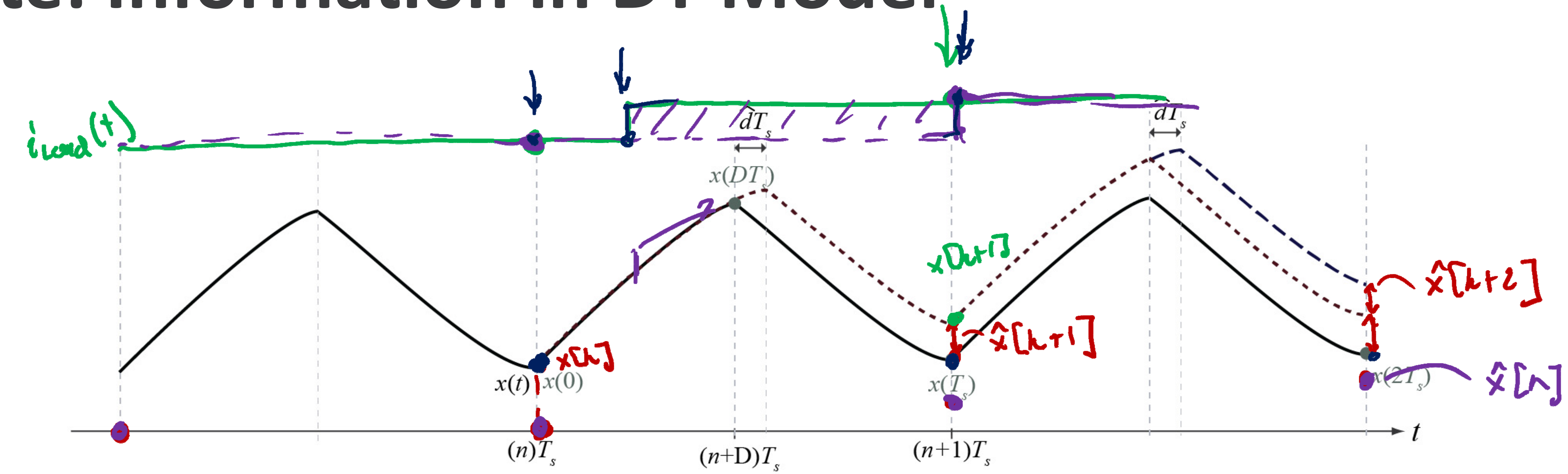
Note: Information in DT Model



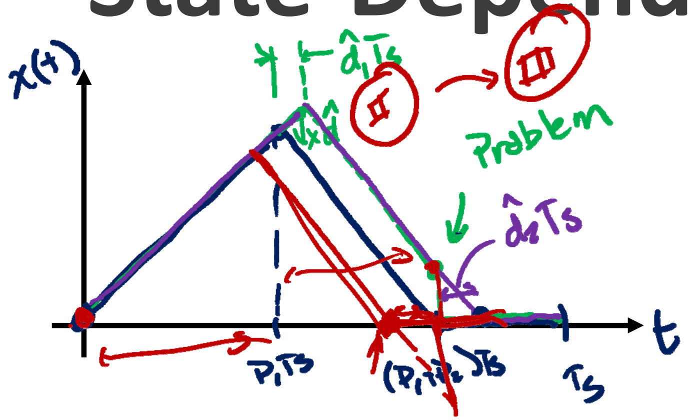
Small-signal DT model

$\hat{x}[k+1] = \Phi\,\hat{x}[k] + \Psi\,\hat{u}[k] + \Gamma\,\hat{d}[k]$

- DT model (sample rate $= f_s$)
- Small-signal model, $\hat{x}$ is perturbation from steady-state

DT sampling aligns w/ ADC & PWM sampling effects
→ No additional approximation in control feedback loop model
→ But, will be inaccurate for perturbations in $\hat{u}$, because they will have false sampling effect

