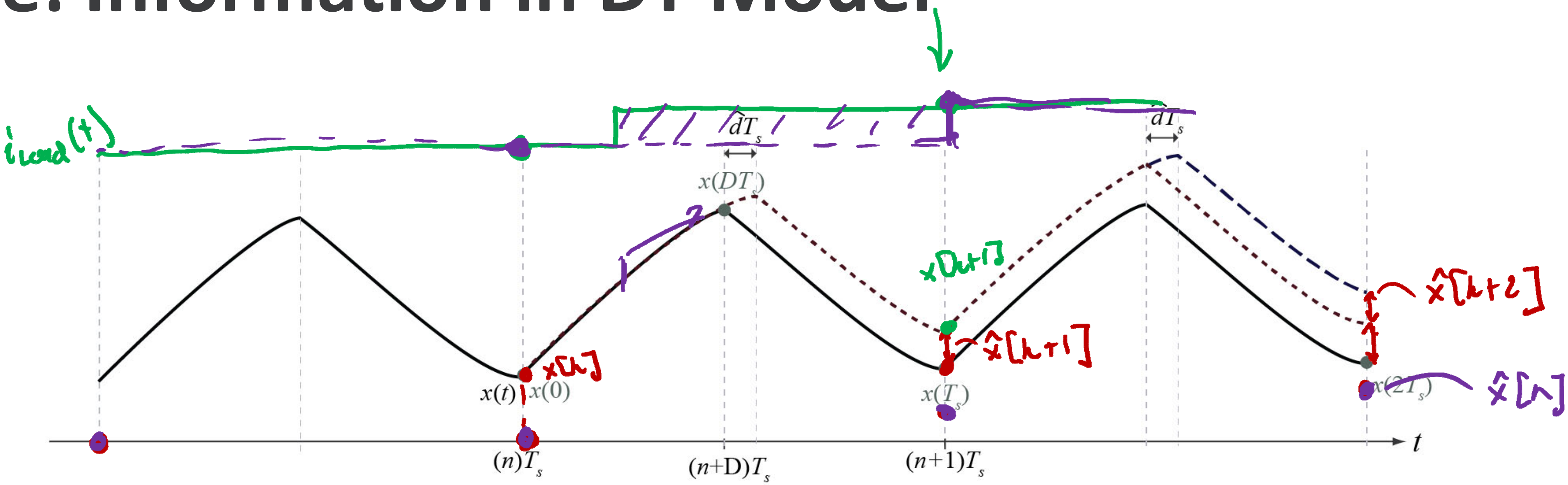
ote: Information in DT Model



all-signal DT model

$\hat{x}[k+1] = \Phi\,\hat{x}[k] + \Psi\,\hat{u}[k] + \Gamma\,\hat{d}[k]$

- DT model (sample rate = f_s)
- Small-signal model, $\hat{x}$ is perturbation from steady-state

DT sampling aligns w/ ADC & PWM sampling effects

→ No additional approximation in control feedback loop model

→ But, will be inaccurate for perturbations in $\hat{u}$, because they will have f[...] sampling effect

ugmented State Space Model

get ψ → need to calculate $\int_0^{t_i} e^{A_i(t_i-\tau)} B_i u(\tau) d\tau$ for every interval

① $A_i^{-1}(e^{A_i t_i} - I) B_i u$ when A_i non singular

② $\tilde{A}_i = \left[\begin{array}{c|c} A_i & B \\ \hline \emptyset & \emptyset \end{array}\right]$ $\quad e^{\tilde{A}_i t_i} = \begin{bmatrix} e^{A_i t_i} & \underbrace{\int_0^{t_i} e^{A_i(t_i-\tau)} B_i}_{} \\ \emptyset & \emptyset \end{bmatrix}$

ded if we want $\boxed{\hat{x}[k+1] = \Phi \hat{x}[k] + \psi \hat{u}[k] + \Gamma \hat{d}[k]}$

don't need to retain $\hat{u}[k]$ as inputs

$$\hat{\tilde{x}}[k+1] = \tilde{\Phi} \hat{\tilde{x}}[k] + \tilde{\Gamma} \hat{d}[k]$$

$$\tilde{\Phi} = \begin{bmatrix} \Phi & \psi u \\ \emptyset & \emptyset \end{bmatrix}$$

$\tilde{A}_i = \begin{bmatrix} A_i & Bu \\ \emptyset & \emptyset \end{bmatrix}$, $\quad \tilde{\Phi}_i = e^{\tilde{A}_i t_i}$, $\quad \rightarrow \quad \tilde{\Phi} = \prod_{i=k}^{1} e^{\tilde{A}_i t}$

$$\hat{\tilde{x}}[k+1] = \tilde{\Phi} \hat{\tilde{x}}[k] + \left(\prod_{i=k}^{p+1} e^{\hat{A}_i t_i} (\tilde{A}_p - \hat{A}_{p+1}) x_p \right) \hat{d}[k]$$

