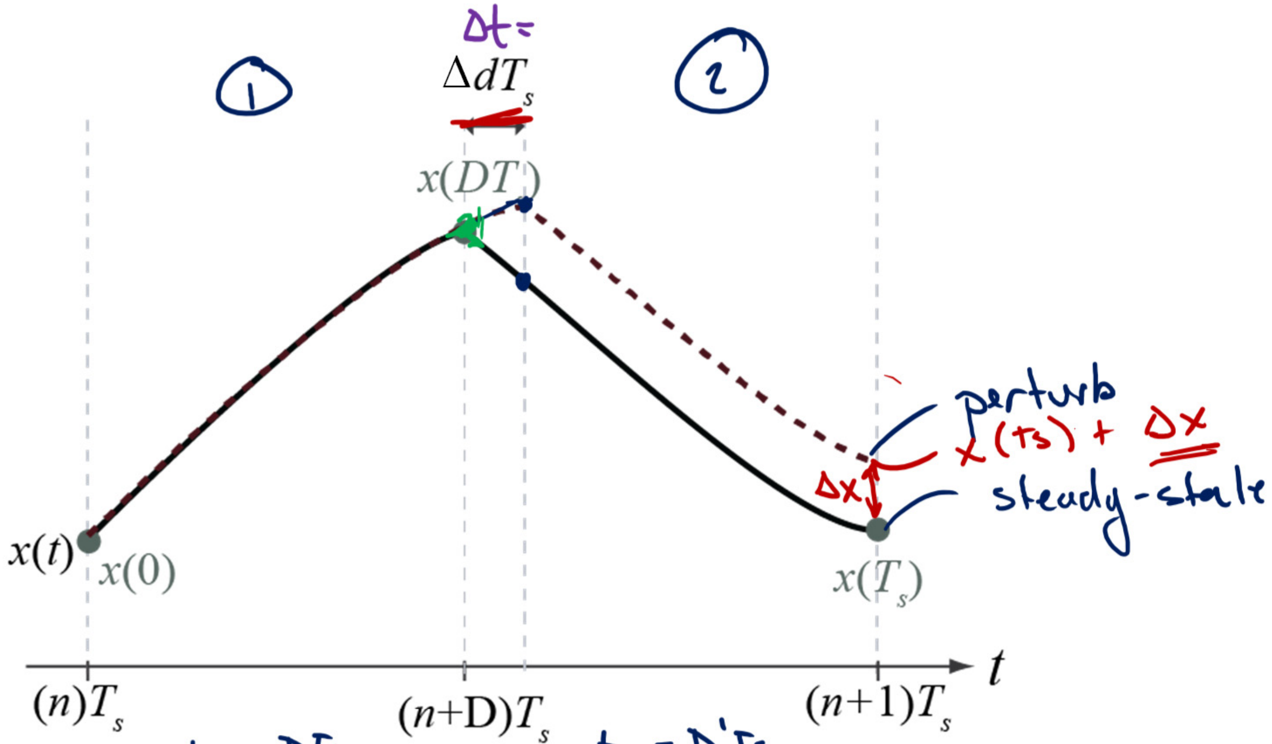
Converter Small Signal Model



$t_1 = DT_s$ $\quad t_2 = D'T_s$

$$\Gamma = \frac{\Delta x}{\Delta d}$$

Large-signal $\quad x[k+1] = \Phi(t_1)\, x[k] + \Psi(t_1)\, u[k]$

$$\Gamma = \frac{\partial x}{\partial d}$$

Small-signal model. $\quad \hat{x}[k+1] = \underline{\ \Phi\ }\, \hat{x}[k] + \underline{\ \Psi\ }\, \hat{u}[k] + \underline{\ \Gamma\ }\, \hat{d}[k]$

First, in steady-state

$$x[k+1] = e^{A_2 t_2} e^{A_1 t_1} x[k] + e^{A_2 t_2}\left(A_1^{-1}\left(e^{A_1 t_1} - I\right) B_1 u\right) + A_2^{-1}\left(e^{A_2 t_2} - I\right) B_2 u$$

when DTs is perturbed $t_1 \to t_1 + \Delta t$ $t_2 \to t_2 - \Delta t$

$$\Delta x + x(T_s) = e^{A_2(t_2 - \Delta t)} e^{A_1(t_1 + \Delta t)} X_0 + e^{A_2(t_2 - \Delta t)}\left(A_1^{-1}\left(e^{A_1(t_1 + \Delta t)} - I\right) B_1 u\right) + A_2^{-1}\left(e^{A_2(t_2 - \Delta t)} - I\right) B_2 u$$

$$e^{A(t+\Delta t)} = e^{At} e^{A\Delta t} = e^{A\Delta t} e^{At} \to kA \Leftrightarrow Ak$$

$$\Delta x + x(T_s) = \underline{e^{A_2 t_2}} e^{A_1 t_1} \underline{e^{(A_1 - A_2)\Delta t}} X_0 + e^{A_2 t_2} e^{-A_2 \Delta t}\left(A_1^{-1}\left(e^{A_1 t_1} e^{A_1 \Delta t} - I\right) B_1 u\right) + A_2^{-1}\left(e^{A_2 t_2} e^{-A_2 \Delta t} - I\right) B_2 u$$

Make small-signal linearization $\Delta t \to \hat{t}$

$$e^{At} \approx I + At + \frac{A^2}{2} t^2 + \cdots \approx \underline{I + At}$$

$$\hat{x} + x(T_s) = e^{A_2 t_2} e^{A_1 t_1} \left(I + (A_1 - A_2)\hat{t} \right) x_0 + e^{A_2 t_2} \left(I - A_2 \hat{t} \right) \left(A_1^{-1} \left(e^{A_1 t_1} (I + A_1 \hat{t}) - I \right) \right) B_1 u$$
$$+ A_2^{-1} \left(e^{A_2 t_2} \left(I - A_2 \hat{t} \right) - I \right) B_2 u$$

$$\underline{\hat{x}} + \underline{x(T_s)} = \underline{e^{A_2 t_2} e^{A_1 t_1} x_0} + \underline{e^{A_2 t_2} e^{A_1 t_1} (A_1 - A_2)\hat{t} x_0} + e^{A_2 t_2} \left(-A_2 \hat{t} \right) A_1^{-1} \left(e^{A_1 t_1} + e^{A_1 t_1} A_1 \hat{t} - I \right) B_1 u$$
$$+ e^{A_2 t_2} A_1^{-1} \left(\underline{e^{A_1 t_1}} + \underline{e^{A_1 t_1} A_1} \hat{t} - \underline{I} \right) B_1 u$$
$$+ A_2^{-1} \left(\underline{e^{A_2 t_2}} - \underline{e^{A_2 t_2} (-A_2 \hat{t})} \underline{- I} \right) B_2 u$$

$$\hat{x} = e^{A_2 t_2} e^{A_1 t_1} (A_1 - A_2) x_0 \hat{t} + e^{A_2 t_2} A_1^{-1} \left(e^{A_1 t_1} A_1 \hat{t} \right) B_1 u + e^{A_2 t_2} \left(-A_2 \hat{t} \right) A_1^{-1} \text{---}$$
$$+ A_2^{-1} \left(e^{A_2 t_2} \left(A_2 \hat{t} \right) \right) B_2 u$$

$$\hat{x} = \Gamma \cdot \hat{t}$$