$$\hat{x} = e^{A_2 t_2}(A_1 - A_2)e^{A_1 t_1} X_0 \hat{t} + e^{A_2 t_2} A_1^{-1}(e^{A_1 t_1} A_1 \hat{t}) B_1 u - e^{A_2 t_2} A_2 \hat{t} A_1^{-1}(e^{A_1 t_1} + e^{A_1 t_1} A_1 \hat{t} - I) B_1 u - A_2^{-1}(e^{A_2 t_2} A_2 \hat{t}) B_2 u$$

Note: $A e^{At} \Leftrightarrow e^{At} A$ $\quad \left(A \sum_{k=0}^{\infty} \frac{A^k}{k!} = \left(\sum_{k=0}^{\infty} \frac{A^k}{k!} \right) A \right)$

$$\hat{x} = \hat{t}\, e^{A_2 t_2} \Big[(A_1 - A_2) e^{A_1 t_1} X_0 + \cancel{A_1^{-1}} e^{A_1 t_1} \cancel{A_1} B_1 u - A_2 A_1^{-1}(e^{A_1 t_1} - I) B_1 u$$

$$\cancel{- A_2 A_1^{-1}(e^{A_1 t_1} A_1 \hat{t}) B_1 u}$$

By 1st order expansion/ linearization

$$- \cancel{A_2 A_2^{-1}} B_2 u \Big]$$

$$\hat{x} = \hat{t}\, e^{A_2 t_2} \Big[(A_1 - A_2) e^{A_1 t_1} X_0 + A_1 A_1^{-1}(e^{A_1 t_1} - I) B_1 u + B_1 u - A_2 A_1^{-1}(e^{A_1 t_1} - I) B_1 u - B_2 u \Big]$$

$$\hat{x} = \hat{t}\, e^{A_2 t_2} \Big[(A_1 - A_2) e^{A_1 t_1} X_0 + (A_1 - A_2)\left[A_1^{-1}(e^{A_1 t_1} - I) B_1 u \right] + (B_1 - B_2) u \Big]$$

$$\hat{x} = \hat{t}\, e^{A_2 t_2} \Big[(A_1 - A_2) \left(e^{A_1 t_1} X_0 + A_1^{-1}(e^{A_1 t_1} - I) B_1 u \right) + (B_1 - B_2) u \Big]$$

$X(DT_s) \equiv X_p$

$$\frac{\hat{x}}{\hat{t}} = e^{A_2 t_2} \left[(A_1 - A_2) X_p + (B_1 - B_2) u \right]$$

$$\Gamma = e^{A_2 t_2} \left[(A_1 - A_2) X_p + (B_1 - B_2) u \right] T_s \Big] - \frac{\hat{x}}{\hat{d}}$$

unit conversion

Converter Small Signal Model

Expedited derivation:

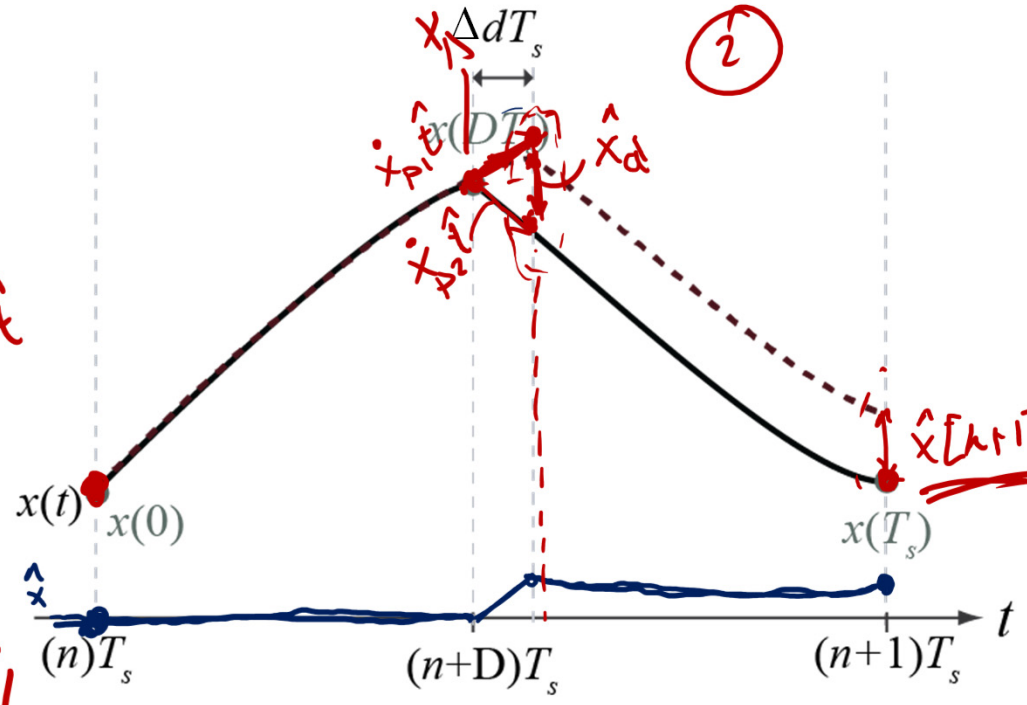
$$\hat{x}_a = [A_1 x_p + B_1 u]\hat{t} - [A_2 x_p + B_2 u]\hat{t} = [(A_1 - A_2)x_p + (B_1 - B_2)u]\hat{t}$$

