Benefits of DT models in control:

1. Correct over all frequencies
2. Suitable for Digital Control
3. DT models are more suitable for advanced converters

Section 2

DYNAMIC MODELING

Discrete Time Model

steady-state solution

$$X_0 = \left(I - \prod_{i=k}^{1} e^{A_i t_i}\right)^{-1} \sum_{i=1}^{k} \left(\prod_{j=k}^{i+1} e^{A_j t_j}\right) A_i^{-1}\left(e^{A_i t_i} - I\right) B_i u_i$$

came from

$$X(T_s) = \underbrace{\left[\prod_{i=k}^{1} e^{A_i t_i}\right]}_{\Phi} X_0 + \underbrace{\left[\sum_{i=1}^{k} \left(\prod_{j=k}^{i+1} e^{A_j t_j}\right) A_i^{-1}\left(e^{A_i t_i} - I\right) B_i\right]}_{\Psi} u_i$$

$$x[k+1] = \Phi\, x[k] + \Psi\, u[k]$$

→ No assumptions,
LTI discrete time dynamic model

Discrete Time Model

$$\begin{cases} x[k+1] = \Phi x[k] + \Psi u[k] \\ y[k] = \delta x[k] + \beta u[k] \end{cases}$$

if we change modulation:

$$x[k+1] = \underline{\Phi(t_i)}\, x[k] + \underline{\Psi(t_i)}\, u[k]$$

Nonlinear DT state space model wrt t_i

Discrete Time state space model

- Analyze with z-transform & difference equations

CT analogue : (Laplace) (differential)

z-Transform:

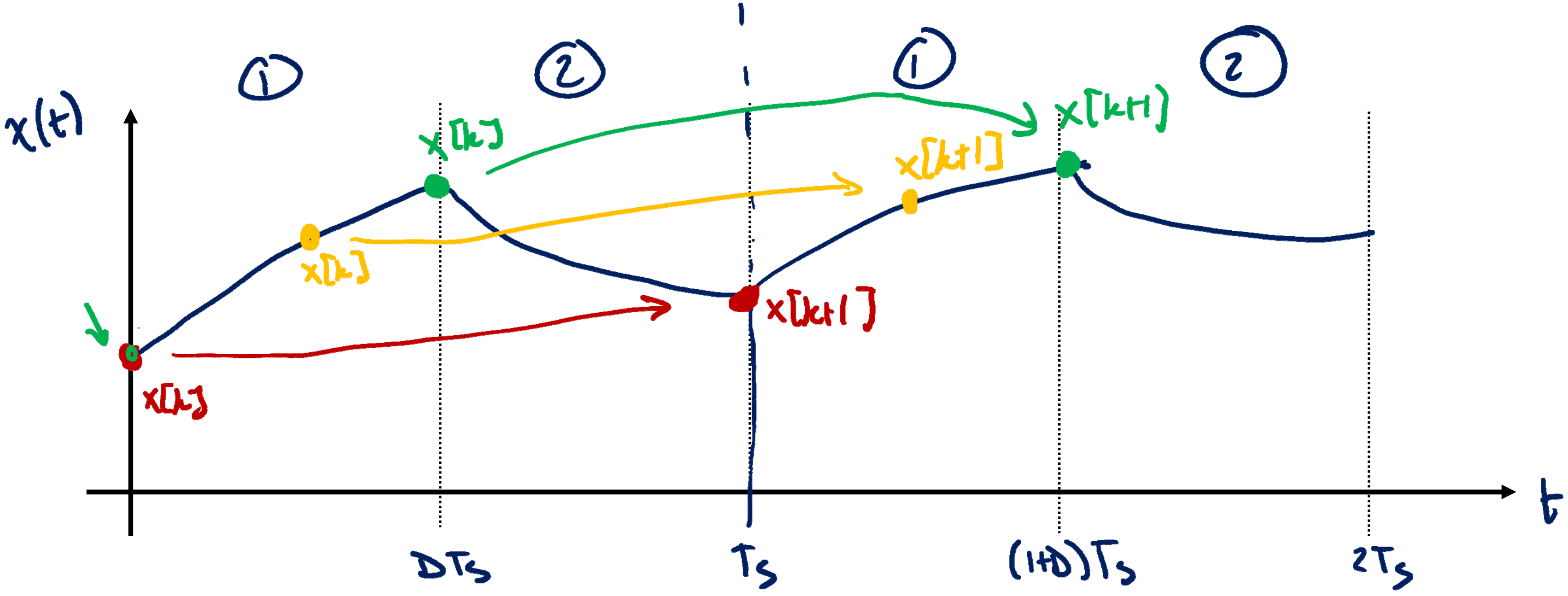
z^{-1} is the unit delay at the sampling rate (nominally f_s)

$$\begin{cases} zX(z) = \Phi X(z) + \Psi U(z) \\ Y(z) = \delta X(z) + \beta U(z) \end{cases}$$

$$\frac{X(z)}{U(z)} = (zI - \Phi)^{-1}\Psi \quad \text{or} \quad \frac{Y(z)}{U(z)} = \delta(zI - \Phi)^{-1}\Psi + \beta$$