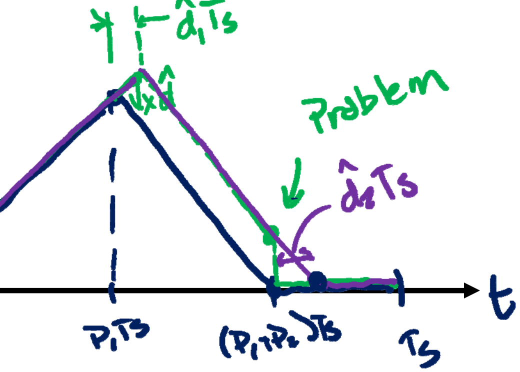
mall-Signal Linearization

$$x[k+1] = \Phi(t_i)\,x[k] + \Psi(t_i)\,u[k] = f(x[k], u[k], t_i)$$

small signal linearization:

$$\hat{x}[k+1] = \underbrace{\left(\frac{\partial f}{\partial x}\right)\Bigg|_{F_0}}_{\Phi} \hat{x}[k] + \underbrace{\left(\frac{\partial f}{\partial u}\right)\Bigg|_{F_0}}_{\Psi} \hat{u}[k] + \underbrace{\left(\frac{\partial f}{\partial t_i}\right)\Bigg|_{F_0}}_{\Gamma} T_s \hat{d}[k]$$

tate-Dependent Switching



$$\begin{cases} \hat{x}[k+1] = f(\hat{x}[k], \hat{u}[k], \hat{d}_1[k], \hat{t}_2) \\ 0 = \sigma(\hat{x}[k], \hat{u}[k], \hat{d}_1[k], \hat{t}_2) \end{cases}$$

in this DCM buck

$$0 = \sigma(\cdot) = C_{I_L}\left(e^{A_2t_2}e^{A_1t_1}\hat{x}[k] + e^{A_2t_2}A_1^{-1}(e^{A_1t_1}-I)B_1\hat{u}[k] + A_2^{-1}(e^{A_2t_2}-I)B_2\hat{u}[k]\right.$$

$$C_{I_L} = [1 \;\; 0] \quad \text{if} \quad x(t) = \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix}$$

constraint: $i_L(t) = 0$ @ $t = (D_1+D_2)T_s$

eneralized State Space Model

$x[k+1] = f(x[k], u[k], d[k], w[k])$

states · indep. inputs · control inputs · $w[k] \rightarrow$ some vector of auxiliary variables

$\emptyset = \sigma(x[k], u[k], d[k], w[k])$

arize both simulteneously

$$\begin{cases} \hat{x}[k+1] = \left.\frac{\partial f}{\partial x}\right|_{F_0} \hat{x}[k] + \left.\frac{\partial f}{\partial u}\right|_{F_0} \hat{u}[k] + \left.\frac{\partial f}{\partial d}\right|_{F_0} \hat{d}[k] + \left.\frac{\partial f}{\partial w}\right|_{F_0} \hat{w}[k] \\ \emptyset = \left.\frac{\partial \sigma}{\partial x}\right|_{F_0} \hat{x}[k] + \left.\frac{\partial \sigma}{\partial u}\right|_{F_0} \hat{u}[k] + \left.\frac{\partial \sigma}{\partial d}\right|_{F_0} \hat{d}[k] + \left.\frac{\partial \sigma}{\partial w}\right|_{F_0} \hat{w}[k] \end{cases}$$

2nd equation for $\hat{\omega}[k]$ & plug into 1st equation

$$\hat{\omega}[k] = -\frac{\partial\sigma}{\partial\omega}^{-1}\left[\frac{\partial\sigma}{\partial x}\hat{x}[k] + \frac{\partial\sigma}{\partial u}\hat{u}[k] + \frac{\partial\sigma}{\partial d}\hat{d}[k]\right]$$

$$\hat{x}[k+1] = \underbrace{\left[\frac{\partial f}{\partial x} - \frac{\partial f}{\partial \omega}\left(\frac{\partial\sigma}{\partial\omega}^{-1}\right)\frac{\partial\sigma}{\partial x}\right]}_{\Phi_{eq}}\hat{x}[k] + \left[\frac{\partial f}{\partial u} - \frac{\partial f}{\partial \omega}\left(\frac{\partial\sigma}{\partial\omega}^{-1}\right)\frac{\partial\sigma}{\partial u}\right]\hat{u}[k] + \left[\frac{\partial f}{\partial d} - \frac{\partial f}{\partial \omega}\left(\frac{\partial\sigma}{\partial\omega}^{-1}\right)\frac{\partial\sigma}{\partial d}\right]\hat{d}[k]$$

$$\hat{x}[k+1] = \Phi_{eq}\,\hat{x}[k] + \Psi_{eq}\,\hat{u}[k] + \Gamma_{eq}\,\hat{d}[k]$$