State-Dependent Switching



$$\to \begin{cases} \hat{x}[k+1] = f(\hat{x}[k], \hat{u}[k], \hat{d}_1[k], \hat{t}_2) \\ \phi = \sigma(\hat{x}[k], \hat{u}[k], \hat{d}_1[k], \hat{t}_2) \end{cases}$$

or $\hat{d}_2[k]$

ex/ in this DCM Buck

$$\phi = \sigma(\cdot) = C_{I_L}\left(e^{A_2 t_2} e^{A_1 t_1} \hat{x}[k] + e^{A_2 t_2} A_1^{-1}(e^{A_1 t_1} - I) B_1 \hat{u}[k] + A_2^{-1}(e^{A_2 t_2} - I) B_2 \hat{u}[k]\right)$$

$C_{I_L} = [1 \; 0]$ if $x(t) = \begin{bmatrix} i_L(t) \\ v_0(t) \end{bmatrix}$

constraint : $i_L(t) = \phi$ @ $t = (D_1+D_2)T_s$

Generalized State Space Model

$$\begin{cases} x[k+1] = f(x[k], u[k], d[k], w[k]) \\ 0 = \sigma(x[k], u[k], d[k], w[k]) \end{cases}$$

$x[k]$: states; $u[k]$: indep. inputs; $d[k]$: control inputs

$w[k] \rightarrow$ some vector of auxiliary variables

if $\hat{w}[k] = \xi_2[k]$ as in DCM Buck

looks like Γ

Linearize both simultaneously

$$\begin{cases} \hat{x}[k+1] = \left[\left.\frac{\partial f}{\partial x}\right|_{F_0} \hat{x}[k] + \left.\frac{\partial f}{\partial u}\right|_{F_0} \hat{u}[k] + \left.\frac{\partial f}{\partial d}\right|_{F_0} \hat{d}[k] \right] + \left.\frac{\partial f}{\partial w}\right|_{F_0} \hat{w}[k] \\ 0 = \left.\frac{\partial \sigma}{\partial x}\right|_{F_0} \hat{x}[k] + \left.\frac{\partial \sigma}{\partial u}\right|_{F_0} \hat{u}[k] + \left.\frac{\partial \sigma}{\partial d}\right|_{F_0} \hat{d}[k] + \left.\frac{\partial \sigma}{\partial w}\right|_{F_0} \hat{w}[k] \end{cases}$$

from second equation, solve for $\hat{w}[k]$

$$\hat{w}[k] = -\left[\frac{\partial\sigma}{\partial w}\right]^{-1}\left(\frac{\partial\sigma}{\partial x}\hat{x}[k] + \frac{\partial\sigma}{\partial u}\hat{u}[k] + \frac{\partial\sigma}{\partial d}\hat{d}[k]\right)$$

then plug into first equation & group terms:

$$\boxed{\begin{aligned}\hat{x}[k+1] = &\left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial w}\left[\frac{\partial\sigma}{\partial w}\right]^{-1}\frac{\partial\sigma}{\partial x}\right)\hat{x}[k] + \left(\frac{\partial f}{\partial u} - \frac{\partial f}{\partial w}\left[\frac{\partial\sigma}{\partial w}\right]^{-1}\frac{\partial\sigma}{\partial u}\right)\hat{u}[k] \\ &+ \left(\frac{\partial f}{\partial d} - \frac{\partial f}{\partial w}\left[\frac{\partial\sigma}{\partial w}\right]^{-1}\frac{\partial\sigma}{\partial d}\right)\hat{d}[k]\end{aligned}}$$

which is now an LTI small-signal discrete time model of the form

$$\hat{x}[k+1] = \underline{\Phi_{eq}}\,\hat{x}[k] + \underline{\Psi_{eq}}\,\hat{u}[k] + \underline{\Gamma_{eq}}\,\hat{d}[k]$$

Note that the constant matrices Φ_{eq}, Ψ_{eq}, & Γ_{eq} usually must be found purely numerically and doing so may, again, require some numerical iteration.

Similar to the Augmented State Space comments, this Generalized State Space modeling can be applied <u>either</u> at the system level (as done here) or can be done for each subinterval seperately before combining them into the overall system.