Note: $u(z)$, $u[k]$ contains all independent sources, but modulation/control not present in this model

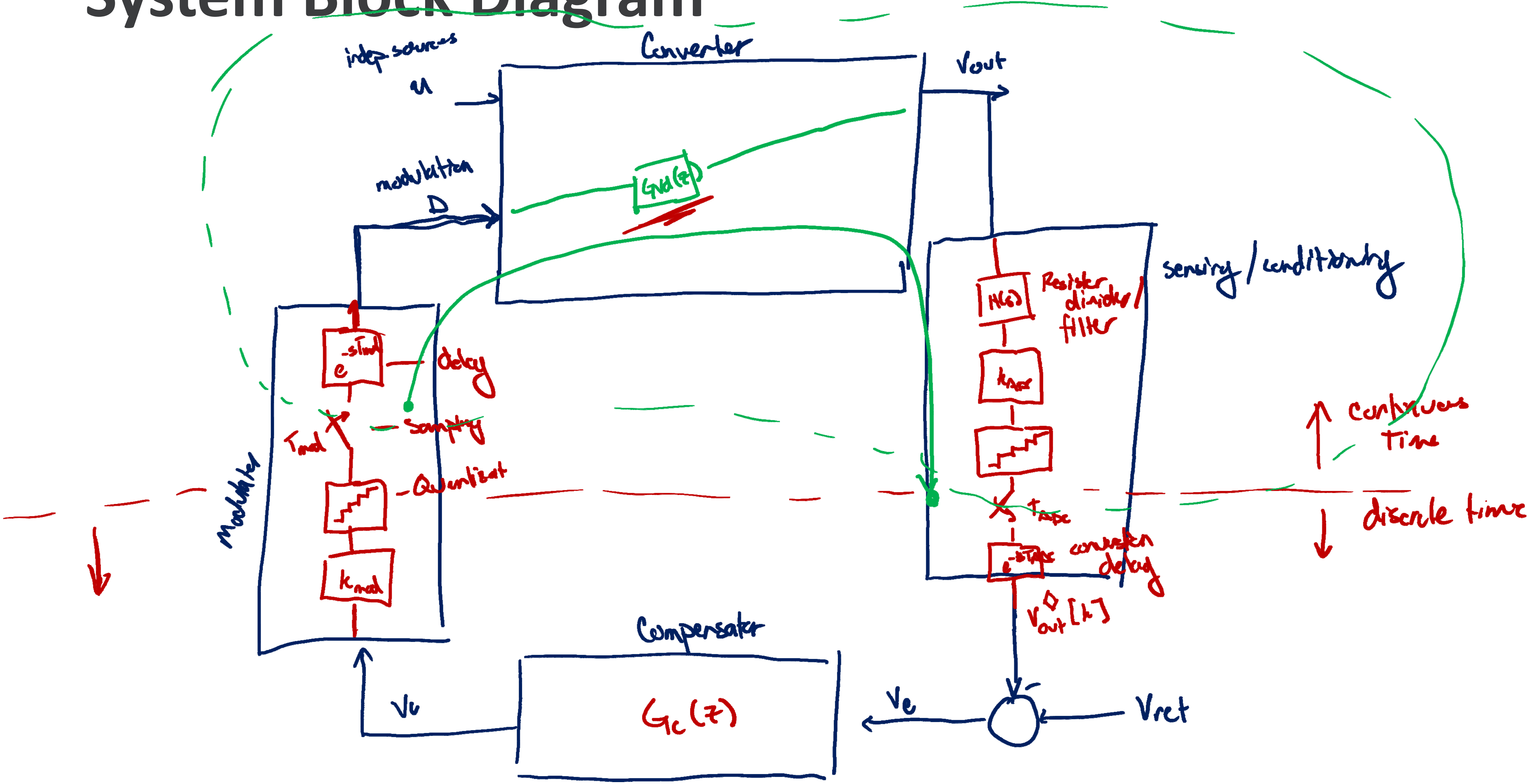
Sample Timing



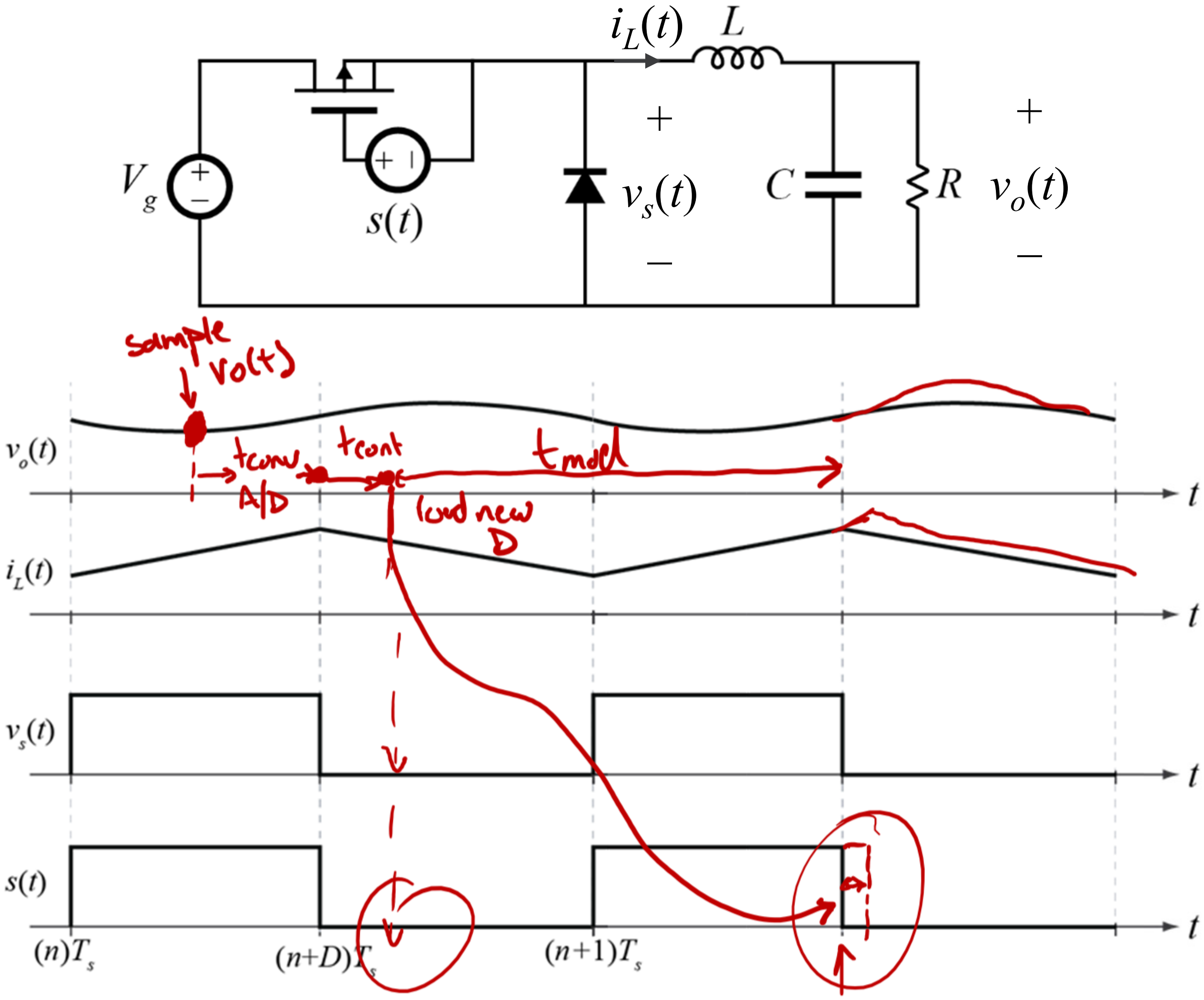
Does it matter?

