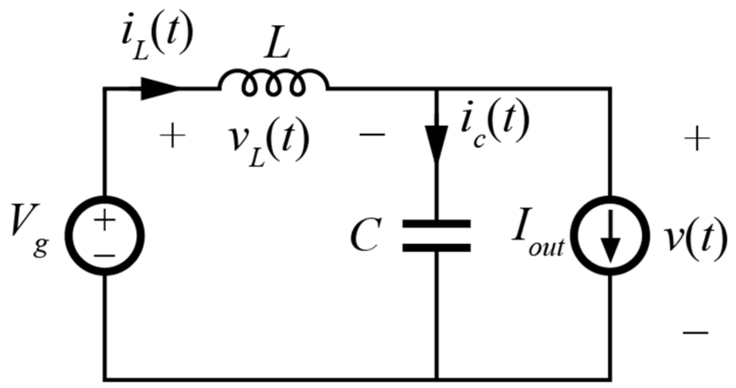
Component Partitioning



Incidence Matrix

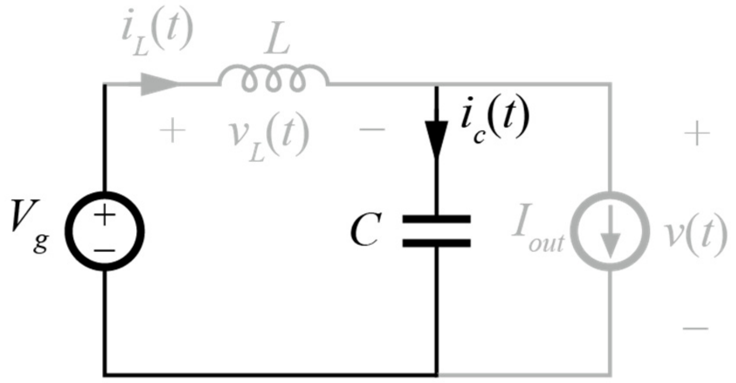


Loop Matrix

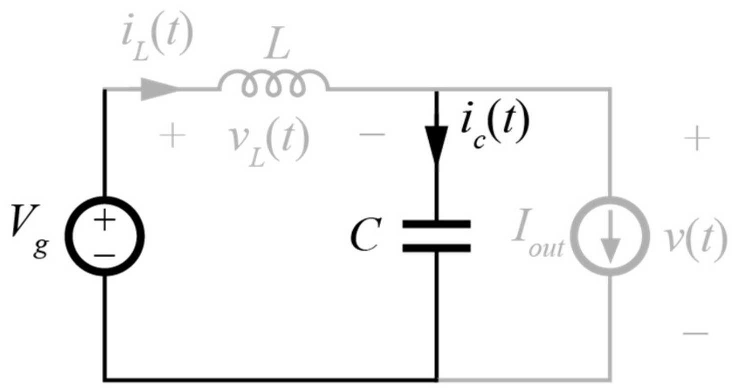


Partitioning

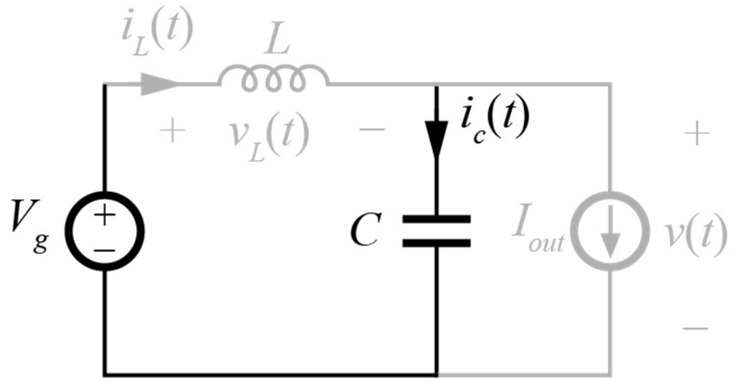
Selecting a Tree



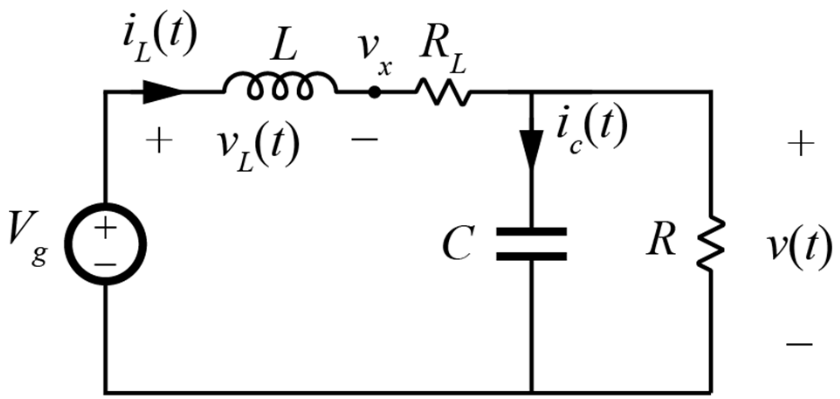
Loop and Cutset Matrices



Final State Space



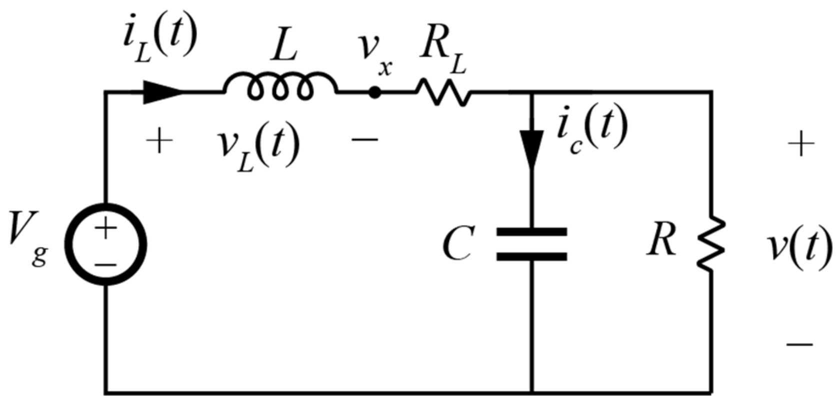
Including Resistances



Components

Nodes	V_g	C	R	R_L	L
in					
x					
out					
gnd					

Including Resistances



- Additional step to eliminate all resistor currents/voltages
  - solve voltage on tree resistors and current in link resistors

$$\begin{array}{ccccc} V_g & C & R_L & R & L \end{array}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} = [\, I \mid D_L \,]$$

$$D_L = \begin{bmatrix} D_{EG} & D_{EJ} \\ D_{RG} & D_{RJ} \end{bmatrix}$$

$$\begin{bmatrix} I_{V,C} \\ V_{I,L} \end{bmatrix} = \begin{bmatrix} -(D_{EG} Z^{-1} D_{EG}^T) & D_{EG} Z^{-1} D_{RG}^T Z_R D_{RJ} - D_{EJ} \\ D_{EJ}^T - D_{RJ}^T Y^{-1} D_{RG} Y_G D_{EG}^T & -(D_{RJ}^T Y^{-1} D_{RJ}) \end{bmatrix} \begin{bmatrix} V_{V,C} \\ I_{I,L} \end{bmatrix}$$

$$\begin{bmatrix} i_g(t) \\ i_c(t) \\ v_L(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -\frac{1}{R} & 1 \\ 1 & -1 & -R_L \end{bmatrix} \begin{bmatrix} V_g \\ v_c(t) \\ i_L(t) \end{bmatrix}$$

L. O. Chua and P. M. Lin, Chapter 6, *Computer-Aided Analysis of Electronic Circuits: Algorithms & Computational Techniques*, 1975