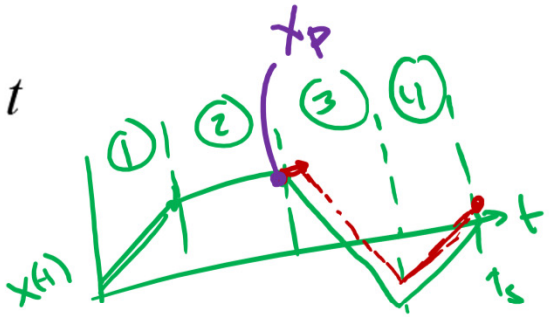
$$\hat{x}[n+1] = e^{A_2 t_2}\hat{x}_d$$

$$\Gamma = e^{A_2 t_2}\left[(A_1 - A_2)x_p + (B_1 - B_2)u\right]$$

e.g. solely increase t_1 without decreasing t_2 (variable f_s modulation)

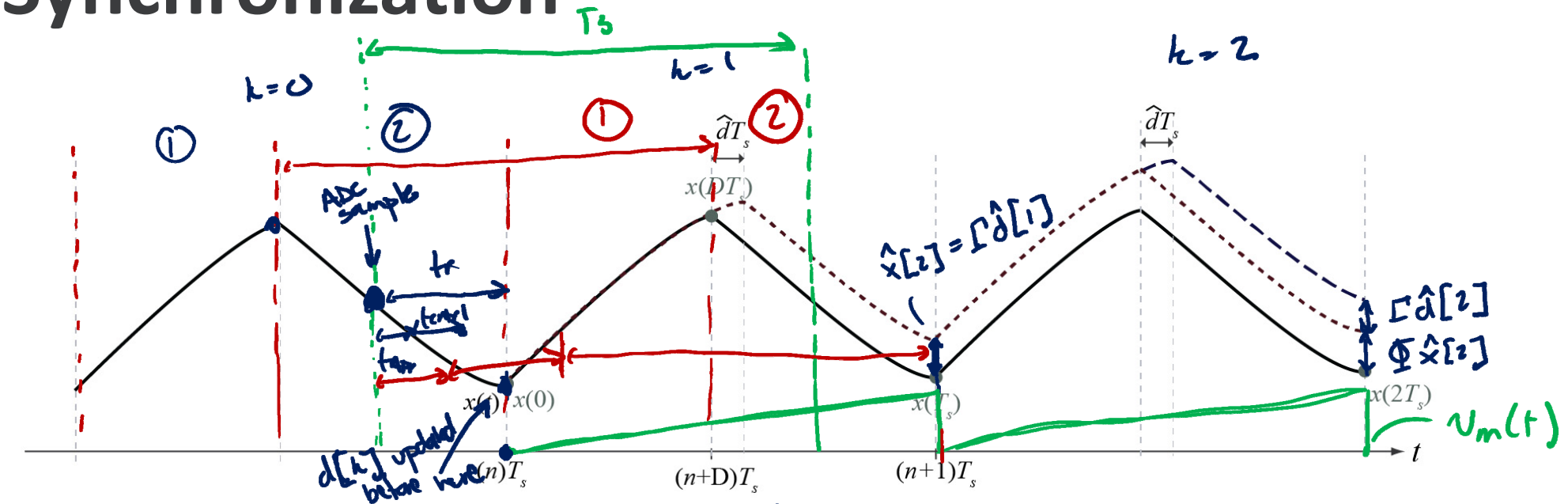
$$\Gamma = e^{A_2 t_2}\left[A_1 x_p + B_1 u\right]$$





$$\Gamma = e^{A_4 t_4} e^{A_3 t_3}\left[(A_2 - A_3)x_p + (B_2 - B_3)u\right]$$

ADC Synchronization



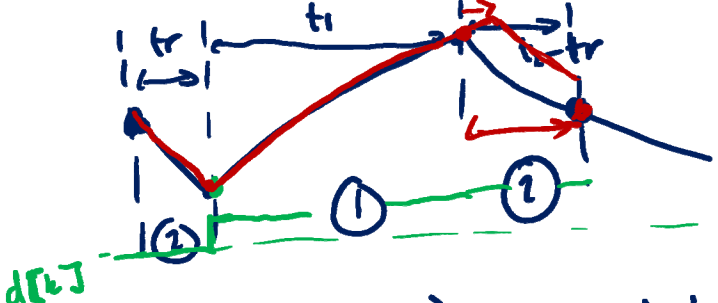
$$\hat{x}[k+1] = \bar{\Phi}\hat{x}[k] + \Psi\hat{u}[k] + \Gamma\hat{d}[k]$$

t_r is time between ADC sample & next PWM update

t_{ADC} → ADC conversion time

t_{ctrl} → control delay to compute next $d[k]$

as long as $t_r \geq t_{ADC} + t_{ctrl}$, this works, otherwise additional T_s delay



$$\begin{cases} \Phi = e^{A_2(t_2-t_r)}\, e^{A_1 t_1}\, e^{A_2 t_r} \\ \Gamma = e^{A_2(t_2-t_r)}\left[(A_1-A_2)X_p + (B_1-B_2)u\right]T_s \end{cases}$$

mode of plant with ADC, PWM sampling & delay included

$$\Psi = \sum \Pi(\cdots)$$