- In steady-state, it didn't matter

System Block Diagram

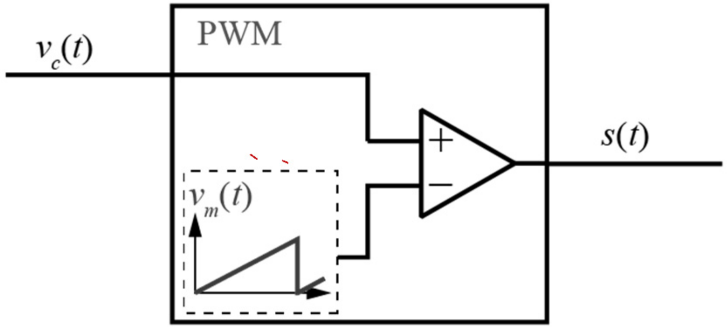


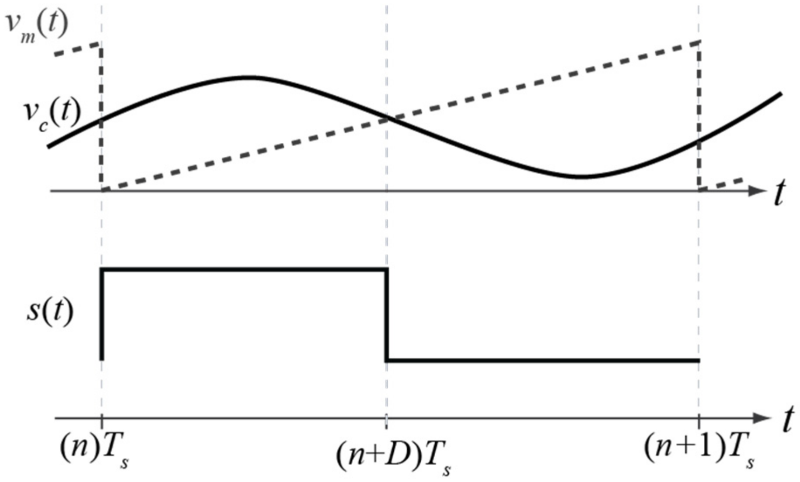
Control Timing



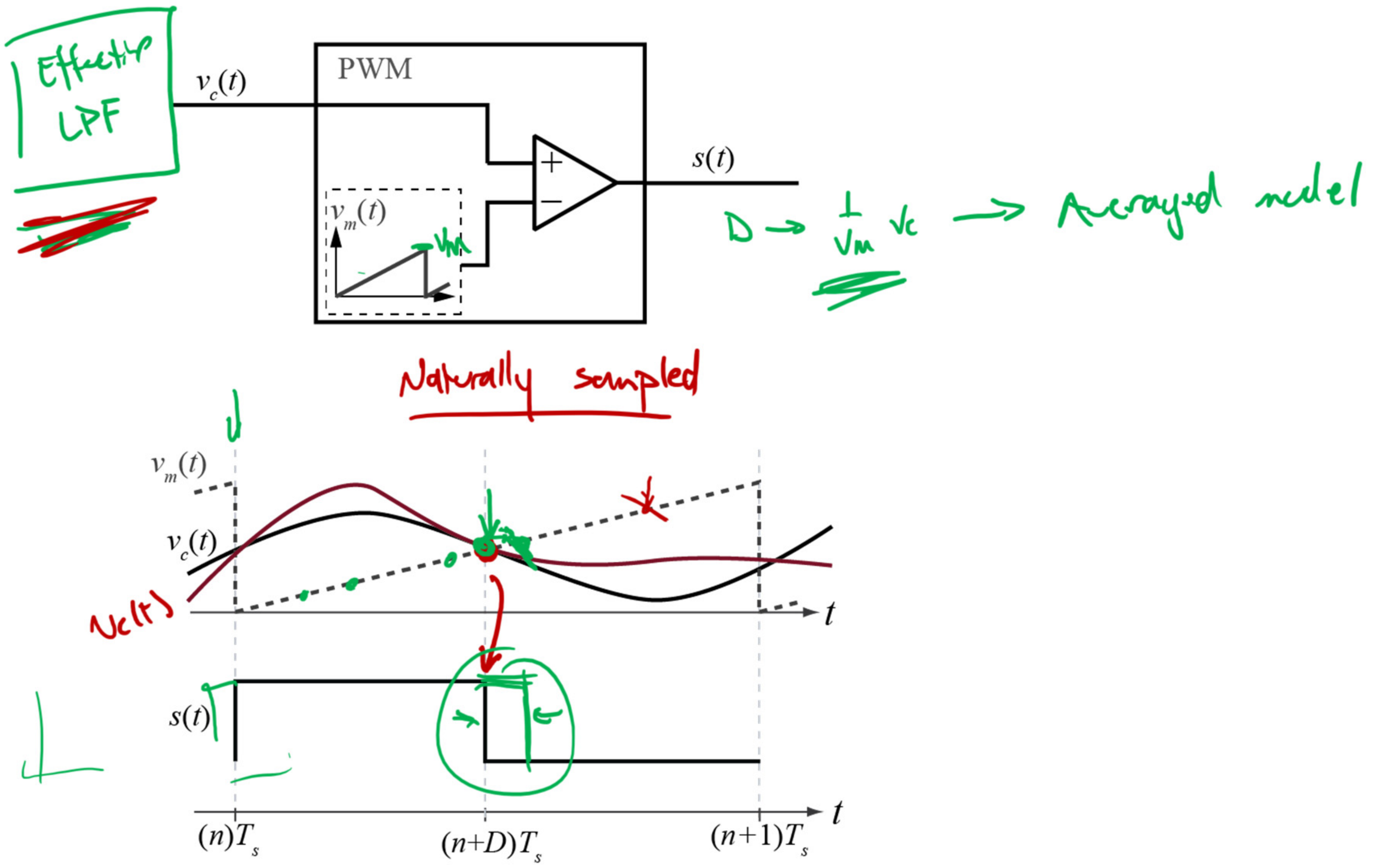
Assume ADC samples @ fs

Modulator Model

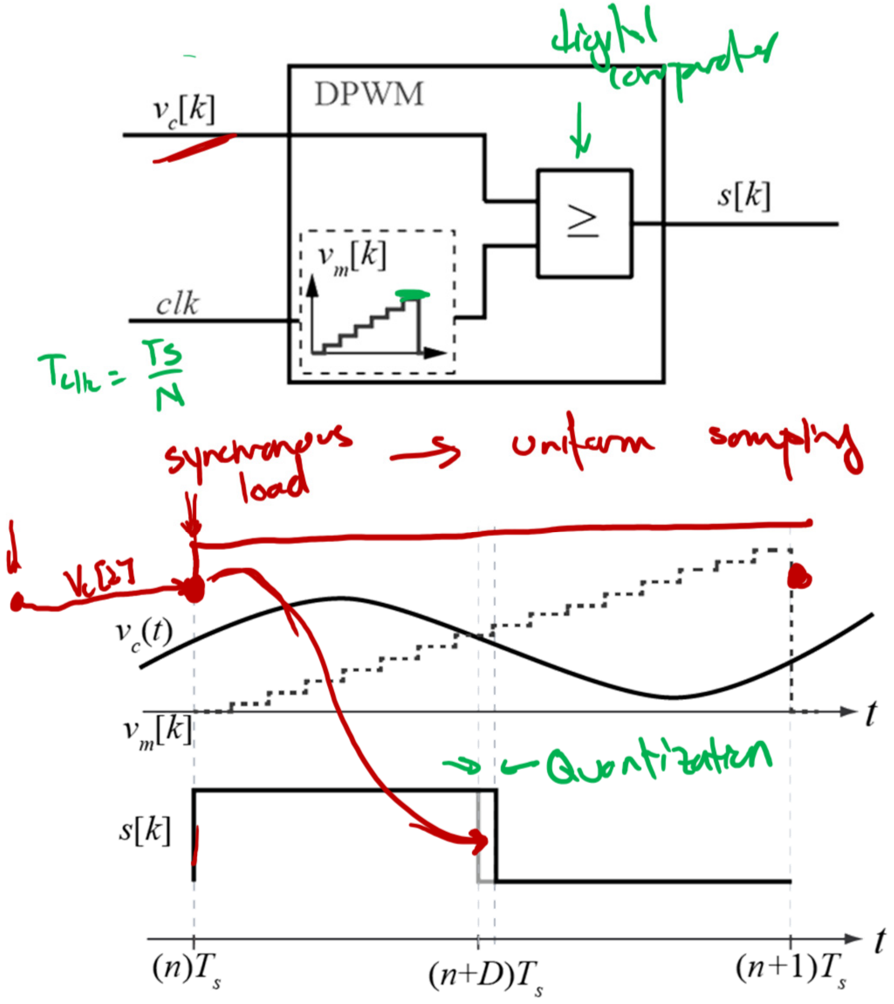




Modulator Model



Digital PWM


digital comparator
DPWM
$v_c[k]$
$s[k]$
$\geq$
$v_m[k]$
clk
$T_{clk} = \frac{T_s}{N}$
synchronous load → uniform sampling
$v_c[k]$
$v_c(t)$
$v_m[k]$
t
← Quantization
$s[k]$
t
$(n)T_s$
$(n+D)T_s$
$(n+1)T_s$